ON DAWSON-MODELS FOR DEONTIC LOGIC

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1. *Introduction.* In a highly interesting article (A Model for Deontic Logic, *Analysis* 19.4 (1959), pp. 73-78) E. E. Dawson shows that Lewis's system S4 of alethic modal logic, supplemented with the formula CMLpLMp as an additional axiom, satisfies A.R. Anderson's requirements for a normal deontic logic (¹) when the deontic operators O and P (read 'obligatory' and 'permitted') are defined respectively as ML and LM ('possibly necessary' and 'necessarily possible'). The main purpose of the present note is (i) to make some further observations about the deontic logic forthcoming in this system S4.2, i.e. S4 + CMLpLMp, and (ii) to show that the standard systems S4 and S3 (²), unsupplemented by deontic axioms, also are normal deontic logics in the Andersonian sense under suitable definitions of O and P in terms of L and M. Some reflections on commitment and a discussion of an important objection will be appended.

2. *Deontic logic in S4.2.* Some preliminary remarks have now to be made. The Andersonian conditions on a normal deontic logic, D, are that, besides the theses and rules of procedure of the classical propositional calculus and the principle of intersubstitutability of tautologous equivalents, D is to contain as theses the formulae COpPp, EPApqAPpPq and EOpNPNp, whereas CPpp, CpPp and, if D is an extension of an alethic modal logic, CMpPp are to be unprovable in D. Now Dawson proves CMLpLMp, ELMApqALMpLMq and EMLpNLMNp to be theses of S4.2, and shows CLMpp, CpLMp and CMpLMp to be unprovable, which yields the result that O-S4.2, i.e. S4.2 with O = ML and P = LM, is a normal deontic logic. He also observes that CKPpPqPKpq is not a thesis; in fact its unprovability could well have been included among Anderson's conditions, since that formula is obviously invalid as a deontic law.

Our first problem concerns reduction theses. S4.2 is known to possess the following ten irreducible *alethic* modalities: Lp, MLp,

(¹) A. R. Anderson, *The Formal Analysis of Normative Systems*, New Haven: Yale Sociology Department, 1956, p. 29 f.

(²) As our foundations for the systems S4 and S3 we may conveniently take those provided by E. J. Lemmon, «New Foundations for Lewis Modal Systems», *Journal of Symbolic Logic*, vol. 22 (1957), pp. 176-186.

LMp, Mp, p, and their negations. But what irreducible *deontic* modalities are there in Dawson's model O-S4.2 ? The answer is simple: already in S3 we have EMLMLpMLp and ELMLMpLMp, yielding EOOpOp and EPPpPp in OS4.2; in S4 we have CMMLpMLp and CLMMLpMMLp, yielding CLMMLpMLp and CPOpOp; and in S4.2 CMLpLMLp (from CMLpLMp, substituting Lp for p and using ELLpLp) and CLMLpLMMLp (using EMMpMp), which yields CMLpLMMLp, i.e. COpPOp. We thus have in O-S4.2 EPOpOp (and, consequently, EOPpPp), so there is an S5-reduction to six *deontic* modalities in Dawson's model, only Op, Pp, p, and their negations being left.

Dawson notes that the expression OCOpp is a thesis of O-S4.2. The following is perhaps a simpler way of proving the formula MLCMLpp already in S4:

1	CLMMLpMLp	
2	AMLLMNpMLp	(1, ECpqANpq, modal equivalents)
3	MALLMNpLp	(2, EAMpMqMApq)
4	MLALMNpp	(3, CALpLqLApq, $\vdash C\alpha\beta \rightarrow \vdash CM\alpha M\beta$) (3)
5	MLCMLpp	(4, EApqCNpq, modal equivalents)

We conclude this section by making a few comments on deontic rules of inference in O-S4 . 2. Being a normal deontic logic this system admits the substitutability of tautologous equivalents. But the following stronger rules, for instance, are easily derivable: $\vdash \alpha \rightarrow \vdash O\alpha$, (using $\vdash \alpha \rightarrow \vdash L\alpha$ and CpMp), and $\vdash C\alpha\beta \rightarrow \vdash CO\alpha O\beta$ (using $\vdash \alpha \rightarrow \vdash O\alpha$ and the thesis COCpqCOpOq which is forthcoming in any normal deontic logic). As might be expected, both these rules still hold in our S4- model for deontic logic but not in the S3-one. We return to this topic below.

3. *Deontic logic in S4.* With the interpretation O=ML and P=LM, the system S4 . 2 is, as pointed out by Dawson, the weakest possible system of alethic modal logic that will satisfy Anderson's requirements for a normal deontic logic, since both CMLpLMp, ELLpLp, and $\vdash \alpha \rightarrow \vdash L\alpha$ are needed in the proofs of the deontic theses. If S4, for instance, had been taken as our modal basis, we could no longer be able to prove COpPp, since CMLpLMp is not in S4. However, there are other candidates for O and P, which are worthy of consideration. We are going to prove that a system O-S4, i.e. S4 with O=LML and

(3) Throughout this paper we use the notation '$\vdash \alpha \rightarrow \vdash \beta$' to mean 'from a thesis α of a considered system to infer a thesis β of that system'.

P=MLM, fulfills Anderson's conditions on a normal deontic logic.

It is well known that CLMLpMLMp is a thesis of S4; hence COpPp is a thesis of O-S4. Trivially, we have ELMLpNMLMNp and EOpNPNp. To get EPApqAPpPq we first prove CAMLMpMLMqMLMApq:

1	LCpApq	
2	LCqApq	
3	LCMpMApq	(1, using the S3-thesis CLCpqLCMpMq)
4	LCLMpLMApq	(3, using the S3-thesis CLCpqLCLpLq)
5	LCMLMpMLMApq	(4, using CLCpqLCMpMq)
6	LCMLMqMLMApq	(similarly, starting from 2)
7	CMLMpMLMApq	(5, CLpp)
8	CMLMqMLMApq	(6, CLpp)
9	CAMLMpMLMqMLMApq	(C7 - C8 - 9)

The proof of the converse CMLMApqAMLMpMLMq is accomplished as follows:

1	CMLqCLMLpMLKpq	(line 11 in Dawson's S4.2-proof of CLMApqALMpLMq; the proof of this line is valid in S4)
2	CLMLqLCLMLpMLKpq	(1, using $\vdash C\alpha\beta \rightarrow \vdash CL\alpha L\beta$)
3	CLCLMLpMLKpqCLMLpLMLKpq	(CLCpqCLpLq with p/LMLp, q/MLKpq, and using ELLpLp)
4	CLMLqCLMLpLMLKpq	(C2 - C3 - 4)
5	CKLMLpLMLqLMLKpq	(4, ECqCprCKpqr)

5 yields the desired result CMLMApqAMLMpMLMq by elementary steps.

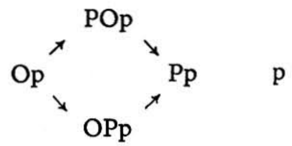
To see that neither CPpp nor CpPp nor CMpPp are provable in O-S4, we observe, like Dawson, that Lewis' matrix group II satisfies S4 but neither CMLMpp (p = 3), CpMLMp (p = 2), nor CMpMLMp (p = 2). Similarly, CKPpPqPKpq is shown to be unprovable in O-S4 by Lewis's group III (e.g. p = 3 and q = 2). Thus, it is established that S4 with O = LML and P = MLM is a normal deontic logic.

As for reduction theses, it is well known that S4 possesses 14 irreducible alethic modalities, viz., Lp, LMLp, MLp, LMp, MLMp, Mp, p, and their negations ([4]). Let it then be noted that O-S4 has just 10 ir-

([4]) See W. T. Parry, «Modalities in the *Survey System of Strict Implication*», *Journal of Symbolic Logic*, vol. 4 (1939), pp. 137-154, sect. vi.

reducible deontic modalities, viz., Op, POp, OPp, Pp, p, and their negations. Already in S3 we have ELMLMLMLMLMLMpMLMLMp and EMLMLMLMLMLMLpMLMLMLp, yielding EOPOPpOPp and EPOPOp POp. By virtue of ELMLLMLpLMLp and EMLMMLMpMLMp we get EOOpOp and EPPpPp in O-S4, where we have at least an S4-reduction to 14 deontic modalities.

But the formulae ELMLMLMLMLpLMLp and EMLMLMLMLMpMLMp are provable in S4 as well, yielding EOPOpOp and EPOPpPp in O-S4, where we thus get 10 irreducible deontic modalities, viz, those in the following diagram of entailments, and their negations:



That these are all follows from Parry's (5) proof that S4 has just the 14 irreducible alethic modalities mentioned above.

The formula OCOpp is a thesis also of O-S4, since we can prove LMLCLMLpp as follows:

1	MLCMLpp	(see above, sect. 2)
2	CCMLppCLMLpp	(CCpqCLpq, with p/MLp, q/p)
3	CLCMLppLCLMLpp	(2, $\vdash C\alpha\beta \rightarrow \vdash CL\alpha L\beta$)
4	CMLCMLppMLCLMLpp	(3, $\vdash C\alpha\beta \rightarrow \vdash CM\alpha M\beta$)
5	MLC1MLpp	(CL - C4 - 5)
6	LMLCLMLpp	(5, $\vdash \alpha \rightarrow \vdash L\alpha$)

We also note that the deontic rules of inference instanced at the end of the preceding section are still derivable in O-S4. The proof is patent.

4. *Deontic logic in S3*. It will now be shown that a system O-S3, i.e. S3 with O=LLML and P=MMLM, is a normal deontic logic in the sense of Anderson. However, instead of presenting S3-derivations of the formulae we are required to prove, we establish this part of our result more easily as follows. By a theorem of S. Halldén (6) we know that a formula is a thesis of S3, if and only if, it is a thesis both of

(5) *Ibid.*

(6) S. Halldén, «Results concerning the decision problem of Lewis's calculi S3 and S6», *Journal of Symbolic Logic*, vol. 14 (1949), pp. 213-235.

S4 and S7, where S7 is the calculus S3 + MMp. Then make the following two observations.

(i) From the fact that S4 with O=LML and P=MLM is a normal deontic logic it follows, by virtue of ELLpLp and EMMpMp, that S4 with O=LLML and P=MMLM is a normal deontic logic.

(ii) We have ELLMLpNMMLMNp in S7, trivially; moreover, any formula beginning with MM is obviously a thesis of S7, whence we immediately get CLLMLpMMLMp, CAMMLMpMMLMqMMLMApq and its converse in S7.

Since the formulae just mentioned in (ii) are provable both in S4 and S7, they must be provable in S3 as well, by Halldén's result. Hence, O-S3 fulfills the positive part of the definition of normal deontic logic. To verify that O-S3 meets the negative one, we still use Lewis's group II and note that the same value-assignments that failed to satisfy CPpp, CpPp, and CMpPp in O-S4 also fail to satisfy these formulae in O-S3. Similarly, using group III, we reject CKPpPqPKpq in O-S3 by the same value-assignment that failed to satisfy this formula in O-S4.

O-S3 turns out to have the same structure of deontic modalities as O-S4, which may be seen as follows. By the results of Parry [7] we have the following equivalences in S3:

(1) ELLMMpLLMp
(2) ELLMLLpLLMLp
(3) ELLMLMpLLMp

Consider then the formula LLMLLLMLp. By two applications of (2) it is equivalent to LLMLMLp, which, by (3), is equivalent to LLMLp, i.e. we have EOOpOp (and EPPpPp, dually) in O-S3. Further, consider LLMLMMLMLLMLp. Applying (3), then (1), then (3) again, then (2), and then (3) again, we obtain the result that this formla is equivalent to LLMLp. Hence, we get EOPOpOp (as well as EPOPpPp) in O-S3. We are left with Op (=LLMLp), POp (=MMLp), OPp (=LLMp), Pp (=MMLMp), p, and their negations. That the entailments are those and only those in the above diagram for O-S4, and that no further deontic reduction is possible, follows from Parry's results concerning S3.

A feature distinguishing O-S3 from the two systems previously considered is that no expression of the form Oα is provable in it. This follows from the familiar fact that there are in S3 no theses beginning

(7) See Parry, *op.cit.*, sect. iii.

with LL. Hence, the formula OCOpp, which was provable both in O-S4 and O-S4.2, is not a thesis of O-S3, nor is the rule $\vdash \alpha \rightarrow \vdash O\alpha$ derivable in this system.

On the other hand, the rule $\vdash C\alpha\beta \rightarrow \vdash CO\alpha O\beta$ is obtained in O-S3 as follows. In S4 we derive the rule $\vdash C\alpha\beta \rightarrow \vdash CLLM考L\alpha LLML\beta$, using the rules $\vdash C\alpha\beta \rightarrow \vdash CL\alpha L\beta$ and $\vdash C\alpha\beta \rightarrow \vdash CM\alpha M\beta$. In S7 we get this rule simply from the fact that any expression of the form $CLL\alpha\beta$ is a thesis of S7. Assume then the hypothesis that $C\alpha\beta$ is provable in S3. By Halldén's aforementioned result this is equivalent to the assumption that $C\alpha\beta$ is a thesis both of S4 and S7. Having the rule $\vdash C\alpha\beta \rightarrow \vdash CLLML\alpha LLML\beta$ in both these calculi, we then get $CLLML\alpha$ $LLM\beta$ as a thesis common to S4 and S7, i.e. as a thesis of S3. Hence our result.

We conclude this consideration of deontic logic in S3 by mentioning —though not proving — that there are in it at least two other normal deontic logics besides O-S3, viz. S3 with O=LML and P=MLM, and S3 with O=LMLL and P=MLMM. Among other things these systems differ from O-S3 as to their structure of deontic modalities. Another point of difference will be briefly touched upon in section 6 below.

5. *Commitment*. A problem to which some attention has been payed by deontic logicians is that of formalizing our intuitive notion of 'commitment'. A familar interpretation — originally proposed by von Wright — consists in taking OCpq as meaning 'p commits us to q'. Together with such a «paradox» of any normal deontic logic as CONpOCpq the suggested interpretation yields the result that if p is forbidden, then p commits us to q, i.e. anything. Writers not willing to admit this have then been looking for some relation stronger than the one expressed by OCpq in order to arrive at an adequate formalization of commitment. Anderson has suggested as a candidate the formula LCpOq which turns out to meet the following conditions (8): (i) CONpLCpOq is unprovable in any of his deontic-supplemented alethic modal logics OM, OM′, OM″, (based respectively on von Wright's M, M′ (= S4), and M″ (= S5)); (ii) CLCpOqOCpq is provable in these systems whereas its converse fails to be; (iii) CKpLCpOqOq and CKOpLCpOqOq are both provable in these systems.

I take the opportunity to note that the formulae mentioned in these conditions turn out to be provable, or unprovable, respectively, in the

(8) See ANDERSON, *op.cit.*, sect. vi, and his 'On the Logic of «Commitment»', *Philosophical Studies*, vol. 10 (1959).

systems O-S4.2 and O-S4 as well. As I have observed ([9]) elsewhere, however, the fact that a deontic logic fulfills condition (iii) constitutes a reason *against* rather than *for* interpreting 'p commits us to q' as LCpOq in that logic, in view of the following argument:

1	Op	
2	Np	
3	LCpOq	(«p commits us to q»)
4	LCNpONq	(«Np commits us to Nq»)
5	Oq	(1, 3, using CKOpLCpOqOq)
6	ONq	(2, 4, using CKpLCpOqOq, with p/Np, q/Nq)

We get the result the conjunction of premisses 1-4 is contradictory, since it entails the incompatible conclusions 5 and 6. As the conjunction 1-4 seems perfectly consistent under the reading 'p commits us to q' of LCpOq, this result is counterintuitive and constitutes an argument against the proposed analysis of commitment in any system satisfying condition (iii) above — whether the Andersonian ones, O-S4, O-S4.2, or any others.

Another circumstance may be mentioned. Saul A. Kripke ([10]) has pointed out that in the strongest Andersonian system OM″ the formula LCLCpOqLCMpOq is provable, whereas it fails for the weaker systems OM and OM′. Clearly, the counterintuitive force of this result under the suggested interpretation constitutes an argument against OM″ as a system where LCpOq could be taken as meaning 'p commits us to q'. Something similar is true of the Dawson systems, where LCLCpOqLCMpOq turns out to be a thesis of O-S4.2 (by virtue of LCLCpMLqLCMpMMLq and EMMpMp), though not of O-S4, which is then in a somewhat better position from this particular point of view.

6. *An objection.* In his paper Dawson emphasizes that his model provides a straightforward means of resolving the status of combined deontic-alethic formulae, noting e.g. the theoremhood in O-S4.2 of such plausible principles as COpMp, CPpMp, CLpOp and CLpPp. They are all provable in O-S4 as well, but in O-S3 the second and the third one fail (still by Parry's results). I come next to a more troublesome point not discussed by Dawson. In O-S4.2 we get the result that all deontic statements of the form Op are either necessary or impossible ,or in other words, that there are no synthetic (contingent)

([9]) L. Åqvist, 'A Note on Commitment', *Philosophical Studies,* vol. 14 (1963).
([10]) See Anderson's paper in *Philosophical Studies.,*

deontic statements Op. For we have in O-S4.2 CMLpLMLp, i.e. COpLOp, and CNMLpNMMLp, i.e. CNOpNMOp, yielding CAOpNOp ALOpNMOp. Detaching AOpNOp we get the result ALOpNMOp. In O-S4 and O-S3 we only have COpLOp (by virtue of CLpLLp and CLLpLLLp respectively), but not CNOpNMOp. If, like S. Kanger [11], we are inclined to assume the existence of synthetic, or at least non-necessary, statements of the form Op, we should have to reject the Dawson-models for deontic logic dealt with in this paper. The only way of answering the present objection seems to be the one mentioned by Kanger: by appealing to the vagueness of the notion of obligatoriness, which probably excludes a definite decision of the question concerning the existence of synthetic ought-statements.

To conclude this discussion, I might indicate another possibility of getting rid of the difficulty, which, however, I do not believe in. As was mentioned above, S3 can be proved to contain at least two other normal deontic logics besides O-S3, viz. S3 with O=LML and P=MLM, and S3 with O=LMLL and P=MLMM. In these new systems we have neither COpLOp nor CNOpNMOp, which looks attractive. However, it is known — by another theorem of Halldén [12] — that the system S3, as well as all systems intermediate between S3 and S4, are «semantically incomplete» in a sense implying that not all valid modal formulae are provable. By a result of McKinsey [13] S4 is known not to be incomplete in Halldén's sense. If in view of this circumstance we are forced to go back to O-S4, we are still faced with the difficulty under consideration.

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([11]) S. KANGER, *New Foundations for Ethical Theory*, Stockholm 1957, sect. 2.6.

([12]) S. HALLDÉN, 'On the semantic non-completeness of certain Lewis calculi', *Journal of Symbolic Logic*, vol. 16 (1951), pp. 127-129.

([13]) J. C. C. MCKINSEY, 'Systems of Modal Logic which are not unreasonable in the sense of Halldén', *Journal of Symbolic Logic*, vol. 18 (1953), pp. 109-113.