# MODALITY CONCEIVED AS A STATUS

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## I. *Introduction*

The object of this paper is to put forward a conception of the «alethic» modality of *necessity* (and correspondingly of *possibility*) which differs from any construction of modality to be found in the recent literature of the subject. The fundamental idea of this approach is to conceive of modality as representing an instrumentality for attributing *ab extra* a certain classification or *status* to a statement (on analogy with truth, or falsity, or theoremhood) — rather than to construe modality as an operator (like negation or conjunction) which enters explicitly into the formulation of a compound statement as an intrinsic constituent thereof (as do the standard modal logics of the present day). This «new» approach to modality, treating it as essentially metalinguistic rather than notationally immanent, is inspired by Aristotle's treatment of modal syllogisms in the *Prior Analytics*. For, as I have elsewhere argued, the best possibility for making sense of Aristotle's modal syllogistic lies in interpreting his modalities in just this way, as *ab extra* assignments of epistemological status (1).

My purpose in the present discussion is not at all historical. The principal interest of the conception of modality as status is, as will appear, as a means for circumventing serious (but usually ignored) difficulties in the introduction of counterfactual assumptions into reasoned analysis.

## II. *Modal Categories*

By a *series of modal categories* (relative to some concept of propositional assertion designated by «$\vdash$») we shall understand any group of *sets of statements*, $M_0$, $M_1$, ..., $M_n$ which satisfies the following conditions:

(1) N. Rescher, «Aristotle's Theory of Modal Syllogisms and its Interpretation,» in M. Bunge (ed) *The Critical Approach: Essays in Honor of Karl Popper*, Glencoe, Ill. (The Free Press), 1963.

(o) The statements of $M_1$ are mutually consistent (though they need not be consistent with statements of *other* $M_j$).

(i) If $\vdash P$, then $P \varepsilon M_0$.

(ii) If $P \varepsilon M_i$, then $P \varepsilon M_j$, for all $j \geqslant i$.

(iii) If $P \vdash Q$ and $P \varepsilon M_i$, then $Q \varepsilon M_i$.

(iv) If $P \varepsilon M_i$ and $Q \varepsilon M_j$, then $(P \& Q)\ \varepsilon M_{max}(_{i,\ j})$

(v) If $P \varepsilon M_i$ and $Q \varepsilon M_j$, then $(P \vee Q)\ \varepsilon M_{min}(_{i,j})$

Observe, that these rules have the consequence:

(vi) If $P_1, P_2, \ldots, P_n \vdash Q$ then $Q \varepsilon M_{max\ i}$',
where the $i'$ are given by the condition that $P_i \varepsilon M_i$.

We shall speak of these $M_i$ as *modal categories*, and shall occasionally write mod(P) = i (read: «P belongs to the i-th modal category») instead of $P \varepsilon M_i$.

It is a significant fact that, given mod(P) = i, we are not, on the basis of the foregoing rules, entitled to make any claims about the modal category of $\sim P$ (when $i > 1$). Furthermore, we have made no general assumption about the consistency of the sets $M_i$. Nothing we have said precludes the existence of a P such that $P \varepsilon M_i$ and $(\sim P) \varepsilon M_i$. Thus the foregoing rules do *not* completely determine the modal category of a statement in terms of the modal categories of its parts. Indeed, the rules do not preclude the possibility that some statements fail to fall into any modal category whatever (of course only when $M_n$ is consistent); but we may, without loss of generality, suppose that all statements have been put into one final (if need be, additional) «catch-all» category.

Each modal category $M_i$ may be divided into two groups - comprising those member-statements that are true and those that are false, respectively. Let us write,

«T (P)» for «P is true».

We may then introduce the idea of a statement «necessary with respect to the i-th modal category», or for short «i-necessary»:

$N_i(P)$ if and only if: Both $P \varepsilon M_i$ *and* T(P)

Correspondingly we may speak of a statement as «possible with respect to the i-th modal category», or for short «i-possible» when its negation is not i-necessary:

$P_i(P)$ if and only if: Either *non* $(\sim P) \varepsilon M_i$ *or* T(P)

(Note that this definition has the consequence that a true statement is i-possible with respect to every modal category.) These relationships establish the bridge between the standard notions of propositional modality on the one hand, and the schematism of modal categories on the other.

## III. *An Illustration*

It is desirable to outline one concrete instance in which a set of «modal categories» as envisaged in the preceeding section can be built up. Such an illustrative example serves two purposes: (1) to give more familiarity to the ideas at issue, and (2) to provide a concrete exemplification of the feasibility of the preceeding schematism.

Let us suppose as given a formal system of some sort, based on certain axioms and rules of inference which provide a corresponding notion of a theorem, $\vdash P$. Consider now some sequence of further (non-theorematic) statements: $S^1_1, S^1_2, \ldots, S^1_{n1}, S^2_1, S^2_2, \ldots, S^2_{n2}, S^k_1, S^k_2, \ldots, S^k_{nk}$. We here assume that the first $n_1$ statements are mutually consistent. Now we construct the modal categories $M_i$ as follows:

(o) $M_0$ is the set of all P such that $\vdash P$

(1) $M_1$ is the set of all P such that $S^1_1, \ldots, S^1_{n1} \vdash P$

(2) $M_2$ is the set of all P such that $S^1_1, \ldots, S^1_n, S^2_1, \ldots S^2_{n2} \vdash P$

Et cetera

It is readily seen that, given this construction of the sets $M_i$, each of the conditions (o) - (v) of the preceeding section is at once satisfied.

## IV. *Ad Absurdum Reasoning in the Context of Modal Categories.*

We inevitably have, for any reasonably orthodox concept of assertability or demonstrability, that the following rule obtains:

Rule 1: If $P_1$, $P_2 \vdash$ Contradiction, then $\vdash \sim(P_1 \& P_2)$, or equivalently $\vdash \sim P_1 \vee \sim P_2$.

The leading idea behind the construction of modal categories along the lines of present discussion is the prospect of supplementing this rule by:

Rule 2: If $P_1$ is self-consistent, $P_1$, $P_2 \vdash$ Contradiction, and if $\text{mod}(P_1) < \text{mod}(P_2)$, then $\vdash \sim P_2$.

In other words, we wish to be in a position to hold that *in cases of a logical conflict among statements, the statement of weaker (i.e. numerically higher) modality gives way to statements of stronger (numerically lower) modality.*

Our wish to have this amended principle of *ad absurdum* reasoning at our disposal means that we must modify the Deduction Theorem (for material implication) in the case of application to inconsistent premisses (but *only in this case* !). For suppose

(1) $P_1, P_2, \ldots, P_n, Q \vdash$ Contradiction

Then by the (unmodified) Deduction Theorem:

(2) $P_1, \ldots, P_n \vdash \sim Q \vee$ Contradiction

And thus:

(3) $P, \ldots, P_n \vdash \sim Q$

But now this line of reasoning is altered, since (1) would not yield (2) but rather,

(4) $P'_1, \ldots, P'_n \vdash \sim Q' \vee$ Contradiction,

where $Q^1$ is the (or any) weakest-modality element of the set $\{P_1, P_2, \ldots, P_n, Q$ and $P'_1, \ldots, P'_n$ are the remaining elements of this set. Thus Rule 2 has the important consequence that (1) leads, not to (3), but to:

(5) $P'_1, \ldots, P'_n \vdash \sim Q'$.

Rule 2 entails various other corresponding changes in the usual machinery of inference in the special case of deductions from inconsistent premisses (but in this case only). For example, we must modify the rule:

Rule 3: If $P_1, P_2, \ldots, P_n \vdash Q$
and $P'_1, P'_2, \ldots, P'_m \vdash R$,
then $P_1, P_2, \ldots, P_n, P'_1, P'_2, \ldots, P'_m \vdash Q \& R$.

For if Q and R are self-consistent but mutually incompatible, then if (say) $mod(Q) < mod(R)$, then the conjoined premisses would not yield Q & R, but only Q.

Perhaps enough has been said to indicate the general character of the changes for the usual approach induced by the shift from Rule 1 to Rule 2.

To repeat: In cases of logical conflict, statements of weaker modality yield to statements of stronger modality.

As these considerations indicate, the ranging of statements within modal categories on the conception of modality as a status amounts, in effect, to a *preferential ranking* of propositions. The category-assignments serve as an indication of the extent to which we feel ourselves to be committed to certain propositions — indicating the extent to which, in cases of a logical conflict with other propositions, we are willing or loath to yield them up. We conceive of modality as indicative of an epistemological status of propositions representing the extent to which they have become embedded in the groundwork of our knowledge. The question of what constitues a reasonable epistemological basis for making such category assignments lies outside the scope of the present discussion.

## V. *Consequences of Inconsistent Assumptions.*

Suppose we wish to study the consequences of a series of statements (assumptions) belonging to different modal categories, say

$A^1_1, A^1_2, \ldots, A^1_{n1}, A^2_1, A^2_2, \ldots, A^2_{n2}, \ldots$

where the first $n_1$ statements are assumed to be mutually compatible, and the $A^i_j$ belong to modal categories as follows:

(o) $M_0$ includes all P such that: $\vdash P$.

(1) $M_1$, includes all P such that: $A^1_1, A^1_2, \ldots, A^1_{n1} \vdash P$

(2) $M_2$ includes all P such that: $A^1_1, A^1_2, \ldots, A^1_{n1}, A^2_1, A^2_2, \ldots A^2_{n2} \vdash P$

(i) Et cetera

Note, that no problems arise for this machinery when all the $A^i_j$ are mutually compatible. Note further that, when this is not the case, then (since the first group $A^1_1, A^1_2, \ldots, A^1_{n1}$ is mutually consistent), the logical conflict must arise *across a categorial boundary,* with the result that our rule of *ad absurdum* reasoning (Rule 2 of section IV above) establishes *the negation of the statement of weaker modality.* Thus the possibility of outright contradiction is *localized,* being confined within non-vicious limits. Whereas we usually have the rule,

If the set of Assumptions is inconsistent, then:

Assumptions $\vdash$ Anything,

this principle is now abrogated. Given the conception of modality as a status, the question of the consequences of an inconsistent set of assumptions becomes nontrivial.

## VI. *Counterfactual Suppositions.*

We now turn to a consideration of the logical difficulties inherent in counterfactual assumptions. Suppose that $P_1, \ldots P_n$ are all true statements, and that we wish to investigate the consequences of assuming $\sim P_1$.

Obviously we cannot simply add $\sim P_1$ to our list of (assumed) truths. For then this list, viz. $\sim P_1, P_1, P_2, P_3 \ldots, P_n$ is inconsistent, and thus any investigation of the consequences becomes logically uninteresting.

Nor will it do simply to modify this list by deleting $P_1$, thus examining the consequences of $\sim P_1, P_2, \ldots, P_n$. For $P_2$ might well be $P_1 \vee R$, and $P_3$ be $\sim R$, so that $P_1$, while overtly dropped from the list, continues as tacitly included in the «residual» list (as a consequence of $P_2$ and $P_3$), with the result that the trimmed list continues inconsistent. This states of affairs would, of course, not obtain if we assumed each of the statements $P_1$ to be *logically independent* of the remainder. But this assumption would clearly be unworkable. For how are we — plausibly and justifiably — to break up «our knowledge» of the facts in any particular concrete case, and split it apart into logically independent units ?

The mechanisms of our foregoing discussion enable us to overcome difficulties of this sort; for we can now proceed by studying, for example, the consequences of the assumptions:

$$\sim P_1, P^2_1, P^2_2, \ldots, P^2_m / P^3_1, P^3_2 \ldots, P^3_{(n-m)} /$$

where (i) the $P^2_i$ are any of the $P_i$ that are mutually (and conjointly) compatible with $\sim P_1$, (ii) the $P^3_i$ are the remaining $P_i$ (including of course $P_1$ itself), and (iii) each time we reach a slash / we are to take all preceeding statements as the «generating assumptions» (in the sense of section III above) of a new (higher) modal category. We are also at liberty to study the results of modifying this set up by adding new slashes as desired, and permuting the slash-groups among each other.

In the interests of clarity let us take a concrete example. Consider as «accepted truths» the statements:

$P_1 = p$
$P_2 = q$
$P_3 = p \vee \sim r$
$P_4 = r$

(for simplicity we assume that the lower case letters represent logically independent statements). Suppose we wish to test the consequences of assuming $P_5 = \sim P_1$ (i.e. $\sim p$). We can now study the results of assuming $P_1$ - $P_5$, with respect to the following possible sets of modal categories (specified in terms of their «generating premisses»):

$P_1 / P_2, P_3, P_4 / P_5 /$ (This case clearly «violates the spirit of the assumption».)
$P_5 / P_2, P_3 / P_4 / P_1 /$ or equivalently $P_5 / P_2, P_3 / P_4, P_1 /$ [2]
$P_5 / P_2, P_4 / P_3 / P_1 /$ or equivalently $P_5 / P_2, P_4 / P_3, P_1 /$

Thus we are now in a position to say what surely accords with our intuitive understanding if the matter, namely that the consequences of introducing the «inconsistent» assumption «$\sim p$» are as follows:

(i) Not simply everything whatsoever.
(ii) «$\sim p$» itself, thus forcing us to reject «p».
(iii) Various «unaffected» statements such as «q», «$q \vee r$», etc.
(iv) We «have a choice» beween retaining one or the other of «r» and «$p \vee \sim r$».

In short, the machinery of modal categories appears to afford a logical resource capable of handling contrary-to-fact assumptions in

(2) Note that a slash may be deleted whenever this involves no loss of opportunities for conflict avoidance.

a manner which, on our accustomed view of the matter, seems greatly more «natural» and satisfactory than the standard mechanisms otherwise afforded by logic.

## VII. *Counterfactual Conditionals.*

The discussion of the foregoing section has an important bearing on the study of the so-called *counterfactual-conditionals,* that is statements of the form «if ... were so, then --- would be so» where «...» is (or is believed to be) false. For example consider hypothetical: «If this coin (a penny) were a dime, then it would be made of silver». Such counterfactuals inevitably fall into the pattern of (1) a series of consistent and accepted statements $P_1$, $P_2$, ..., $P_n$, namely our actual beliefs with respect to the matter in hand, (2) a hypothetical with an antecedent that negates one of these $P_i$, and (3) a consequent that draws some «appropriate» conclusion from the «residual» $P_i$.
Specifically, in our example we have

Accepted Beliefs:

$P_1$. This coin is a penny.
$P_2$. This coin is not a dime.
$P_3$. This coin is made of copper.
$P_4$. This coin is not made of silver.
$P_5$. Pennies are made of copper.
$P_6$. Dimes are made of silver.

Belief-Contravening Assumption.

$P_7$ ($= \sim P_2$) This coin is a dime.

We are now in the situation of having a choice between:

| *Course 1* | | *Course 2* | |
|---|---|---|---|
| *Retain* | *Reject* | *Retain* | *Reject* |
| $P_7$ | $P_1$ | $P_7$ | $P_1$ |
| $P_5$ | $P_2$ | $P_5$ | $P_2$ |
| $P_6$ | $P_3$ | $P_3$ | $P_6$ |
| | $P_4$ | $P_4$ | |

In short, we are forced to a choice between $P_3$ - $P_4$ on the one hand, and $P_6$ on the other. And if we are willing to accept that $P_6$ has the august status of a «law», so that we accept Course 1 instead of Course 2, we are led to the «natural consequence», asserted by our initial counterfactual conditional, that «This coin is made of silver» (i.e. $\sim P_4$) (8).

This entire way of approaching the analysis of counterfactual conditionals fits smoothly into the discussion of the preceeding section. Thus the alternative between Course 1 and Course 2 comes down to the alternative modal categorizations:

(1) $P_7, P_5 / P_6 / P_3, P_4 / P_1, P_2 /$, or equivalently
$P_7, P_5, P_6 / P_3, P_4, P_1, P_2 /$

(2) $P_7, P_5 / P_3, P_4 / P_6 / P_1, P_2 /$, or equivalently
$P_7, P_5, P_3, P_4 / P_6, P_1, P_2 /$

The question at issue, regarded from this standpoint, is seen to come down to this: Is «stronger» modal status to be accorded to $P_3 - P_4$, or to $P_6$.

To summarize: If the relevant beliefs are partitioned (sufficiently finely) into modal categories, then when antecedent of a counterfactual conditional is added as a supposition, the counterfactual as a whole may be accommodated perfectly naturally by the machinery of counterfactual suppositions in the context of modal categories as outlined in section VI.

To be sure this approach only provides the formal mechanisms for analyzing counterfactual conditionals. Its concrete application hinges on the availability of the needed epistemological instrumentalities for assigning statements to modal categories of an appropriate type — i.e. in such a way as actually to reflect the degree of «rational attachment» we are entitled to have towards statements on the basis of the supporting grounds for them. It warrants mention, however, that the familiar nomenclature of «law», «theory» and «hypothesis» points to a ranking of just this kind.

## VIII. *Conclusion*

The foregoing discussion points to two principal conclusions:

(i) The conception of modality as the *status* of a statement (rather than as an operator explicitly into its formulation) represents a *feasible* notion of modality.

(8) The approach to the analysis of counterfactuals sketched in this paragraph is presented in considerable detail in N. Rescher, «Belief-Contravening Suppositions», *The Philosophical Review*, vol. 70 (1961), pp. 176-196.

(ii) This approach is not merely feasible but also *useful* in connection with the logical analysis of contrary-to-fact assumptions and counterfactual conditionals.

The conception of modality as a status of assertions can be elaborated in various ways which we have not even envisaged here. For example, our concept of status-modality is such that the modality of a conclusion corresponds to that of the «weakest» premiss. This principle departs from Aristotle's interesting conception that — under certain restricted circumstances — the modality of a conclusion can be «stronger» than that of the «weakest» premiss. The development of the conception of modality as a status in the direction of such ideas might well prove to possess substantial interest.

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