## SOME REMARKS CONCERNING STATEMENTS, TRUTH-VALUES, AND CATEGORIES OF PREDICATES

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In many textbooks of logic statements are spoken of in such ways that it seems reasonable to define: a statement is anything that can significantly be said to be true or false. In this paper I take it for granted that this definition (whether expressly formulated or not) implies the following presuppositions: a) statements may be linguistically formulated sentences, b) statements are something that is either true or false, and c) one must know the meaning of the words 'true' and 'false' in order to know what a statement is. To these presuppositions I would make a few comments which are of some importance, I believe.

ad a) Statements need not be linguistically formulated. The linguistic formulation of a statement is inessential from a logical point of view. One and the same statement may be expressed in different languages and even in different sentences within the same language. Besides, one may often be in doubt as to how to formulate a certain statement in the most appropriate way, even when knowing very well what statement one wishes to formulate. This indicates that the linguistic formulation is inessential to a statement, and that the essential thing is the meaning connected with the possible formulations or expressions, i.e. the apprehension or conception (often called the proposition or thought), which the apprehending or judging person has in his mind. One may, for instance, recognize a thing or a colour or a melody without formulating any designation of them or any sentence describing them. On the other hand, many sentences, for instance interrogative, imperative and exclamatory sentences, are not expressions of statements. Thus, to state something and to use a language seem to be two processes that are in principle independent of each other, although they are very often connected in organisms that have an extensive training in using a language.

But what then is a statement?

To answer this question seems to be very difficult, and I shall therefore restrict myself to a very short description of what I consider the most essential characteristics of a statement. First, I think a statement is the result of a psychological process taking place in the person making the statement and nowhere else. Next, this process is directed towards an *object* in the wide sense of this word in which it may be

said that a concept is a concept of something, a fear fear of something, hate hate of something etc. Further, the process makes one or more of the objects features appear, or appear more distinctly than before. The object towards which the process is directed is called the subject of the statement, and the features emphasized are called the predicate(s) of the statement. The statement does not contain the subject as a part, but it 'has' the subject, i.e. it is directed towards the subject. In so far the process is a process of analysis: a given content of consciousness is articulated in such a way that certain features of it are emphasized as features or parts of the given totality. Which features are emphasized depend partly on the apprehending person and partly on the object apprehended. The judging individual cannot arbitrarily decide what features are to appear, i.e. to be found by the analysis. Although the individual may wish that the object should posses certain features he will be unable to create such features by analysis, since he can thereby only ascertain the existence of features that the object actually possesses. It may therefore, perhaps, be best to define: to judge (or to make a statement) is to predicate, i.e. by means of a more or less thoroughgoing analysis to ascertain that a given object contains, or has certain features, e.g. a certain quality, a certain magnitude, a certain position in space and time, certain relations between its parts, etc. etc.

If one wants to communicate one's statement to other persons, then one has to give expression to it by naming which object the statement concerns and what is asserted about that object. Often the most convenient way to do this is to use the usual linguistic expressions designating the object and its ascertained features. Since these designations are in principle different from the object and its features, they should never be called 'subject' and 'predicate' but always 'subject-designation' and 'predicate-designation'. In the statement 'the earth is round' the subject is the earth itself and the predicate a property of the earth, while the word 'earth' is subject-designation and 'round' predicate-designation. This is, of course, obvious, but as we shall see later on, it is a thing which it is important to remark and bear in mind.

ad b). It is generally agreed that statements may be true or false. Nevertheless it seems incorrect to define statements as anything that may be true or false. Indeed, a general definition of a statement ought to state the characteristics that are to be found in *any* statement. But truth cannot be found in false statements, and falseness not in true statements: it is merely the alternative characteristic 'true or false' that can be found in any statement. Just because these

alternatives are different (and moreover mutually exclusive) they merely give an 'extensional' definition of statements, namely as an aggregate of two different classes, each of which is to be defined in its special way. Thus truth and falseness are inessential characteristics of statements (even if they play an important part in logic). Therefore I find it better to define: a statement is an ascertainment of something (the subject) containing such and such features (the predicates). Such statements may then be either true or false. But what does that mean? Under which conditions is a statement true, and under which is it false?

ad c). Usually a statement is said to be true, if and only if it agrees with reality. What then is reality? The word 'reality' has, as is well known, various senses, the most important of which seem to be the following: In everyday-life it signifies the usual reality, and it is true to state that grass is green, the heaven blue, church bells noisy, roses flagrant, sugar sweet, etc. But in science the word 'reality' signifies the objective (physical) reality, and it is true to state that things are composed of colourless atoms, that light rays are electromagnetic waves, that sounds are air waves, etc. In psychology, however, the word 'reality' signifies the psychological or phenomenological reality (i.e. what is immediately given), and it is true to state that the full moon is larger on the horizon than in zenith, that a thing's quality of being warm or cold varies with my temperature, that my images are colourless, etc. Further, in the world of literature, art, or imagination, the 'imagined' fantasies or day-dreams are the 'real' works of art or imagination, and it is true to say that Hamlet killed Polonius, that Ophelia went mad, that Pegasus is a winged horse, etc. And finally, in the sphere of mathematical objects the word 'reality' signifies the mathematical concepts, and it is true to say that in an Euclidean triangle the sum of the angles equals two right angles, that in a square the diagonal is incommensurable with the side, that continous functions that cannot be differentiated exist, etc.

If desiring to give the word 'true' an unambiguous sense one thus cannot define the truth of a statement as its conformity or agreement with reality. But, if desiring to follow common usage, one cannot deny that all the above examples of statements are true, everyone of them expressing a correct or true apprehension of the object about which it predicates something, and its negation expressing an incorrect or false apprehension of the same object. The question then arises whether it is possible to find a definition of 'truth' that comprises all the various examples of true statements.

This question can, in my opinion, be answered in the affirmative. Common to all the statements mentioned is the fact, that the object about which they state something actually contains the predicate which they ascribe to it, i.e. that that predicate can be found by an analysis of the object: grass is something that is green, physical things are composed of atoms, the phenomenon of the full moon is larger on the horizon than in zenith, the imaginary animal Pegasus has wings, and the sum of angles in an Euclidean triangle equals two right angles, etc. In one word: in every true statement the predicate is contained in the subject of the statement, - no matter whether the subject is an everyday thing, a physical object, an immediately given content of an experience, an imaginary object, or a conceptual object. And, conversely, a statement cannot be true, if the object does not contain the features signified by the designation of the predicate. If the subject of a statement does not contain the features predicated of it, then the statement is false. If so, the result of a thoroughgoing analysis will show the falseness of the statement. Therefore, truth, respectively falseness, may be defined thus:

A statement is true, if and only if the predicate of the statement is contained in the subject of the statement: and a statement is false, if and only if the predicate of the statement is not contained in the subject of the statement.

If calling statements whose predicates are contained in their subjects 'analytic statements' one may assert that any analytic statement is a true statement, and that only analytic statements are true statements. In this connection it must be emphasized that what we are here concerned with is the subject and the predicate of the statement, and neither with the designation of the subject or the predicate, nor with the judging individual's concept of the subject or the predicate. The fact that a predicate of a statement is contained in the subject of the same does not imply that the concept of the predicate is also contained in the concept of the subject. Whether this is the case or not depends on how ample and correct a concept of the subject the individual has formed. Usually, however, the properties, relations or other features of the subjects of statements are not found by an analysis of the concepts of these subjects, but by an analysis of the subjects themselves, i.e. the objects about which something is predicated. Where, as, for instance, in mathematics, something is stated about objects that only 'exist' by virtue of a definition, the subjects of the statements are concepts, the properties of which are to be found by a conceptual (logical) analysis of the definition (no matter whether this definition is formulated explicitly in a definitional equation, implicitly in a postulate-system, or merely understood).

According to Albertus Magnus the above-mentioned definition of the truth of statements was first proposed by the Arabian philosophers Avicenna, Alfarabi, and Alhazen ab. 1000 A.D. and was later taken up by Hobbes and Leibniz. As these thinkers did not, however, distinguish clearly between the subjects of statements and the concepts of these subjects, the statements were conceived as relations between the concept of the subject and the concept of the predicate, and the analysis was conceived as a purely conceptual one. In consequence, it lay near at hand for Kant later to introduce the presumably untenable distinction between what he called analytic and synthetic statements that play so important part in his whole philosophy. In my sense of the word there are no true synthetic statements. All true statements are analytic, but the analysis from which they spring varies in character as to the sphere of objects to which the subjects of the statement belongs. If the subject belongs to the sphere of everyday objets, the analysis will be a usual observation; if it belongs to the sphere of physical objects, the analysis will be a more or less complicated investigation by means of various apparatuses; if it belongs to the phenomenological sphere, the analysis will consist in an awareness of what is immediately given; if it belongs to the sphere of imagination, the analysis consists in observing the characteristics of the imaginary beings or occurrences (by the way, one may here easily find examples of non-mathematical, undecidable statements, as for instance, 'Ophelia wore a pigtail'); and finally, if it belongs to the conceptual sphere, the analysis will be of a logical character and often consists in a proof or disproof based on the given or presupposed definition.

The five spheres of objects here mentioned overlap, of course, and it may be very difficult to find precise criteria of each of them. But this is not essential for my present argument, no more than is the question as to whether they may be reduced to a smaller number or not. What matters is merely, that the decision of the question as to whether a given statement is true or not depends on some kind of analysis of the subject of the statement, no matter how many spheres of objects are concerned.

The next question that arises is: How many kinds of statements can be distinguished? This seems a very difficult question to answer, and I shall therefore restrict myself to mention a few examples that are of importance in this connection. First, it may be convenient, however, to distinguish between statements on the one hand and

designatory conventions, that are not statements, on the other. Such conventions are, of course, necessary presuppositions for linguistic formulations of the statements, and, unfortunately, these conventions are themselves formulated in sentences that are identical with sentences expressing statements, e.g. 'this is a tree', or 'this is Mr. N. N.'. Being conventions such sentences are, however, not expressions of ascertainments and can neither be true nor false. They may always be conceived as answers to the question: 'What is this called?' or 'What is the name of this?', and they should always be formulated by means of the words 'called' or 'named': 'This is called a tree', or 'N. N. is the name of this person' (if conceived as statements concerning the usage of the language they are statements, but they are not statements concerning the designated objects, which remain unchanged independently of any arbitrary change of the designation).

Returning now to the real statements a most important distinction seems to be the one between primary statements and secondary statements. Statements the subject of which is not statements, but some other kind of objects, are called primary statements. And statements the subjects of which is one or more primary statements are called secondary statements. Consequently, the statement 'this is red' is a primary statement, and 'the statement 'this is red' is false' is a secondary statement. If necessary, this classification may be continued by the introduction af tertiary, quaternary, etc. statements.

Of other possible fundamental fundamenta divisionis may be mentioned: a) the object-sphere to which the subject of the statement belongs; b) the number of possible predicate categories of the subjects (about which something will be said further on), and c) the extension of the subject.

These distinctions may be crossed so that a rather complicated system of different kinds of statements results. Here, however, I shall confine myself to some remarks concerning the first-mentioned distinction, occasionally adding a little about the other ones.

The simplest and most fundamental of the primary statements are presumably those, the subject of which is a present object being analysed in such a way that either different features, or different parts of it and relations between such parts are ascertained. Statements in which the features are predicated of the subject may be called attributive statements, while statements in which relations between parts of the subject are predicated of the same may be called relational statements. Usually the linguistic expression of an attributive statement designates the subject by a single designation, while the several parts of the subject of a relational statement must

have separate designations in the linguistic expression of such a statement. Both kinds of statements have, however, only *one* subject, i.e. the predicate(s) is (are) ascribed to one and only one subject irrespective of this having parts or not.

These two kinds of primary statements may also result from a process of recognition, in which a present object is ascertained to be the same as an earlier experienced object, or to be similar to such object. Such statements of recognition are necessary presuppositions for the introduction of designations of objects at all. Without recognition of objects one cannot introduce and apply designations of them. But later on, when the application of the object-designations has been learnt, these designations often substitute the subject in linguistic formulations of statements, and these then appear to be purely linguistic affairs. The connection with the non-linguistic reality that alone gives the sentences a meaning is interrupted, and the designation of the subject or the concept of the subject may then easily be mistaken for the real subject, which mistake has caused serious errors in logical theory. The introduction of subjectdesignations is, of course, an enormous practical advantage, because by means of them one can talk about objects that are not present, - their names represent them. But as the names do not generally resemble the designated objects, one is, as a rule, forced to found one's predications on one's memories or on one's concepts of the objects concerned; and as one's memories or concepts are usually rather vague or meagre (and moreover often incorrect) one is, of course, easily led to rather unimportant or misleading statements. The name or the concept easily comes to function as the subject of the statement instead of merely representing it. Under such conditions the argument, at worst, develops into a purely verbal affair or, at best, into a purely conceptual construction (as is the case in all formal systems). Conversely, the subject-designation may be omitted from the linguistic formulation of the statement in cases where the subject is present to the conversing persons, or it may be referred to by a pronoun. Such statement-expressions without a subject-designation are in German called 'Subjektlose Urteile' (subject-less statements), but this designation is misleading, since, as a matter of course, every statement has a subject - whether designated or not. A predication cannot be made without a subject. The sentence 'it rains' is merely a linguistic shortening of another sentence that may be formulated somewhat as follows: 'my out-door environment is in a rainy state'. On the other hand a plurality of predicates of earlier formulated statements may be condensed in a

single subject-designation of a new statement, such as for instance, 'the old man that lived in this house has died', which statement presupposes the truth of the statements 'the man was old', and 'the man lived in this house'. And here it is immaterial whether the man belonged to the sphere of everyday things or to the sphere of imagination.

Returning to the classification of statements it is important to remark that primary statements (as well as statements of higher order) may be affirmed or negated. Affirmation of a primary statement is called a positive (or affirmative) statement, and negation of a primary statement is called a negative statement. Positive and negative statements are of the order next above the statements affirmed or negated. Linguistically a negating statement may be shortened by means of the word 'not' as, for instance, 'this rose is not red'. This shortened form of the linguistic expression has led to the untenable conception that negative statements ascribe a 'negative' predicate to the subject. But 'negative predicates' seem unthinkable, and thus all statements ascribe a positive predicate to some subject. If, however, the subject does not have the ascribed predicate, then the statement is false, and the negation of it is then true.

A positive (secondary) statement is true, if and only if the predicate 'true' can be found by analysis of its subject, which is a primary statement. The positive statement 'the statement 'this rose is red' is true' is itself true, if and only if the statement 'this rose is red' is true, and this last-mentioned statement is true, if and only if the present rose actually has the quality red. The truth of the secondary statement is thus ascertained by an analysis of the corresponding primary statement, and the truth of this primary statement by an analysis of its subject. On the other hand, the property 'true' can, of course, never be found by analysing merely the linguistic expressions of statements, since these linguistic expressions have but the property 'true' in relation to their subjects, that can never occur as a part of the linguistic expressions (unless, exceptionally, the statement is a statement about a linguistic expression, such as, 'the word 'statement' contains nine letters').

Analogously, a negative (secondary) statement is true, if and only if the predicate 'false' can be found by an analysis of the corresponding primary statement. If so, the subject of the negative (secondary) statement (i.e. the primary statement) has the property falseness, and it is consequently true to ascribe this property to it. This is done in the negative statement, and therefore this statement is true.

True positive or negative statements presuppose true, resp. false,

primary statements. And the fact that the truth or falseness of primary statements depends on the properties of their subjects, which properties are independent of the person making the statements, presumably show that truth and falseness cannot be introduced by convention, but are based on the ascertainment of facts. And analogously, where false positive or negative statements are concerned.

In so far as the truth or falseness of secondary statements depends on the truth or falseness of the corresponding primary statements, the firstmentioned may be said to be truth-functions of the latter. Thus truth-functions are a special kind of secondary statements. Not all secondary statements are truth-functions. The statement 'the statement p is complicated' is, for example, a secondary statement, but it is not a truth-function. In passing it may also be remarked, that not every statement containing another statement as part of it, is a truth-function. For example, the statement 'A believes p' is not a truth-function, since it is not even a secondary statement. Its subject is not a statement, but the belief of A which is a psychogical fact that may contain the statement p, but that is not itself a statement.

The relation between primary statements on the one hand, and positive, resp. negative, statements on the other may be expressed in a so-called *truth-table*:

If and only if the primary statement p is true, resp. false, then the positive statement 'p is true' is true, resp. false, and the negative statement 'p is false' is false, resp. true.

In this way we get a truth-table for affirmation corresponding to the usual truth-table for negation, and it is shown that the truth-value of the truth-function eventually depends on the 'content' (or 'intension') of the primary statement, viz. on the properties of the subject, which properties are designated by the predicate-designations. If one does not take this 'content' in account, then one does not know the sense of the words 'true' and 'false' at all, and the whole procedure change into a meaningless play.

Affirmation and denial need not concern merely single statements, as was the case in the examples given above. Such statements may be combined either conjunctively or disjunctively, and by affirming or denying these combinations one gets new kinds of truth-functions, customarily called 'conjunctions', resp. 'disjunctions'.

As regards the other usual truth-functions the matter seems to be more complicated. However, space do not allow me to go further into this question here and I, therefore, will conclude by making a few remarks concerning what I call categories of predicates.

Consider a simple everyday thing, e.g. a table, and let the analysis of same result in the finding of the following properties: rectangular, four-legged, brown, glossy, hard, wooden, combustible, large, etc.

All the properties found by analysis of the various tables may now be classified in different groups which I shall call categories of predicates, because each property corresponds to the predicate in one of the statements that may be made concerning one or another of the tables. The principle according to which the classification is carried out is, that predicates that are mutually exclusive belong to one category. A category of predicates thus comprises the predicates that exclude one another in statements concerning the same subject. No subject has more than one predicate from any category. Experience shows which predicates are incompatible.

In the above-mentioned statements concerning the tables, predicates from the following categories are *i. alia* contained:

the category of form: rectangular, round, triangular;

the category of colour: brown, white, grey;

the category of leg-number: four-legged, three-legged, one-legged; the category of reflection: glossy, dull;

the category of material: wood, marble, stone.

If we do not confine ourselves to the analysis of tables, but include all possible objects, we shall find a lot of qualities belonging to various categories of which there is an immense number: the category of size, of weight, of duration, of stability, of sound, of taste, of various kinds of value, etc. etc.

Every subject (object) may have predicates belonging to various categories, but there hardly exists an object having predicates from all categories. Quadratic numbers, for example, cannot be red or hot, tables not hard-hearted, atoms not green, etc. Predicates belonging to a category the members of which cannot be ascribed to a certain subject may be said to be foreign to this subject. And statements ascribing such foreign predicates to a subject may be said to be meaningless, in contradistinction to false, false statements ascribing a predicate to a subject to which it is not foreign, but merely actually not belonging. Of a quite concrete subject it is always meaningless to predicate properties that it actually does not possess (a concrete swan cannot be both white and black allover). But of a more or less abstract subject it is always meaningful to predicate the various properties belonging to a category of predicates of which a member can be ascribed to the corresponding concrete subject (swans, generally, may be either white or black).

Of statements concerning abstract subjects the following seem to be true: predicates belonging to one and the same category may be disjuncted, but not conjuncted, i.e. of the same subject it will always be true to assert that it has either the first, or the second, or the third, etc. predicate of one of its possible categories, every category comprising also the negation of the other predicates within the category, but this negation not being identical with any predicate outside the category. For instance, 'shapeless' is neither identical with 'colourless', nor with 'immaterial', nor with 'thoughtless', nor with any other predicate belonging to another category than the one to which the predicate 'shape' belongs. Every category forms so to say a universe of predicates, and it is such a predicate-universe that is split up when represented as the logical sum of a term in it and the negation of that term (this negation being the disjunction of all the other terms in the universe). Thus the universe of truth-values, for instance, is exhausted by the predicates 'true 'and 'false', because 'false' is the negation of 'true' and vice versa. Every statement the predicate of which is the disjunction of all possible predicates within a certain category is a tautology in Wittgenstein's sense. Such statement must always be true, because one of the predicates must belong to the subject. If this was not the case, all the predicates would be foreign to the subject, and the statement consequently meaningless. So is the nature of our conceptual universe and of our corresponding significations of predicates. The fact that the predications must be restricted to a definite category, Wittgenstein, however, seems not to have observed. But it can hardly be denied that it is highly artificial to predicate of a given subject a disjunction of predicates from different categories, as e.g. 'the table is either rectangular, or brown, or four-legged, etc.'. Indeed, these predicates do not exclude one another, and in common usage they are not disjuncted. Corresponding to the above-mentioned statement concerning the disjunction of predicates it may thus be stated: predicates belonging to different (but possible, i.e. not subject-foreign) categories may be conjuncted, but not disjuncted. While it is contrary to usage to say 'the table is either rectangular, or brown, or four-legged, etc.', it is quite in order to say 'the table is rectangular, and brown, and fourlegged, etc.'. This last-mentioned statement would, however, be contradictory, if the negation of a predicate was not restricted to the corresponding category of predicates, i.e. if 'four-legged', for instance, belonged to the negation of 'rectangular'. Whether the last-mentioned conjunctive statement is true or false must be decided by an analysis of the subject concerned. Statements, however, in which

a conjunction of predicates belonging to the same category is predicated of a subject, are always false and thus have a certain affinity to contradictions in Wittgenstein's sense.

Actually, the characterization here given of tautologies and contradictions seem to show that the present concepts of same are generalizations of the corresponding Wittgensteinian concepts. According to Wittgenstein tautologies and contradictions are respectively disjunctions and conjunctions of statements, and what matters are the truth-values of these statements, while here we have to do with disjunctions and conjunctions of predicates generally of which the truth-values are special instances. And, of course, statements may have predicates belonging to other categories than those of truth-values; they may, for instance, be more or less complicated, be primary or secondary, etc., be attributive or relational, etc. But in the statement 'any statement must be either true or false', merely a category of truth-value is considered. As this category contains merely the two predicates 'true' and 'false', it may properly be said to be divalent, and its two members to be contradictory. Other categories of predicates may be trivalent, tetravalent, or generally polyvalent, and in all these cases the members of the categories are said to be contrary. Contrary predicates exclude one another, but merely taken all together they exhaust the category: An inference from the falsity of a statement to the truth of its negation therefore becomes more and more indefinite, the more comprehensive the category of the statement's predicate is. If the category is divalent it is possible to infer from the absence of one of its members to the presence of the other member, e.g. from the absence of the predicate 'false' to the presence of the predicate 'true', and vice versa (tertium non datur). But if the category is trivalent, then the absence of one of its members will merely allow an inference to the presence of the one or the other of the two remaining members. And in case the number of predicates in the category increases, the indefiniteness also increases.

As to polyvalent propositional logics, they are, in my opinion, either purely formal (uninterpreted) games, or dealing with predicates not belonging to the category true-false. If a third truth-value, e.g. 'indefinite', is introduced, the two other values should presumably not be named 'true' or 'false', but rather 'definitely true' and 'definitely false', and the subject of these predicates will then be the judging person's knowledge of the truth-values of the statements, not these statements themselves. Similarly, in case a propositional logical formalism with infinitely many truth-values is interpreted as a logic of probability, the extreme members of the cate-

gory of probability are not 'true' and 'false', but 'true with the probability 1', and 'true with the probability 0', and between these two extreme values all the other probability-values are situated. That the statement p has the probability m/n  $(m \le n)$  means that the predicate of p may be ascribed to n subjects, but that merely m of these subjects have this predicate, i.e. that p is true merely in mof the n possible cases. Or more exactly: when I ascribe the probability-predicate m/n to a statement p, then I am merely predicating of p that it belongs to a group of n statements of which m and not more than m are true. If m=n, then p has the probability 1, and it is possible to infer that p is true, because p must then belong to the group of true statements. But nevertheless the concept 'true' and the concept 'true with the probability 1' are quite different, and the last-mentioned of these concepts can merely be defined by means of the first-mentioned. And analogously as regards the concepts 'false' and 'false with the probability 0'.

Finally, I may add that the various categories of predicates mentioned in this paper are but a small selection of a vast number of such categories which have mutually numerous and complicated relations. As far as I know, this vast field has until now been but little investigated. Presumably the linguistic semanticists have done more in this direction than the logicians. And possibly such investigations can throw some light on the Hegelian-Marxist 'dialectics', which are actually a continuation of some of Aristotle's ideas that have been widely neglected by later logicians. If my modest suggestions in this paper could stimulate the interest in the problems here touched on, they would not have been written in vain.

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