

THE LAW OF NON-CONTRADICTION

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By 'The Law of Non-contradiction' I mean, of course, the Law of Contradiction; I use this terminology because the standard usage seems in principle misleading, although perhaps in practice it is innocuous. It is customary to distinguish a 'logical' version of the Law of Non-contradiction, to the effect that no proposition can be both true and false, from an 'ontological' version, to the effect that nothing in the real world requires to be described in such a way as to violate the 'logical' version. The 'ontological' version is often expressed by saying that no object can both have a certain property, and not have it. But of course, there is nothing peculiarly ontological about this statement in itself, since it could perfectly well be used to express a quantificational variant of the 'logical' version. If 'Not both p and not- p ' is a way of expressing the 'logical' version, so also is 'For all x , not both φx and not φx '.

Now there are consistent formal systems containing the theorem that every proposition is both true and false. An example is the one whose only axiom is ' p and not- p ' and whose only rule of inference is that if α is a theorem and β is a theorem, then α and β is a theorem. That this system is Post-consistent is shown by the fact not every proposition is deducible from the axiom by means of the rule of inference. That it is consistent in the classical sense follows from the fact that it has at least one model. For if ' p ' be interpreted as any integer n , 'not- p ' as $-n$, and 'and' as $+$, and if 'theorem' be interpreted as any expression having the numerical value of 0, then the axiom just stated is a theorem, and the rule of inference permits no propositions except theorems to be derived from propositions that are themselves theorems.

Thus if the 'logical' version of the Law of Non-contradiction means that there are no consistent formal systems that contain ' p and not- p ' as a theorem, that version is false. The falsity of the 'logical' version, as interpreted in this way, moreover, tends to raise questions about the 'ontological' version. For the region from which models must be drawn if they are effectively to test the consistency of systems is 'the real world'. But it is precisely 'the real world' to which the Law of Non-contradiction is alleged to apply. It follows that the very models that establish the consistency

of systems in which ' p and not- p ' is a theorem must be drawn from a region of which the 'ontological' version of the Law of Non-contradiction is alleged to be true.

Consider the model that I have already used as an example. This model is a set of integers n . The consistency of the system is established by the fact that in 'the real world', $n + -n = 0$. In particular, this fact exemplifies ' p and not- p '. But if ' p and not- p ' is a breach of the 'logical' version of the Law of Non-contradiction, then any situation in 'the real world' that exemplifies ' p and not- p ' is a breach of the 'ontological' version.

The following objection is now certain to arise: the fact that the integers n have the property that $n + -n = 0$ has no tendency to show that they both have a certain property and do not have it. In order to show that the 'ontological' version actually has exceptions, one would have to point to an object to which a certain property both belongs and does not belong; to an integer, for example, that is both even and odd.

This objection seems to me to rest upon a misunderstanding of the relation between a formal system and any of its models. It is unreasonable to expect a model to 'show' that anything at all is true or false; this is just not the function of a model. In particular, no model could possibly 'show' either that an integer both has a certain property and does not have it or the contradictory of this. Only a system is capable of 'showing' the truth or falsity of either proposition. The model is just a situation in 'the real world' interpreted as exemplifying the system. So if a system violates the 'logical' version of the Law of Non-contradiction, and a situation is discoverable that can be interpreted as exemplifying the system, then the situation itself violates the 'ontological' version of the Law of Non-contradiction, in the only way in which that version can be violated or satisfied at all. It may be that a situation interpretable as exemplifying a system that violates the Law of Non-contradiction can also be interpreted as exemplifying *another* system that does *not* violate the Law of Non-contradiction. This possibility is illustrated, in fact, by the objection to which I am now trying to reply, since that objection implies that it is possible to interpret the property that $n + -n = 0$ as exemplifying a system that does not violate the Law of Non-contradiction. But from the fact that this interpretation is possible, it does not follow that it is necessary.

The remarks I have made so far seem to raise questions about both the 'logical' and the 'ontological' versions of the Law of Non-contradiction. Is there any further version of this Law that is not

open to such questions? A possibility is suggested by the role of *consistency* in what I have said so far. My whole discussion presupposes the existence of *consistent* systems that violate the 'logical' version. Indeed, the entire range of logical enquiry presupposes the same thing. If consistency is impossible; there is no point to doing logic. So perhaps the Law of Non-contradiction operates as a norm rather than as a general description of formal systems or of the 'real world'. In this version, it might be expressible as the imperative, 'Avoid inconsistency!' I shall call this version 'regulative'.

The difficulty with this version is that it seems hopeless to attempt to specify what it is that one is to avoid. If finite models were the only ones that could be used to establish consistency, one could adequately formulate this version by saying, 'Avoid systems for which there are no finite models!' But there are consistent systems the establishment of whose consistency requires infinite models. Nor can this difficulty be overcome by simply deleting the word 'finite' from the formulation I have just given. For when a system is unlike those for which models are acknowledged to exist, no one is entitled to deny *a priori* the possibility of suitable models of a new kind, drawn from the 'real world' in a different way from that in which the existing models are drawn from it. In other words, the injunction to avoid systems for which there are no models is useless, because in advance of the construction of novel systems it is not clear what this injunction permits or rules out. If, in the effort to make it clear what is permitted and what is ruled out, one were to state a criterion for the suitability of any possible model, this would simply have the effect of halting progress in the construction of new formal systems at a limit that could not be shown to be other than entirely arbitrary.

If the remarks I have made so far are cogent, they may be taken to recommend a certain scepticism with regard to the Law of Non-contradiction in all the versions I have considered. But scepticism in itself does not constitute an adequate assessment of the Law of Non-contradiction. This Law has been so generally regarded as a fundamental axiom of reasoning that whoever doubt its general applicability or usefulness ought to try to be very sure that he has understood its real point. It is all too easy to criticize where one misses the point.

The point, as I see it, is that the Law of Non-contradiction functions not as a statement about logical systems or about 'the real world', nor yet as a directive, but rather as a theory of possible evidence. The person who objects that ' $n + -n = O$ ' does not vio-

late the 'ontological' version supposes that it is *necessary* to interpret the equation just given as exemplifying a system that does not violate this version. Why does he suppose this? He does because he supposes that no report of evidence could possibly take the form '*p* and not-*p*', by the very nature of evidence. But anyone who takes such a position with regard to the scope and limits of possible evidence must sooner or later use this position itself as the criterion according to which he stigmatizes alleged evidence against this position as not constituting evidence at all. After all, if a person who supposes that evidence of a certain type is impossible permits himself to be worried by evidence of this very type, that raises questions as to whether he really does suppose that evidence of this type is impossible. If he is to be true to his own theory of evidence, he must refuse to accept what it requires him to refuse to accept. Of course, the position taken by the person who defends the Law of Non-contradiction is only one of a large number of theories of possible evidence that have been adopted in the course of the history of philosophy. If I mention just one more, it will be easy to extrapolate to many others. The one I have in mind is the theory that evidence in the form of abstract entities is impossible. It is the standard — and necessary — strategy of the nominalist, who maintains this theory in one form or another, always to reinterpret any alleged evidence of abstract entities in such a way that the entities turn out no longer to be abstract. He uses his own position as the criterion for determining whether what is offered in evidence against his own position is evidence at all. If he did not do this, he would not be a nominalist.

Perhaps, then, we ought to abandon all theories of possible evidence, and simply be open to evidence in whatever form it may come: abstract or concrete, self-contradictory or consistent. I am willing to concede the possibility of such a theoryless position. But the position must take the shape of a theory as soon as it is challenged. If it ignores the challenge, this can only mean that it regards the challenge as falling beyond the scope of the evidence with which it is obliged to deal, so that the position really was at least implicitly a theory of evidence after all. But it can meet the challenge only if it can formulate the contention that the evidence to which it is open is always good evidence. The result of this formulation will be a theory of evidence. All that needs to be added is that any position that succeeds in avoiding commitment to a theory of possible evidence *will* be challenged, because in accepting evidence of all kinds it is sure to accept evidence that some people

will call unreal. A theoryless position is possible only if there are no theories of evidence. But there are theories of evidence. Therefore, a theoryless position is impossible. Therefore it is necessary that there should be theories of evidence.

The Law of Non-contradiction, then, can be regarded as asserting a theory of evidence. The question arises whether this is not just the 'ontological' version over again. Evidence, after all, must be drawn from 'the real world'. So when one claims that only evidence of a certain type is possible, is he not *ipso facto* characterizing 'the real world' in just such a way as it is characterized by the 'ontological' version? The difference is that the latter is a report allegedly based on actual evidence, not a theory of possible evidence. It is the claim that 'the real world' is actually such that no object in it can both have a certain property and not have it. This claim presupposes that evidence to the contrary is possible, even though never actual. Thus the 'ontological' version could, in principle, be disconfirmed through counter-evidence, while the present version could not.

Of course, not much is said about the nature of possible evidence by the mere statement that it is not self-contradictory, so that it may be more accurate to think of the Law, in its role as a theory of possible evidence, as a principle common to many theories of evidence, none of which is a full theory unless it asserts other principles regarding possible evidence as well. This point does not concern me now. I am more concerned at the moment to argue that the position I have reached so far is a more adequate account of the status of the Law of Non-contradiction than any treatment of it as a 'logical', 'ontological', or 'regulative' principle. For I think I have shown that if the Law had any of these roles, it would either function as a false statement ('It is a fundamental theorem common to all formal systems...' or 'The real world is such that...') or as an otiose directive ('Avoid systems without models'). But if the Law functions as a theory of evidence, then it at least cannot be shown to be false in any ordinary sense of the word 'false', since this would require an appeal to evidence of a type explicitly rejected by the theory under attack. In addition, it is clear that a theory that enables one to decide what evidence is possible must be far from otiose. Aristotle was certainly right to regard the Law of Non-contradiction as among those principles constituting 'what a man must know if he knows anything', although if he held that the Law is a general principle of logic or ontology, he was wrong. If Aristotle made any sound recommendations about dealing with

people who ignore this Law, they do not pertain to its 'logical', 'ontological', or 'regulative' versions.

Aristotle comes into the picture here because his recommendations suggest a way out of a quandary implicit in the line I am taking. To make the quandary explicit it is sufficient to repeat that it would be impossible to show the falsity of the Law of Non-contradiction as a theory of possible evidence, and to add that for a precisely similar reason it would be impossible to show the falsity of the contradictory of the Law in this role. In either case, the attempt would involve a *petitio*. For that matter, any attempt to show the *truth* of either this version of the Law or its contradictory would be the question, since it could not but appeal to the theory to be proven in order to find a sanction for the very evidence used to prove the theory. So the situation is even worse than that envisaged by Aristotle at the beginning of his discussion of the Law of Non-contradiction. It is not merely an impasse in which the responsible thinker is unable to use any deductive means of showing Heraclitus and Protagoras that they are ignoring one of the basic axioms of responsible thinking. It is rather a nightmare in which Heraclitus and Protagoras are, after all, fully as responsible in their thinking as is Aristotle himself. Indeed, those who ignore the Law of Non-contradiction may even be a little more responsible than Aristotle, since their position is capable of endorsing its own contradictory as well as itself, instead of being obsessively concerned with evidence of just one type. In any event, we seem to be left with two radically distinct positions, each denying the other, but separated by an abyss such that each is wholly unable to touch the other. Aristotle's recommendations seem to me to offer the only possible hope of crossing the abyss between the positions. These recommendations, furthermore, appear to be no less useful as means for dealing with the situation that I envisage than they are as means for dealing with the more limited situation that Aristotle envisaged. Of course, by 'dealing with the situation', I mean 'crossing the abyss', not 'establishing the Law of Non-contradiction'. Aristotle's recommendations, in my view, make it possible to cross the abyss in both directions, and thus render Aristotle just as vulnerable to criticism by Heraclitus and Protagoras as they are to his criticism. But it is precisely the vulnerability of the position that defends the Law of Non-contradiction which needs to be shown. The whole trouble so far has been that a defendant who can always interpret evidence against his position as constituting evidence for it seems *invulnerable* to refutation.

What Aristotle recommends is to find a way of reminding one's antagonist that he is *already* committed to the Law of Non-contradiction (and does not merely endorse it as a gratuitous consequence of having endorsed its contradictory). Point out that he does not in fact suppose that it is every bit as worth-while to walk to Megara as not to walk there. Call it to his attention that his success in making you understand anything that he has said presupposes that he has not used language in such a way that a given adjective can both qualify a given noun and not qualify it. Even though he has, in effect, denied the possibility of non-self-contradictory discourse, his use of it is an admission that it exists. If his denial of the Law arises from the belief that change is ultimately real, get him to see that the reality of change actually implies the truth of the Law, because if all things already had all properties, there would be no way in which anything could change *into* anything else. (On the other hand, the philosopher who doubts the ultimate reality of change cannot deny the Law in order to reach this conclusion, since, as Aristotle shows elsewhere, it is precisely his trust in the Law of Non-contradiction that gives rise to this doubt). Aristotle's own summary of his recommendations is that 'he who wants to convince an opponent who makes opposite statements that he is wrong must obtain from him an admission which shall be identical with the proposition that the same thing cannot at one and the same time be and not be, but shall seem not to be identical with it' ⁽¹⁾. If it seemed *identical*, of course, the opponent could not even be *claiming* to deny the proposition.

Recommendations of other types are also made by Aristotle in this connection, but they do not seem very useful to me. The dialectical ploy of telling one's opponent that if all propositions are both true and false, then the Law of Non-contradiction itself is true as well as false is likely to backfire, because, as I noted before, the reply may be that only this tolerant inclusiveness avoids an obsessive preoccupation. It is nearly as much of an *ignoratio elenchi* as it would be to accuse one's opponent of being inconsistent — a rejoinder that Aristotle sometimes seems to be suggesting. Nor does it help to go patiently over the opponent's arguments with him. When Aristotle pronounces that sensory phenomena do not constitute evidence against the Law of Non-contradiction, because no sense contradicts itself at the same moment about the same object, his opponent is always in principle able to name a respect in which a sense does

⁽¹⁾ *Metaphysics*, Bk K, 1062^a6-9. Tredennick translation.

contradict itself at the same moment. The dialectic of this discussion has been plotted by Nagel in 'Logic Without Ontology' ⁽²⁾.

On the other hand, the recommendations that are actually summarized by the summary I quoted are so effective that by acting upon them one could cogently criticize Aristotle's own advocacy of the Law of Non-contradiction. I would only be necessary to wring from him some admission tantamount to a denial of the Law. I would be inviting irrelevant scholarly controversy if I were to try to specify exactly what admission ought to be wrung from him (similarly, scholarly controversy invited by Aristotle's characterizations of the positions of Heraclitus and Protagoras has tended to obscure what Aristotle was actually recommending). It is probably innocuous enough, however, to remark that a philosopher for whom form and privation are fundamental categories of change is at least skating on thin ice. Also, it might be difficult to make good on a doctrine of potentiality without permitting certain perhaps rather unobtrusive contradictions. The perpetrators of 'the Megaric Fallacy' may have had a point, after all.

The abyss, then, can be crossed in both directions. This is not, however, to deny that it is an abyss. Indeed, it is only because the abyss can be crossed that it is an abyss, rather than an inter-galactic space. If the position that endorses the Law of Non-contradiction and the position that denies it were totally unable to affect each other, each would be without importance. It is important to assert this Law, or its contradictory, only because it is important to rule something out. A position incapable of ruling anything out is also incapable of asserting anything. But merely to say, for example, that self-contradictory evidence is impossible, is not to rule out evidence of this sort. To rule it out is effectively to attack the position that maintains its possibility. This means crossing the abyss.

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⁽²⁾ *Logic Without Ontology*, reprinted from Y. H. KRIKORIAN, *Naturalism and the Human Spirit*, in FEIGL and SELLARS, *Readings in Philosophical Analysis*, New York, Appleton-Century-Crofts, Inc., 1949.