DECISION PROCEDURES FOR STRUCTURE IN NATURAL LANGUAGES (1)

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The rules of formation of a logistic system are by definition (2) such that the notion of formula, well-formed formula or sentence, determined by these rules, is effectively decidable. However, I am not convinced that the arguments brought forth by Church (3) to the effect that sentencehood *has* to be an effectively decidable notion for *any* system that may be used for communication purposes are conclusive. I therefore regard it to be a serious problem whether the syntactic structure of a natural language such as English can always be adequately described by a set of formation rules that guarantee the decidability of the notion of sentence, or for that matter, of any syntactical structures such as phrases etc. Inasmuch as there exist good reasons for doubting whether the answer to this problem is affirmative, the prospects for completely automatic, high-quality machine translation from one natural language into another natural language look dimmer than many workers in the field of machine translation would like to think. This since not even one necessary, though by no means sufficient, condition for this process, namely the mechanical determination of the syntactical structure of any given sentence in the source language, could possibly be completely fulfilled. Though applicability to machine translation is often in the back of my thinking on the description of the syntax of natural languages, I shall refer here no longer to this application, having dealt with it elsewhere at some length (4).

The seriousness of our problem has apparently not been sufficiently recognized so far because many linguists explicitly, and most, if not all, of them as well as most logicians, implicitly believed that the syntactical structure of natural languages is adequately describable by an *immediate constituent model*, or a *phrase structure model*

(1) Report presented at the Colloque de Logique, on September 6th 1958.

(2) See, e.g., A. CHURCH, *Introduction to mathematical logic*, I, Princeton, 1956, p. 51. There exist, however, less demanding conceptions.

(3) *Ibid.*, p. 53.

(4) In *Some linguistic obstacles to machine translation*, forthcoming in the Proceedings of the Second International Congress of Cybernetics, held in Namur, September 1958.

according to the term recently introduced by Chomsky (1). It is indeed true that if natural languages were adequately describable in terms of such a model, there would exist a decision procedure for structure, as I have shown in effect, though not with full rigor, in a paper published some five years ago (2).

Before I proceed to present some arguments for the fact that the phrase structure model is not fully adequate, let me spend some time in presenting again, in briefer and I hope improved form, an informal outline of this proof. The basic idea behind the immediate constituent model is that every sentence can be regarded as a result of the operation of one continuous part of it upon the remainder such that those constituent parts which in general are not sentences themselves, but rather phrases, are themselves again the product of the operation of some continuous part upon the remainder, etc., until one arrives at the final constituents, say words or morphemes. To illustrate:

Young John slept soundly

would be regarded as the result of the operation of *slept soundly* upon *young John*; *slept soundly* in its turn wold be considered the result of the operation of *soundly* upon *slept* and *young John* the result of the operation of *young* upon *John*. All this so far is nothing but reformulation in somewhat unfamiliar terms of the procedure well known from school days as parsing. As linguists put it, *young John* and *slept soundly* are the immediate constituents of the sentence under discussion, *young* and *John* the immediate constituents of the first immediate constituent of the sentence, *slept* and *soundly* the immediate constituents of he second immediate constituent. Hence all together *young, John, slept* and *soundly* are the final constituents of the given sentence.

Another basic feature of the model is that all operator constituents must be contiguous with their argument constituents. Both these features are exemplified in our illustration, but this of course is by no means a proof that this model can be carried through all of language. On the contrary, linguists have realized that occasionally discontinuous constituents have to be taken into account, but they seem to have believed that these were exceptions which did not se-

(1) See N. Chomsky, *Three models for the description of language*, IRE Transactions on Information Theory, Vol. IT-2, N° 3 (1956) and *Syntactic Structures*, 's-Gravenhage, 1957.

(2) *A quasi-arithmetical notation for syntactic description*, Language 29: 47-58 (1953).

riously affect the validity of the model with which they were used to work.

In most language systems invented by logicians, the two mentioned features were automatically incorporated into their respective rules of formation. The problems arising in connection with discontinuous expressions were, to my knowledge, never explicitly discussed by logicians.

According to the immediate constituent model, every word — and we shall for our purposes consider words to be the basic syntactical elements — of a natural language belongs to one or more syntactical category. Among these categories some will be *pure argument categories,* by which term I denote a category whose members always serve as arguments and never as operators, as well as *operator categories* whose members may operate upon other words though they may perhaps also be operated upon by other operator expressions. *John,* for instance, in as much as it belongs to the syntactic category of *nominals,* is always an argument and never an operator. *Slept,* inasmuch as it belongs to the category of *intransitive verbals,* may operate upon a nominal such as *John* to form the sentence *John slept,* but may also be operated upon by the adverbial *soundly* to form the intransitive verbal expression *slept soundly.* A word may belong to more than one category not only because it may be regarded as homonymous — as would be the case with regard to *sleep,* which clearly belongs to the category of nominals as well as to the category of intransitive verbals — but also because, for instance, many adverbials operate upon intransitive verbals as well as upon transitive verbals: *soundly,* for example, in the sentence

Belgium soundly defeated the Netherlands

(in the last soccer game, of course) operates upon the transitive verbal *defeated,* forming the transitive verbal expression *soundly defeated,* and has therefore a different kind of argument as well as a different kind of value than has *soundly* when operating upon *slept.*

In order to exhibit the decision procedure or constituent structure let us denote, following Lesniewski and Adjukiewicz ([1]), the category of nominals by «*n*» and the category of declarative sentences by «*s*». (Since I am engaged in presenting an outline only, I shall not here go into the very difficult question to what degree these two argument

([1]) See K. ADJUKIEWICZ, *Die syntaktische Konnexitaet,* Studia Philosophica, 1:1-27 (1935-36); cf. A. A. FRENKEL and Y. BAR-HILLEL, *Foundations of set theory,* Amsterdam, 1958, pp. 169-170.

categories would have to be refined and expanded in order to get even the beginnings of a reasonably working model.) Operator categories will be denoted by symbols that will indicate both the categories of their arguments and the category of the resulting expression. In addition, since arguments may be positioned either at the immediate left or at the immediate right of their operator, these positions too will have to be indicated in the symbolism. Therefore, I shall, for instance, denote the category of *slept* by «$n\backslash s$» — read: *n* sub *s* — and the category of *young* by «n/n» — read: *n* super *n* [1] —, where the direction of the slash indicates in an obvious fashion whether the argument is to the left or to the right. *And*, for instance, *qua* sentence connective will be assigned to the category $s\backslash s/s$, since in this function it is a word that out of a sentence to its immediate left and a sentence to its immediate right forms a sentence. *Soundly* will belong to the categories $(n\backslash s)\backslash(n\backslash s)$ — to be abbreviated in a self-explanatory way as $n\backslash s\backslash\!\backslash n\backslash s$ — and $n\backslash s/n/\!/n\backslash s/n$ — as well as to a few other categories.

Assume now that we have a complete category list of all English words, i.e. a list which gives all the categories to which every English word may belong. In order to arrive by a completely mechanical procedure at the constituent structure of any given English sentence, one would only have to copy from the category list the category symbols for all the words in this sentence, write them down in columns and go to work on them according to the following rule: *Replace a sequence of three symbols, having respective the form* α, $\alpha\backslash\beta/\gamma$ *and* γ, *with* β. This rule comprises as limiting cases the following two subrules: (1) Replace the sequence of symbols of the form α and $\alpha\backslash\beta$ by β. (2) Replace the sequence of symbols of the form β/γ and γ by β.

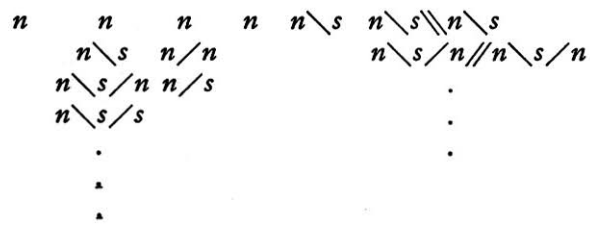
Instead of going into a detailed but rather obvious description of the decision procedure let us illustrate through a somewhat more elaborate example. Assume that the word sequence to be tested for sentencehood as well as for its constituent structure is

Paul thought that John slept soundly.

Assume further that copying from the category list yields the following result:

[1] In the paper mentioned in note 1, p. 18, I used a less convenient symbolism. The present symbolism is due to J. LAMBEK, *The mathematics of sentence structure*, American Mathematical Monthly 65:154 (1958).

Paul thought that John slept soundly



(the three dots indicating that the complete list would probably contain further entries which shall, however, be here disregarded for the sake of simplification). The reader will do well to envisage contexts in which *thought* and *that* will belong to each of the given categories. He might as well try to find out to which categories *thought* would belong in such contexts as *John had thought of ...*, *... thought processes,* and *... thought provoking ...* .

Now, taking into account only the categories explicitely indicated we have twenty-four initial symbol sequences to which we will apply our rule. Starting for instance with

$$n \quad n \quad n \quad n \quad n\backslash s \quad n\backslash s\backslash\backslash n\backslash s$$

we see that sub-rule (1) can be applied for the fourth and fifth symbols, yielding s. The resulting sequence is now

$$n \quad n \quad n \quad s \quad n\backslash s\backslash\backslash n\backslash s,$$

which obviously cannot be further operated upon. The same sub-rule operating upon the fifth and sixth symbols yields $n\backslash s$, hence the sequence

$$n \quad n \quad n \quad n \quad n\backslash s,$$

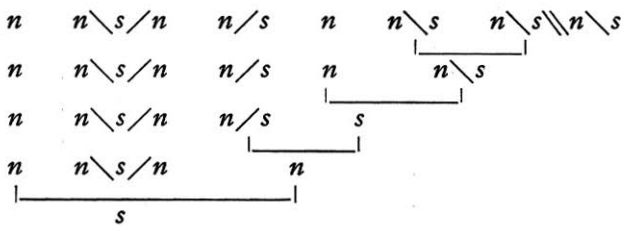
which has once more to be operated upon by the same sub-rule, yielding

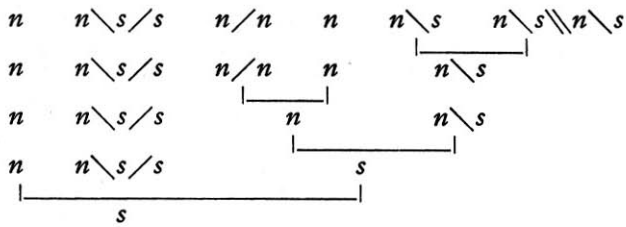
$$n \quad n \quad n \quad s,$$

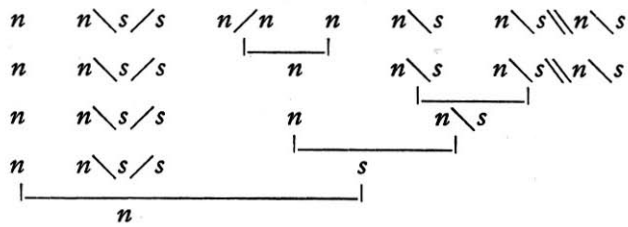
which cannot be processed any further.

Performing these operations upon all the twenty-four initial symbol sequences through all possible continuations, we would find that there exist exactly three *derivations* as we shall call columns of symbol sequences each of which (with the exception of the first, of course) results from the preceding line by one application of the rule — whose final line, or *exponent,* consists of a single symbol which in both cases is «s».

Here are the derivations:





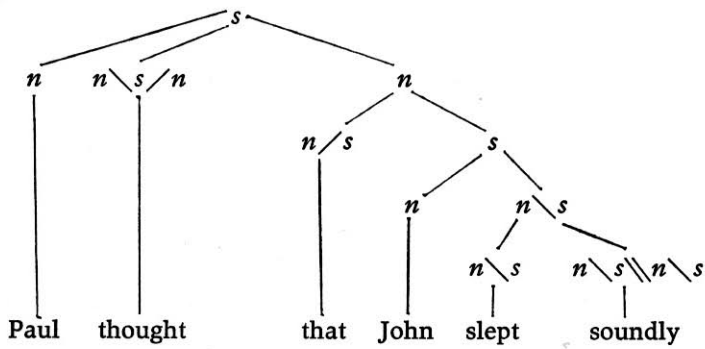


The last two derivations being equivalent, in a rather obvious sense of the word, we have only two essentially different derivations before us, indicating, probably to the surprise of many readers — and to my own surprise some six years ago when I came across this situation simulating a machine processing of this illustration —, that the sentence under discussion is *syntactically ambiguous* or *constructionally homonymous*. The reader will do well to read out aloud this sentence according to its two essentially different constituent structures which in this case make the sentence also semantically ambiguous as such, though one constituent structure is much less likely to be used than the other.

I hope that this illustration is sufficient to show that under the essential and, as we shall see, highly problematic assumption that a complete and completely adequate category list is available, there exists indeed a wholly mechanical procedure to determine whether

a given word sequence is a declarative sentence under one of its constituent structures as well as what all of its constituent structures are.

For certain purposes it is worthwhile to look upon our derivation procedure upside down, i.e. to deal with *expansion* rather than with derivation. The expansion corresponding to the first derivation exhibited above of our sample sentence would look like the following tree:



(Two derivations, by the way, are equivalent if they correspond to the same tree.)

How well, then, does the immediate constituent model work? Apparently quite well for relatively short sentences such as those discussed so far, but even there not too well. The number of categories to which the English words will have to be assigned to make the category list reasonably adequate will occasionally have to be rather large, and the categories themselves rather complex. In addition, it is quite clear that not only will one have to work with highly complex refinements of the categories mentioned so far in order to take care, for example, of the fact that *John sleeps* is a sentence but not *John sleep,* but that one will also have to refine the sentence category and distinguish between declarative sentences, imperative sentences, yes-or-no question sentences, *wh*-question sentences, etc., these various types not being reducible to each other under our model. These refinements may result in such piling up of category symbols assigned to the words occurring in a given sentence that the number of derivations would easily run into the trillions, hence be beyond the practical capacity of even the fastest electronic computers. For instance, if the average number of categories of the twenty words of a given English sentence is four, we will have 4^{20} initial lines only,

and a still enormously higher number of derivations. This means, then, that the indicated method of mechanically resolving the syntactical structure of any given English sentence would certainly be impractical as such. However, were it the case that this is still a theoretically adequate method, one could think of certain improvements which would reduce the required number of operations by many degrees of order. Unfortunately, however, the actual situation seems to be much worse. It is not only a matter of practicality, but it seems that the whole model is just not good enough. Already six years ago I was worried by sentences such as

John, unfortunately, slept soundly

which, so it appears at least, cannot be handled by a model incorporating the two above mentioned basic features. Notice that there is no trouble with the slightly different and semantically, though perhaps not stylistically, equivalent sentence

Unfortunately, John slept soundly.

Assigning *unfortunately* to the category s/s — a wholly natural and intuitive assignment — we arrive at an adequate syntactical analysis. This assignment, however, clearly does not work for *John, unfortunately, slept soundly* as the reader will easily verify for himself. It is of course possible that some other less natural category assignment to *unfortunately,* perhaps combined with some ingenious treatment of the commas (which so far have been completely disregarded in the immediate constituent model), would do the trick. It seems, however, unlikely that such an assignment could be made in a fashion which would not be almost entirely *ad hoc.* And this would not only be esthetically and methodologically repugnant, but also in all likelihood have unpleasant repercussions inasmuch as word sequences which intuitively would not be regarded as grammatical sentences would have derivations with an exponent s.

A similar situation, but even simpler since no commas are involved, arises with regard to the word sequence

He looked it up.

Regarding *he* and *it* as belonging to the categories n — leaving aside once more the clearly required refinements —, *looked* as belonging to the category $n\backslash s/n$, as seems natural, it looks highly implausible that any category assignment of *up* which would not be woefully *ad hoc* would insure the sentencehood of the given word sequence. Assigning *up,* for instance, to the category $s\backslash s$ would obviously result

in a derivation with an exponent s, but this unnatural saving of the phenomena would immediately retaliate with the unwanted imposition of sentencehood to such sequences as

He went home up.

(For further examples of the breakdown of the phrase structure model see Chomsky's *Syntactic Structures* (1), to which I owe much of the present argument.)

Every English speaker, I presume, feels that in our sentence

He gave it up

gave and *up* belong somehow together. Indeed, there is no trouble with such a sequence as

He gave up this idea,

as the reader will easily verify for himself if only *up* is assigned in a completely intuitive fashion to the category $n\backslash s/n\backslash\backslash n\backslash s/n$. This being so, assigning *up* to a different category, whatever it may be, in the sentence

He gave it up

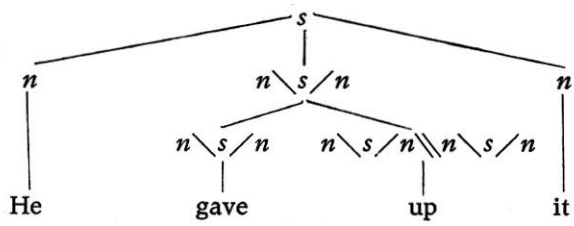
looks now even more artificial than before.

These simple facts indicate, though it cannot be said that they prove in the strong sense used in mathematics, that the immediate constituent model is not an adequate one as such, but has to be supplemented in one way or another.

Let me finish this discussion by presenting a very brief outline of one such supplementation, referring the reader for a fuller discussion to Chomsky's mentioned book and other publications of his (2). The new model, called the *transformational model,* assumes that sentences are generated not only by the procedure we called above expansion, but also, in addition, by so-called *transformations.* Once such transformation, for instance, would transform the so-called *terminal string* of the following expansion

(1) See above, note 4, p. 17.

(2) Viz., to those mentioned above in note 4, p. 17, as well as, for instance, to a forthcoming paper, *A transformational approach to syntax.*



i.e. *He gave up it*, which is, of course, not an English sentence, into *He gave it up* by a certain *obligatory* transformation. This transformation rule, which states in effect that in certain environments certain word sequences have to be turned around, is clearly beyond the reach of an immediate constituent model. On the other hand, this way of looking at how the sentence *He gave it up* was generated has a rather natural appearance, and might well correspond, at least in spirit, to the way old-fashioned, traditional grammar has dealt with the situation.

Other transformations transform two terminal strings into one sentence. One, of these, for instance, would operate upon the sequence of the two terminal strings (which are in this special case sentences in their own right)

Paul thought it. John slept soundly.

and turn this sequence into the sentence

Paul thought that John slept soundly.

This very same transformation operates upon the sequence

Paul thought it. That John slept soundly.

and transforms it into

Paul thought that that John slept soundly.

Yet another transformation to the effect that under certain determined conditions *that* may be omitted, hence an *optional* transformation, would transform this last sentence into

Paul thought that John slept soundly.

This way of looking at the situation results now in a natural and adequate explanation of the constructional homonymy of the last sentence. We also realize, by the way, that transformation may operate upon the results of prior transformations.

The linguists such as Harris, Chomsky and their associates who are at work at the development of this new kind of model (1) have already unveiled a large number of transformations amounting to many hundreds, in English. It is, however, quite clear that the transformations introduced so far are not yet sufficient to account for all intuitively possible English sentences. It is at this state that the question mentioned at the beginning of this paper arises — *whether there exists a decision procedure for structure in English,* or in other natural languages for that matter, since it is unlikely that the natural languages should differ among themselves in this respect. Obviously the answer to our question will depend upon the exact nature of the transformations. Only when we will have a better and more extensive understanding of the kind of transformations at work, will we be in a position to fruitfully attack our problem. At this moment one could only speculate about this answer, and it is doubtful whether such speculations would be worthwhile. In any case, even the possibility that for a certain set of formation rules in English the notion of English sentence would not be a decidable (or general recursive) one seems exciting enough to warrant an increase in interest in our problem among mathematical logicians, who by training are in many respects in a better position to attack it than are linguists. Chomsky has already been able to show that there exist highly interesting connections between the theory of linguistic models and such theories as the theory of automata, recursive function theory (perhaps especially conspicuous in the form of the theory of algorithms) and the theory of Post cannonical systems. This multiple relationship indicates that we have, in all probability, in the theory of language models an interesting new field in which cross-fertilization of mathematical logic and structural linguistics should lead to important results.

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(1) In addition to Chomsky's publications, see Z. Harris, *Cooccurrence and transformations in linguistic structure,* Language 33:238-340 (1957) and the excellent review of Chomsky's *Syntactic structures* by R. L. Lees in Language 33:375-408 (1957).