THE LOGIC OF NORMS (*)

In previous pubications, (1) I have outlined some systems OX of symbolic logic, designed for the formal analysis of *obligation*, *permission*, *prohibition*, and related deontic concepts. These systems were devised mainly in the hope of providing for normative or prescriptive discourse the same sort of formal logic as is provided for scientific or descriptive discourse by the more familiar systems. Just how useful our analyses will be is of course a pragmatic question, to be settled by further study. But viewing the formal systems as explications, or « rational reconstructions » in the sense of Carnap, we can at least suggest some criteria of adequacy which we would expect them to meet.

In the first place, there are requirements of syntactical rigor. The achievement of rigor is of course one of the primary motivations for the study of formal systems: we like to know with as much exactitude as possible just what we are committing ourselves to, i.e., just what consequences our assumptions have. From this point of view, the systems OX are entirely unexceptionable. They are formulated in a manner familiar to students of symbolic logic, and meet current demands for rigor in precisely the usual ways.

Secondly, it is to be hoped that the systems will prove pragmatically useful. As already remarked, this is a question for time to tell. Still, it is worth noting that an attempt is being made to apply these systems in the analysis of empirical findings in sociology. (2) Most contemporary logicians feel that the more familiar systems have been of use in analysing conceptual issues in the philosophy of science. At the moment we can do no more than hope for similar applications of OX in the behavioral sciences, but research in this direction is under way, and gives promise of leading to useful results.

Finally, there are semantic criteria, which will form the principal topic of this paper. Of these we may mention at least the following two:

(a) There should be substantial and important agreement between the

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(1) *The formal analysis of normative systems*, Technical Report No. 2, Office of Naval Research Contract No. SAR/Nonr-609(16), New Haven, 1956. For a discussion of applications of these systems in the behavioral sciences, see Alan ROSS ANDERSON and Omar KHAYYAM MOORE, «The formal analysis of normative concepts», *American sociological review*, vol. 22 (1957), pp. 9-17. Some variant systems based on the approach described here are discussed by A. N. PRIOR in Appendix D of *Time and modality*, Oxford, 1956.

(2) Principally in research directed by Prof. MOORE, under the ONR Contract cited above. Mention should also be made of Mr. LAYMAN ALLEN's studies in the logical analysis of positive law, in the *Yale Law Journal*, vol. 66 (1957), pp. 833-879.

« logical truths » deduced in the formal systems, and our intuitive ideas of the logical relations under investigation. Of course we should not expect to find complete accord between the formal systems and our logical intuitions; recent developments in logic and mathematics provide abundant examples of theorems, issuing from perfectly innocuous-looking assumptions, which are neverthless offensive, at least to our most untutored intuitions. The much-discussed « paradoxes » of material and strict implication are examples, and, on a more sophisticated level, so are the Löwenheim-Skolem theorem and the Banach-Tarski theorem. Still, these are « paradoxes » only in the sense that they are or might be psychologically unexpected results, not in the sense that they lead to formal inconsistency; and in the case of the implicational paradoxes and the Löwenheim-Skolem theorem, it is easy to explain the counter-intuitive appearance away. ([3]) In general, the intuitive plausibility of the overwhelming majority of known theorems in the familiar systems has led most contemporary logicians to re-educate their intuitions so that the counter-intuitive appearance vanishes. In any event, the familiar systems of « descriptive » symbolic logic satisfy at least our more sophisticated intuitions, and we would expect an adequate logic for « prescriptive » discourse to do likewise.

(b) The formal systems should make no commitments of, roughly speaking, a non-logical character. The familiar systems of descriptive formal logic are usually conceded to be neutral to « matters of fact. » Formalization of competing scientific theories does not give one or the other an edge as regards confirmation; it simply makes clearer the logical relations of propositions within each theory. We would expect that a satisfactory formal logic for normative discourse would be just as neutral to important issues in morals and ethics, at least as among those positions which place any premium at all on the explication of moral concepts. The function of such a system, that is, would be to explicate *logical* relations, not to make moral or ethical commitments.

It will be the object of this paper to argue that the systems OX satisfy these semantic criteria. The discussion to follow does not presuppose detailed familiarity with the systems; we will first give a brief characterization of the leading ideas, and then proceed to their defense.

Surely most philosophers would agree that there is *something* wrong with not doing one's duty. Put more explicitly, if x ought to do A, but does not, then something, somewhere in the universe, is the worse off for it. Just *what* is worse off is of course a moot point. For some philosophers the « thing wrong » is pain, for others it is the disapproval of God, for others a deterioration of

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([3]) The implicational paradoxes are discussed in any standard text. For the Löwenheim-Skolem theorem, see John MYHILL's clear account in «On the ontological significance of the Löwenheim-Skolem theorem», in *Academic Freedom, Logic, and Religion*, ed. by Morton White, Philadelphia, 1953.

the character or the will, for others a diminution of the totality of value in the universe, and so on. Of course in some cases, notably statements of positive law, the « thing wrong » is explicitly indicated to be a punishment of some sort. But presumably in any normative system, formal or informal, expressed or tacit, something untoward is held to attend failure to do one's duty.

Moreover, the connection between failure to do one's duty and the attendant disvalue is in some sense a « necessary » one. The pain which follows a Hedonist's lapse is a necessary causal consequence of his failure in his duty, and it is no accident, according to positive law, that a fine is imposed for overparking. The disvalue or « Bad state-of-affairs » is entailed, logically or nomologically, by failure to fulfill obligations.

For purely formal investigations we may ignore the precise character of this « bad state-of-affairs, » simply labelling it « S », and referring to it as a « sanction, » or « penalty, » or « disvalue ». We may note parenthetically that this use of « sanction » is in accord with that common among behavioral scientists; but we do not mean the term here to carry with it, necessarily, the idea of a socially inflicted punishment. The propositional constant «S» is to stand for whatever disvalue is associated with failing to carry out an obligation, and what this disvalue is varies from case to case, depending on the particular normative structure or set of rules in question.

Regardless of the particular sanction involved, however, it is clear both in ethical discourse and social practice that it must be avoidable, or at least thought to be so. If the sanction is to have any force, it must be the kind of thing that might reasonably motivate behavior. Of course we are in fact motivated by lots of considerations besides the hope of avoiding sanctions, social or otherwise; but unless we thought we *could* avoid them, they could not operate as motivating factors.

The fundamental idea is then that a state-of-affairs p is obligatory if and only if its failure leads necessarily to an avoidable disvalue, and this idea may be clothed in a formal system in the following way. To some standard system X of alethic modal logic (logic of possibility and necessity ([4])), we add a propositional constant «S» expressive of the «thing wrong,» or «sanction,» and our sole axiom governing «S» says that the sanction is avoidable, i.e., possibly false. Using the usual notation, we have the

Axiom: ([5]) $\Diamond\sim S$.

Deontic modes are then defined as follows;

Df. O. «Op» is short for «$\sim p \prec S$.»

Df. F. «Fp» is short for «$O \sim p$.»

Df. P. «Pp» is short for «$\sim F p$.»

([4]) The systems of LEWIS and LANGFORD, *Symbolic logic*, New York, 1932, or G. H. von WRIGHT, *An essay in modal logic*, Amsterdam, 1951, for example.

That is, a state-of-affairs p is obligatory («O»), relatively to the normative system whose sanction is S, if the falsity of p entails (strictly implies) the sanction; it is forbidden if its falsity is obligatory; and it is permitted if it is not forbidden.

With this axiom and these definitions, numerous theorems of deontic interest arise in any of the standard systems of alethic modal logic. For example, the following principles suggested by von Wright ([6]) are provable:

Th. 1. $Pp \vee P \sim p$. (Either p is permitted, or not-p is permitted.)

Th. 2. $P(p \vee q) \equiv (Pp \vee Pq)$. ($p$ —or—q is permitted if and only if p is permitted or q is permitted.)

It follows that von Wright's *Deontic logic* is a subsystem of the systems OX. But in OX we can prove theorems not available in von Wright's logic, for example: ([7])

Th. 3. $O(p \supset Pp)$. (Only permitted things ought to happen. ([8])).

Th. 4. $O(Op \supset p)$. (Obligations ought to be fulfilled.)

Th. 5. $Op \supset \Diamond p$. (What is obligatory is possible; or «ought implies can.»)

Our purpose here, however, is not to list theorems, but rather to contend that the leading ideas behind these systems are generally in accord with our intuitions, and neutral to substantive issues in ethics and morals. There appear to be two ways of approaching these questions: we may first scrutinize the theorems for *prima facie* counterintuitive consequences, and secondly argue from common practice in normative discourse.

It is true that some of the theorems of the systems OX are, if not counterintuitive, at least non-intuitive, in that they deal with questions where our logical intuitions tell us little or nothing. We seldom consider necessities or impossibilities from a moral point of view, for example, but our systems do ascribe a deontic character to such states-of-affairs, by way of the following theorems:

Th. 6. $\sim\Diamond\sim p \supset Op$. (What is necessary ought to be the case.)

Th. 7. $\sim\Diamond p \supset Fp$. (What is impossible ought not be the case.)

These theorems may at first sight seem peculiar, but it should be noted that the English use of «ought» and «should» (though not of «obligatory») is ambiguous as between «ought, for normative reasons,» and «ought, for logical or causal reasons.» (E.g., «That ought to be a theorem, but I can't

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([5]) Actually, the axiom can be dispensed with. See the writer's «A reduction of deontic logic to alethic modal logic», *Mind* n.s. vol. 67 (1958), pp. 100-103, or an abstract thereof in the *Journal of symbolic logic*, vol. 22 (1957), p. 105.

([6]) See von WRIGHT, *op. cit.*, or his article «Deontic logic», in *Mind*, n.s. vol. 60 (1951), pp. 1-15.

([7]) Some of these have been discussed by PRIOR *op. cit.*, and in his *Formal logic*, Oxford, 1955. See also the discussion between Prior and R. Feys, *Revue philosophique de Louvain*, vol. 54 (1956), pp. 86-89.

([8]) We avoid artificial renderings like «It ought to be the case that if p then it is permitted that p», in favor of a more natural idiom.

find a proof,» or«He took the six o'clock train— he should arrive about seven thirty.») French *devoir être*, Spanish *deber ser*, and our «O» are similarly ambiguous. «O*p*» should be read «It ought (should) be the case, for normative or logical or causal reasons, that *p*.» But if one wishes to isolate the «purely normative» sense of «ought» in the formal systems, this aim is easily accomplished by defining an operator «O′» as follows:

Df. O′. «O′*p*» is short for «O*p* & $\Diamond\sim p$.»

It is easy to define «P′» and F′» analogously, and these operators, like «O′», in effect have a scope limited to contingent propositions. The upshot is that, faced with cases of this sort, where intuition fails to give us a clear clue, we can adopt either alternative we wish, or, indeed, both. We need not be committed to the view that there is only one « correct » way of using «ought,» «should,» etc.; in some cases the sense will be more accurately reflected by «O», in others by «O′».

Of theorems which have, at least at *very* first blush, a counterintuitive look, we may mention the following:

Th. 8. P*p* ⊃ P (*p* v *q*). (If *p* is permitted, then *p*-or-*q* is permitted.)

An instance: «If smoking is permitted, than smoking or embezzling is permitted.» The counterintuitive ring to this assertion is, however, deceptive. Ordinarily when one says «You may go or not go,» one means to assert *neither* that a tautology (going or not-going) is permitted, *nor* that at least one of the two (going, not-going) is permitted. The intent is clearly to indicate that *both* are permitted, i.e., «You may go, and you may stay — as you like.» It is recognized that «and» is sometimes used in English in a sense more akin to the truth-functional «or» («Doctors and lawyers must register»); we have in Theorem 8 a clear case where English «or» means truth-functional «and».(9) The assertion says that if *p* is permitted, then at least one of the alternatives *p*, *q*, is permitted («in particular,» the antecedent says, «*p*»), which is intuitively unobjectionable. The assertion P*p* ⊃ P (*p* & *q*), which *is* objectionable, and which more accurately reflects the ordinary sense of «or» in «if you may do *p*, then you may do *p* or *q*» is not a theorem.

Limitations of space preclude discussion of further examples, and in any event no list of examples would be conclusive. One can offer only persuasive evidence for thesis (*a*) above, since our intuitions are not well defined. Scrutiny of theorems has thus far led to no serious conflicts with the writer's intuitions, at least, and in view of the following remarks, there is reason to hope that no strongly counter-intuitive theorems will arise.

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(9) Compare the alethic theorem $\sim\Diamond\sim p\supset(\Diamond p \vee \Diamond\sim p)$, «What is necessary might by true or might be false», or the deontic theorem O*p*⊃(P*p* v P-*p*), «If *p* is obligatory, then you may do it or not». These sound odd, but only because «or» is normally used in such contexts as a truth-functional «and». If we read «or» in the truth-functional sense of «or», the appearance of paradox vanishes.

Aside from the fact that the idiom recommended for «ought» in OX *is* the standard way of expressing obligations in certain natural languages ([10]) it is also common as a paraphrase or justification of obligation-statements in English. «Why ought I do A?» «Because if you don't, the law will get you,» or «Because if you don't, Mother will be annoyed,» or «you won't go to heaven,» or even, «you'll be failing in your duty.» As remarked before, we find in practice a wide variety of «things wrong,» or sanctions, or penalties. But unless *something* is wrong with failing to do one's duty, it is difficult to see what «duty» means; and whatever the «thing wrong» may be, it is a candidate as an interpretation for «S». These familiar English usages would seem to lend support to the claim that the systems OX adequately reflect our intuitive ideas.

They also make it apparent that the systems make no substantive commitments regarding ethical or moral matters. The sanction «S» can be interpreted according to one's ethical predilections. In particular, OX does not, as might be thought, commit us to a teleological normative system. True, if «S» is interpreted in such a way that the connection beween *p* and S, when O*p* obtains, is a causal connection, then the resulting normative system is teleological in character. But if we choose to be ethical intuitionists, for example, we are perfectly free to interpret «S» as saying «a wrong has been done,» or the like. The resulting interpreted system will characterize the formal logic of an ethical intuitionist's normative system; the formal systems themselves do not prejudice substantive ethical questions.

«Still,» one might say, «obligations are defined in terms of sanctions, or disvalues. And this means that obligations are grounded in values, rather than the other way about. And that is a substantive ethical commitment.» But the choice of definitions, axioms, and rules, for systems of symbolic logic, is to a certain degree arbitrary. We can for example take «and» and «not» as primitives for a formulation of the two-valued truth-functional propositional calculus, and define «or» in terms of them; or we can take «or» and «not» as primitive, and define «and». It consequently makes little sense to claim that «conjunction is grounded in disjunction,» or the converse. And in the case of the deontic systems, it is perfectly possible to choose a deontic concept, say «F» (forbidden) as primitive, and define the notion of a disvalue «S» in terms of «F». ([11]) Whichever way one's fancies about ontological

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([10]) In Japanese, for example, «X ought to be done», is standardly «*X wo shinakereba, narimaseng*», which retranslates into English roughly as «If X is not done, then things will go wrong». See Bloch and Jordan, *Spoken Japanese*, War Department Education Manual M561, pp. 399 and 415. For a comparable locution in Korean see Horne and Yun, *Introduction to spoken Korean*, New Haven, 1951, pp. 198 and 322. These facts protect the idioms of OX from charges of unnaturalness or artificiality.

([11]) This development is due to A. N. Prior (in correspondence).

priority may lean, they can be accomodated in systems embodying the leading ideas of OX.

The analyses proposed here also accord with practice among jurists and behavioral scientists. In the case of positive law, it is certainly customary to state explicitly the (in this case socially inflicted) punishment to be brought to bear on those who violate the statutes. So common is this practice, in fact, that laws are frequently framed simply in terms of punishments for certain acts. Nothing is said about the act's being forbidden—this is left to be inferred. And the importance of the connection indicated in OX between norms and sanctions has long been recognized by anthropologists and sociologists. It is a platitude that what counts as a norm, in a given society, is what is enforced as a norm; departures therefrom are in some way socially punished.

Ordinary moral discourse, and legal and sociological practice, seem to accord reasonably well to the analyses provided by our systems, at least as far as unanalyzed propositional content is concerned. But it might still be urged that our systems fail to take account of one very important ingredient of normative problems, namely their resolution by compromise. Conflicts among obligations, and among normative systems (as e.g. the laws of a democratic as opposed to a totalitarian society) are ubiquitous, and unlike the case of scientific investigations, it is characteristic of these problems that they are resolved, if at all, by give-and-take. It might appear that the «uncompromising» character of the link between obligations and sanctions fails to take account of this important difference between normative and scientific problems.

But in the first place, one might question the view that compromise is absent in science. As some writers have pointed out, there is a certain amount of juggling in scientific practice; we sometimes accomodate our view of the «facts» to fit our theories, as well as the other way around. Even granting the thesis, however, we should not expect of a normative logic that it would give us material assistance in solving our moral problems, any more than a descriptive logic gives us material assistance in solving scientific problems. If we *have* a scientific theory, the descriptive logic might help elucidate its consequences, and if we *have* a moral code, a normative logic might be useful in the same way. But obviously conflicts of obligation, or clashes among normative systems, cannot be resolved by logical methods alone. Normative logic can only clarify logical relations— and indeed that is all any logic does. But it can do that. And it might well shed light on the informal logic we use continually in, for example, discovering that various sets of obligations are incompatible.

This summarizes evidence for the adequacy of our deontic logics. Of course these systems are calculi of propositions only; no account has yet been taken of acts, agents, patients, etc. To handle such notions effectively

we require quantificational extension of OX. ([12]) Quantificational extensions also enable us to obviate the artificiality of having but one disvalue «S». Of course there are sets of rules where a single penalty is attendant on all rule violations: the only thing wrong with making a forbidden move in chess, for example, is that one is thereby not playing chess. (There may well be wider social sanctions against cheating, and so on, but the standard chess manuals do not contain a rule to the effect that cheating is forbidden. Chess rules simply define the game, and if a person wishes to play chess, he had better obey the rules, or he will be defeating his own purpose.)

But in the general case we obviously require a variety of sanctions, especially if we are to give an account of such phenomena as «rôle conflicts,» i.e., being obliged by one set of norms to behave in one way, and by another in another. The fundamental idea remains the same, however: a state-of-affairs is obligatory if its denial leads necessarily to a disvalue. And it is to this fundamental idea, put in the context of formal logic, that we have tried to bring support from the common practice of students of law and culture, of moralists, and of our everyday discussions of normative systems and their consequences.

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([12]) A preliminary study of such extensions will appear in 1958 as a Technical Report, under the contract referred to in footnote ([1]).