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PUTTING WHITEHEAD'S THEORY OF GRAVITATION IN ITS HISTORICAL CONTEXT

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Introduction

Alfred North Whitehead (1861–1947) is known by many for his *Principia Mathematica* collaboration with Bertrand Russell, and by some for his later philosophical works. This paper, however, does not focus on the mathematics of his Cambridge period (1880–1910), nor on the metaphysics of his Harvard period (1924–1947), but on Whitehead's involvement with relativity during the London period of his professional career (1910–1924). This involvement culminated in an alternative rendering of Albert Einstein's general relativistic theory of gravitation, outlined in a number of publications, most notably in his 1922 book, *The Principle of Relativity with applications to Physical Science*.

If one ignores the broader British reception of Einstein's physics of relativity, Whitehead's alternative appears to be a creation out of nothing, and just as incomprehensible. The aim of this paper is to put Whitehead's theory of gravitation in its historical context.

Like most of his British contemporaries, Whitehead only got acquainted with Einstein's 1905 special theory of relativity in the 1912–1914 timeframe, in Minkowski's format, and via the publications of, among others, Ebenezer Cunningham and Ludwik Silberstein, two men he also knew personally. Whitehead wrote *An Enquiry Concerning the Principles of Natural Knowledge* (1919) and *The Concept of Nature* (1920) with the aim of constructing the special relativistic (Minkowksi's pseudo-Euclidean) space-time from the uniform spatio-temporal texture of our sense perception.

Whitehead learned about Einstein's general theory of relativity, completed a decade after the special one, when it was first presented in England by Cunningham and Eddington in 1916. The astronomer Arthur Eddington, a former student of Whitehead, was made aware of the importance of Einstein's theory of gravity for astronomy by Willem de Sitter, and Eddington, like Silberstein and Whitehead, chose de Sitter's side in the interpretative controversy between Einstein and de Sitter with regard to the question whether or not the cosmic structure of space-time is dependent on the cosmic distribution of matter — Einstein, inspired by Ernst Mach, thought it was; de Sitter held the opposite view.

Whitehead did not only share de Sitter's view, he took an additional step: according to him, space-time is not only uniform and matter-independent instead of variably curved and matter-dependent at the *cosmic* scale, but at *any* scale; moreover, space-time has to be identified with Minkowski's space-time, for that is the one that turns out to be constructible from the texture of our sense perception. Hence, Whitehead wrote *The Principle of Relativity with Applications to Physical Science* (1922) with the aim of reformulating Einstein's theory of gravity in such a way that gravity would no longer be identified with the allegedly variably curved space-time, but with a physical interaction (Whitehead's gravitational impetus) that can be defined against the uniform background of Minkowski's space-time.

Actually, the latter implied a return to Minkowksi's (unsuccessful) attempts to formulate a special relativistic law of gravitation. Inspired by Minkowski's 1908 vision that the gravitational interaction between masses is analogous to the electromagnetic interaction between charges, and that it can also be described in terms of retarded potentials and of the principle of least action; inspired by Cunningham's elegant, special relativistic 1914 expression of such potentials in the electromagnetic case; and inspired by Silberstein's earlier (and unsuccessful) 1918 attempt to undo Einstein's identification of gravitation and space-time; Whitehead reached his aim in the 1920–1922 timeframe.

Whitehead's special relativistic reformulation of Einstein's general relativistic theory of gravitation is quite close to Einstein's 1915–1916 original with regard to mathematical formulae and empirical tests. Only highly sophisticated contemporary tests (such as the ones highlighted by Gary Gibbons and Clifford Will[1]) reveal that the two theories are not empirically equivalent, and justify physicists to favor Einstein's theory with regard to experimental success. However, Whitehead's 1920–1922 alternative came out of season for the physicist, and hence, it never played a significant role in the history of physics. Allowing for a common sense interpretation, its importance is mainly philosophical, but the philosophical interpretation of Whitehead's theory is not included in the scope of the present paper.

[1] Cf. Gibbons & Will 2008.

Why Whitehead was interested in special relativity

The following publications are representative for the British knowledge about Einstein's special theory of relativity in the decade after its conception:[2]

- Edwin Bidwell Wilson & Gilbert Newton Lewis's 1912 memoir "The Space-Time Manifold of Relativity,"
- Ludwik Silberstein's 1914 monograph, *The Theory of Relativity*, and
- Ebenezer Cunningham's 1914 and 1915 books, *The Principle of Relativity* and *Relativity and the Electron Theory*.

Linking Whitehead with the Wilson & Lewis 1912 memoir, and with Silberstein's 1914 monograph, is straightforward. Indeed, in 1919, Whitehead wrote: "In connection with the theory of relativity I have received suggestive stimulus from Dr L. Silberstein's *Theory of Relativity*, and from an important Memoir ('The Space-Time Manifold of Relativity,' *Proc. of the Amer. Acad. of Arts and Sciences*, vol. XLVIII, 1912) by Profs. E.B. Wilson and G.N. Lewis." (*PNK* vii) However, in order to understand *why* the coauthor of *Principia Mathematica* took interest in these writings, as well as in Cunningham's writings, it is important to stress three biographical facts on Whitehead.

First, we should avoid the mistake of reducing Whitehead to a pure mathematician,[3] but take into account Russell's remark that "Clerk Maxwell's great book on electricity and magnetism [was] the subject of Whitehead's Fellowship dissertation," and that "on this ground, Whitehead was always regarded at Cambridge as an applied, rather than a pure, mathematician." (Russell 1959: 33) In fact, looking at Whitehead's Cambridge training, we can notice a remarkable similarity with his near contemporaries J.H. Poynting, J.J. Thomson, and Joseph Larmor — three major proponents of the second generation of British Maxwellians. Poynting, Thomson, Larmor, and Whitehead can be qualified as similar Cambridge products.[4] Poynting did his Cambridge Mathematical Tripos exam in 1876, Thomson and Larmor in 1880, and Whitehead in 1883; all four were coached by Edward Routh,

[2] Cf. Eddington 1918: vi–vii and Henry Brose's Translator's Note in Freundlich 1920: vii.

[3] For example, in *Mathematical visions: The pursuit of geometry in Victorian England*, Joan Richards pictures Whitehead in the early 1900s as moving wholeheartedly into the development of formal or pure mathematics, and away from the descriptive or applied mathematics tradition of Victorian England. Cf. Richards 1988:229 & 241–244.

[4] Cf. Warwick 2003: 333–398 and Lowe 1985: 92–109.

who excelled during the Tripos examination of 1854, and beat Maxwell into second place; and all four attended the intercollegiate courses on Maxwell's 1873 *Treatise on Electricity and Magnetism*, given by Maxwell's friend W.D. Niven. Being slightly younger, Whitehead also attended Thomson's lectures on electromagnetism. As is manifest in his writings, Whitehead developed a life-long interest in Maxwell's theory of electromagnetism, Poynting's theorem on the energy flow of an the electromagnetic field, and Thomson and Larmor's electronic theory of matter. In line with Hermann Minkowski's electromagnetic worldview, Wilson and Lewis in 1912, Silberstein in 1914, and Cunningham in 1914 and 1915, presented Einstein's special theory of relativity primarily as a contribution to electromagnetism, and more specifically, as a contribution to the electronic theory of matter. This constitutes a first explanation of why Whitehead took interest in them.

Secondly, we should avoid the mistake of reducing Whitehead's mathematical research to the *Principia Mathematica* project, and his philosophy of mathematics to Russell's logicism. Prior to his collaboration with Russell, Whitehead's mathematical research had already given birth to his 1898 *Treatise on Universal Algebra with Applications* — a publication that led to Whitehead's election as a Fellow of the Royal Society.[5] In Book VII of his *Universal Algebra*, Whitehead forged a vector calculus from Hermann Grassmann's algebra of extensions, applicable in various branches of physics, especially hydrodynamics and electrodynamics. This was an important first step in Whitehead's career to make the philosophical dream of applied mathematics come true, "that in the future these applications will unify themselves into a mathematical theory of a hypothetical substructure of the universe, uniform under all the diverse phenomena." (*ESP* 285) Whitehead was well aware of the similar approach by Josiah Willard Gibbs at Yale University.[6] Gibbs forged a three-dimensional vector calculus from William Rowan Hamilton's algebra of quaternions, and one might say that it belonged to Whitehead's core business to pay attention to the further development of both Grassmannian and Hamiltonian vector calculus. In line with Minkowski's formal developments, Wilson and Lewis in 1912, Silberstein in 1914, and Cunningham in 1914 and 1915, each presented a tailor-made four-dimensional vector calculus to deal with special relativity. This constitutes a second explanation of Whitehead's interest in them.

Thirdly, we should avoid the mistake of reducing the *Principia Mathematica* project to the three volumes that have been published. Early on in their

[5] Cf. Lowe 1966:137. That Whitehead was an FRS explains his presence at the famous meeting of the Royal Society and the Royal Astronomical Society on November 6th, 1919, a meeting that will be delt with in the remainder of this paper.

[6] Cf. *UA* 573.

collaboration, Russell and Whitehead decided that the latter would write a fourth volume in which all of geometry was going to be based on the symbolic logic of relations.[7] This was an obvious decision, given the prominence of Euclidean and non-Euclidean, projective and descriptive geometry in Whitehead's earlier *Universal Algebra* research. However, following the lure of the applied mathematician, Whitehead's attention shifted from the logical reformulation of all known pure geometries to the search for an answer, in terms of the symbolic logic of relations, to a question that had long occupied him: How is the geometry of physics rooted in experience?[8] Not only did this question lead Whitehead into an area of research that had been dominated by men like Hermann von Helmholtz, Henri Poincaré, and Ernst Mach, hence necessitating Whitehead to position himself with respect to these giants, it also made him hypersensitive to the impact of special relativity, for this theory required the replacement of Euclidean space as the object of physical geometry by Minkowskian space-time. Of course, the Minkowskian unification of space and time was the point of departure of Wilson & Lewis in 1912, Silberstein in 1914, and Cunningham in 1914 and 1915, and hence, constitutes a third explanation of Whitehead's interest in them.

To summarize: For Whitehead, the special relativistic writings of Wilson and Lewis, Silberstein, and Cunningham, represented a threefold attraction. This attraction can safely be called 'Minkowskian,' for it is associated with the imperative unification of space and time, with the mathematics developed to formulate physical laws against the background of this unified space-time, and with the thus reformulated electromagnetic worldview.

Cunningham

According to Whitehead's biographer,[9] in June 1911, Karl Pearson vacated the Goldschmidt chair of Applied Mathematics and Mechanics at University College, London, and Ebenezer Cunningham — by then Pearson's assistant — was asked to continue Pearson's teachings prior to naming a final successor. In July 1911, however, Cunningham was already released to accept a lectureship at Cambridge, and Whitehead — who had moved from Cambridge to London in 1910, and was in search for a job — gladly accepted

[7] Cf. Lowe 1990: 12, 14–15, 92–95, 273.

[8] Cf. *PNK v*.

[9] Cf. Lowe 1990: 6–14.

to replace Cunningham during the interregnum year 1911–1912. Whitehead hoped to be the final successor of Pearson, but mid March 1912, his hopes were destroyed when he learned of the appointment of another applied mathematician (L.N.G. Filon). Yet, Whitehead stayed at University College during the years 1912–1913 and 1913–1914, occupying a chair in pure mathematics, prior to leaving it for the Imperial College of Science and Technology, where he was able to secure a professorship in applied mathematics.

Anyway, the fact that Whitehead succeeded Cunningham in 1911 is one of the factors to conclude that he was familiar with Cunningham's work on special relativity. Other factors are: their similar training and teaching curricula; their common interest in Thomson and Larmor's electronic theory of matter; Cunningham's contribution to the 1911 edition of Pearson's very popular *Grammar of Science*;[10] the text-book status of Cunningham's *The Principle of Relativity*. Moreover, Whitehead and Cunningham met at least once in the context of the annual meetings of the British Association for the Advancement of Science, namely, in 1916, when Whitehead presided over Section A (mathematics and physics), and when Cunningham and Eddington introduced Einstein's general theory of relativity to the British scientific community.[11]

In an interview of Ebenezer Cunningham by John Heilbron on June 19th, 1963, Cunningham remembered that Whitehead was "quite interested" in his 1914 monograph on relativity (American Institute of Science: www.aip.org/history/ohislist); and in his unpublished autobiography, Cunningham wrote that he visited Whitehead at Harvard later in life, and added: "This was a renewal of acquaintance. I had disputed with him about relativity." (Cunningham 1970: 114).

[10] Cunningham was the main author of Chapter X, "Modern Physical Ideas." Whitehead was already familiar with the first (1892) and the second (1900) edition of Pearson's *Grammar* when he wrote his contribution on "Mathematics" for the 11th issue of the Encyclopaedia Britannica (cf. *ESP* 287). Given the obvious (but in the literature neglected) importance of Pearson's *Grammar* for the development of Whitehead's thought, and the fact that Whitehead worked as Pearson's interim in 1911, it is most likely that he also got acquainted with its third (1911) edition.

[11] Cf. *Nature*, Vol. 98, Nr. 2450 (October 12, 1916), p. 120, and Sanchez-Ron 1992: 60 & 76.

Silberstein and the Aristotelian Society

According to Ludwik Silberstein's biographers,[12] this physicist from Polish origin, German student of, e.g., Helmholtz and Max Planck, and Italian lecturer in mathematical physics, moved from Italy to London in 1912, where he obtained a lectureship at University College, London. Consequently, Whitehead and Silberstein became University College colleagues that year. Moreover, it is Silberstein's University College course of lectures on the special theory of relativity, delivered during the academic year 1912–1913, which developed into his 1914 monograph, *The Theory of Relativity*.[13] Hence, Whitehead most likely knew of Silberstein's work prior to Silberstein's 1914 publication. Next to the fact that during a brief period Whitehead and Silberstein were colleagues, there are quite a number of other facts that imply a more personal relationship. These facts are related with Whitehead being elected a member of the London Aristotelian Society in 1915 — in Whitehead's words: "a pleasant center of discussion," where "close friendships were formed." (*ESP* 14)

As from 1912, the Aristotelian Society now and again welcomed Silberstein to take part in the discussions.[14] As for Whitehead, on July 10th, 1912, Bertrand Russell already suggested in a letter to Wildon Carr that Whitehead might be able to present a paper at the Aristotelian Society dealing with relativity, and Russell added: "I know he has been going into the subject." (Passmore 1992: 191) And when Whitehead joined it in 1915, Wildon Carr was its president, Samuel Alexander and Lord Haldane were among its vice-presidents, Percy Nunn was its treasurer, and Charles Dunbar Broad was one of its younger members.[15] That these Aristotelian Society members became Whitehead's friends, even close friends in the case of Haldane and Nunn,[16] is not the only reason for mentioning them here. All these men were deeply

[12] Cf. Duerbeck & Flin 2006: 1087–1089.

[13] Cf. Silberstein 1914: v.

[14] For instance, on November 4th, 1912, when Russell gave a lecture on "The Notion of Cause," and on January 5th, 1914, when a paper was read on "Philosophy as the Coordination of Science." Cf. *Proceedings of the Aristotelian Society*, New Series, Vol. 13, p. 362 & Vol. 14, p. 425. Notice that even though Silberstein participated in the discussions prior to Whitehead, Whitehead became a member prior to Silberstein, for Whitehead was elected in 1915, and Silberstein in 1919, only a year before he left London for New York. Cf. *Proceedings of the Aristotelian Society*, New Series, Vol. 19, p. 310.

[15] Cf. *Proceedings of the Aristotelian Society*, New Series, Vol. 15, p. 437.

[16] Cf. Lowe 1990.

engaged in the philosophical issues with regard to relativity. This involvement with relativity — most likely one of the major reason for the mutual attraction between Carr, Alexander, Haldane, Nunn, Broad, Silberstein, and Whitehead — culminated in Carr's *The General Principle of Relativity, in Its Philosophical and Historical Aspect* (1920), Alexander's *Space, Time and Deity* (1920), Haldane's *The Reign of Relativity* (1921), Nunn's *Relativity and Gravitation* (1923), and Broad's *Scientific Thought* (1923). Whitehead figures in the latter four books, especially in Haldane's, and Silberstein figures in Nunn's book. In fact, Nunn and Silberstein were close. In 1914 Nunn already read the proofs of Silberstein's *Theory of Relativity*,[17] and in 1922 Nunn recalled that he "mixed a good deal with men like Silberstein, who are keen followers and even developers of the theory of relativity when it first came among us."[18]

So: Whitehead and Silberstein were colleagues during the academic year 1912–1913; as from 1915, they were both active in the Aristotelian Society; and they had a close friend in common (Nunn). All this leads to the conjecture that Whitehead and Silberstein knew each other well, and frequently met. The latter is confirmed by the minutes of the meetings of the Aristotelian Society. On January 3rd, 1916, when Whitehead read some explanatory notes on his first relativity paper, "Space, Time, and Relativity,"[19] Silberstein was present, and took part in the subsequent discussion. And on December 18th, 1916, when Whitehead read "The Organization of Thought,"[20] Silberstein was again present, and again joined the discussion.[21] Of course, the hypothesis that Whitehead and Silberstein frequently met is also supported by their joint presences at other Aristotelian Society meetings, e.g., on January 6th, 1919, and on March 3rd, 1919, when both took part in the discussion.[22] And finally, one should not forget that they were

[17] Cf. Silberstein 1914: v.

[18] Letter of Nunn to Haldane, dated July 8th, 1922. Cf. National Library of Scotland, Haldane Papers, Manuscript 5915, Folio 192.

[19] This paper of Whitehead was first read to Section A (mathematics and physics) at the Manchester Meeting of the British Association for the Advancement of Science in 1915. Cf. *OT* 191–228.

[20] This paper of Whitehead was his Presidential Address to Section A at the Newcastle Meeting of the British Association for the Advancement of Science in September 1916. Cf. *OT* 105–133.

[21] Cf. *Proceedings of the Aristotelian Society*, New Series, Vol. 16, p. 364 & Vol. 17, p. 481.

[22] Cf. *Proceedings of the Aristotelian Society*, New Series, Vol. 19, p. 293 & p. 294.

both present at the famous joint meeting of the Royal Society and the Royal Astronomical Society on November 6th, 1919, when Eddington presented the observational data gathered during the May 1919 solar eclipse, and when Silberstein, contrary to Eddington, pointed out that they were insufficient to confirm Einstein's general theory of relativity.[23]

Minkowski's 1908 papers

Whitehead's acquaintance with the work of Wilson & Lewis, Silberstein, and Cunningham — and hence, with Einstein and Minkowski's unification of space and time, with the Minkowskian mathematics to formulate physical laws against the background of Minkowski's unified space-time, and with the Minkowskian reformulation of electromagnetism, naturally led him to the work of Minkowski himself. In May 1941, Whitehead told his biographer, Victor Lowe: "Minkowski's paper was published in 1908, but its influence on me was postponed approximately ten years." (Lowe 1990: 15) Given the fact that Whitehead got to know Minkowski's work via Wilson & Lewis, Silberstein, and Cunningham, accounts for a retardation of the direct influence of Minkowski's 1908 paper on Whitehead, although, as Lowe adds: "*Ten* may be an overstatement by one to three years." Also, it is not immediately clear whether Whitehead pointed at Minkowski's 1908 paper "Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern," or to his famous 1908 Cologne lecture "Raum und Zeit" — the two texts of Minkowski to which Wilson & Lewis, as well as Silberstein, frequently refer.[24]

It was in the *Grundgleichungen* that Minkowski first employed the term "spacetime" (Walter 2007: 219), but it was in his famous "Space and Time" lecture that he said: "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." (Minkowski 1952: 75) The *Grundgleichungen*, with its treatment of space-time vectors of the first and the second kind, and of the matrix-method to operate with these vectors,[25] is more

[23] Cf. *SMW* 10 for Whitehead's presence, and Duerbeck & Flin 2005: 191 & 200–203 for Silberstein's presence and intervention.

[24] E.g., Wilson & Lewis 1912: 391 & 495, and Silberstein 1914: 127 & 129–130 & 143 & 266 & 282. Remarkably, even though Cunningham devotes a whole part of his 1914 book to Minkowski's work (Part II Minkowski's Four-Dimensional World, pp. 85–134), he does not explicitly refer to any of Minkowski's papers or lectures.

[25] Cf. Minkowski 1910: 483–486 & 495–503.

mathematical than "Space and Time," but the latter, with its vision of the whole universe being resolved into world-lines, and of a world-line as "the everlasting career of the substantial point" (Minkowski 1952: 76), is more likely to be remembered by an eighty year old philosopher — Whitehead in 1941 — whose notion of 'historical routes' in *The Principle of Relativity* is a slightly more concrete version of Minkowski's abstract notion of 'world-lines.'[26] Moreover, in "Space and Time" Minkowski holds that "physical laws might find their most perfect expression as reciprocal relations between those world-lines," and after describing the electrodynamical relations between the world-lines of point-charges in terms of the Maxwell-Lorentz electron theory and the Liénard-Wiechert retarded potentials, Minkowski expresses his belief that the resolution of the universe in world-lines of point-charges can be seen as "the true nucleus of an electromagnetic image of the world." (Minkowski 1952: 91) Well, with some exaggeration, one might say that Whitehead's relativity of historical routes of events is the true nucleus of the philosophical image of the world as presented in his later works, again implying that most likely Whitehead pointed at Minkowski's famous 1908 Cologne lecture, "Space and Time," when telling Lowe about Minkowski's influence on him.

The search for a relativistic theory of gravitation

Next to space-time unification, mathematical formalism, and electromagnetic worldview, there is another important aspect in both Minkowski's appendix to the *Grundgleichungen*[27] and his "Space and Time" lecture, as well as in the Wilson & Lewis memoir, and the Silberstein and Cunningham textbooks. Inspired by Poincaré — "Poincaré's scientific output fascinated Göttingen scientists in general, and Minkowski in particular" (Walter 2007: 214) — Minkowski sought to bring gravitation within the purview of Einstein's principle of relativity.

In the appendix to the *Grundgleichungen* Minkowski wrote that "it would be highly unsatisfactory" if Einstein's principle of relativity "could be accepted as valid for only a subfield of physics" (Minkowski 2007: 274), and he proposed a first relativistic law of gravitation, formulated in terms of a 4-scalar gravitational potential, and inspired by his own reformulation of Maxwell's equations in terms of a 4-vector electromagnetic potential.[28]

[26] Cf. *R* 30.

[27] This appendix is titled "Mechanics and the Relativity Postulate." For an English translation, see Minkowski 2007.

[28] Cf. Walter 2007: 224.

In "Space and Time" Minkowski expressed the belief that the gravitational relations between the world-lines of point-masses should be treated just like the electromagnetic relations in the case of point-charges, and he accordingly proposed a second relativistic law of gravitation, expressing the driving gravitational force in terms of a 4-vector gravitational potential.[29]

However, the challenge to solve the problem of the incorporation of gravitation in a relativistic image of the word remained, because, as Scott Walter poignantly puts it: "By proposing two laws instead of one, Minkowski tacitly acknowledged defeat," and "could hardly claim to have solved unambiguously the problem of gravitation." (Walter 2007: 234)

At the end of their 1912 memoir, Wilson & Lewis echo Minkowski's vision that the searched for formulae expressing the gravitational force and potential must be "completely analogous" to the new formulae expressing the electromagnetic force and potential, and suggest — by analogy — the use of the term "gravito-magnetic" instead of gravitational. (Wilson & Lewis 1912: 496) Silberstein — in his 1914 book — mentions Poincaré's 1906 attempt to use the general form of the Lorentz transformations for the treatment of both the dynamics of the electron and universal gravitation, and notices the advantage Minkowski's approach seems to offer for a relativistic theory of gravity.[30] The most elaborate treatment of the search for a relativistic theory of gravity, however, is given in Cunningham's *The Principle of Relativity*.

While dealing with the electron theory in Minkowskian format, and, more specifically, with the Lorentz covariant four-vector expression of the Liénard-Wiechert potentials for the field due to the motion of a single point-charge, Cunningham refers to "the work founded on that of Poincaré for modifying the law of gravitation to conform to the Principle of Relativity." (Cunningham 1914: 109) Contrary to Wilson & Lewis, and to Silberstein, Cunningham does not leave it at a simple suggestion of the gravitation-electrodynamics analogy. An important part of his treatment of the dynamics of a particle is devoted to the search for a relativistic theory of gravitation.[31] Moreover, Cunningham does not only refer to the 1906 paper of Poincaré — "Sur la dynamique de l'électron" — but also treats the 1911 paper of the Dutch astronomer Willem de Sitter — "On the Bearing of the Principle of Relativity on Gravitational Astronomy" — which has been called "the most authoritative account in English of the astronomical importance of the principle of relativity [...] before the appearance of Einstein's general theory." (Warwick 2003: 453)

[29] Cf. Walter 2007: 234.

[30] Cf. Silberstein 1914: 87 & 241.

[31] Cf. Cunningham 1914: 171–180.

None of the relativistic theories of gravitation Whitehead encountered in the writings on relativity at his disposal prior to 1916 was satisfactory. None of the theories which can be found in Poincaré's 1906 paper, Minkowski's 1908 papers, de Sitter's 1911 paper, and Cunningham's 1914 book, were in accordance with the astronomical observations of the secular motion of the perihelion of Mercury, as de Sitter and Cunningham clearly highlight.[32]

It seems to have been Minkowski's opinion that the incorporation of gravitation into relativistic thinking was not a major problem, and — as Minkowski died on January 12, 1909, at the age of 44 of a sudden and violent attack of appendicitis — he never lived to see how elusive and difficult the task would turn out to be.[33] By 1913, Einstein characterized the attempt to find a relativistic generalization of Newton's law of gravitation as "a hopeless undertaking," at least, in the absence of some good physical guiding principles, such as the "laws of energy and momentum conservation," and the "equality of the *inertial* and the *gravitational* mass," and Einstein adds:

> To see this clearly, one need only imagine being in the following analogous situation: suppose that of all electromagnetic phenomena, only those of electrostatics are known experimentally. Yet one knows that electrical effects cannot propagate with superluminal velocity. Who would have been able to develop Maxwell's theory of electromagnetic processes on the basis of these data? Our knowledge of gravitation corresponds precisely to this hypothetical case: we only know the interaction between masses at rest, and probably only in the first approximation. (Einstein 2007: 544)

Contrary to Minkowski, but in line with Einstein, when Whitehead read his first relativity paper, "Space, Time, and Relativity," before the members of the Aristotelian Society, he added the following comment: "We have begun to expect that all physical influences require time for their propagation in space. This generalization is a long way from being proved. Gravitation stands like a lion in the path." (*OT* 225) However, in September 1916, eight months after making this comment, Whitehead first learned about Einstein's general theory of relativity — a theory that claimed to have defeated the lion that blocked the road to an empirically adequate relativistic treatment of gravitation.

[32] Cf. Cunningham 1914: 180.

[33] Cf. Corry 2004: 192 & 227.

Eddington and de Sitter

Linking Whitehead and Eddington is easy, because in 1902, Eddington — thanks to his outstanding ability in mathematics and physics — was granted a natural science scholarship to Trinity College, Cambridge, where he was coached by Robert Herman,[34] and where "among the formal lectures which Eddington and most of his group attended were those of [...] A.N. Whitehead." (Douglas 1957:10) So when Whitehead presided over Section A (mathematics and physics) at the 86$^{\text{th}}$ meeting of the British Association for the Advancement of Science in Newcastle-On-Tyne in September 1916,[35] he already knew Eddington personally, at the very least as his former student. And that the two men met at this section, and, moreover, that their meeting was related to Einstein's general theory of relativity, is made clear by the following account of it in the October 12$^{\text{th}}$, 1916, issue of *Nature*:

> The first of the two organized discussions arranged for this section was on "Gravitation." The discussion followed immediately after Prof. Whitehead's presidential address,[36] and it happened that the arrangement was appropriate, for the president's exposition of the logical texture of geometry had carried us far from the ordinary conceptions of space, and paved the way for the revolutionary ideas associated with the space-time world of Einstein and Minkowski. Mr. E. Cunningham, who opened the discussion, and Prof. A.S. Eddington, who followed, dealt with Einstein's recent work, which brings gravitation within the scope of the principle of relativity. (*Nature*, Vol. 98, Nr. 2450, p. 120)

This *Nature* quote is in harmony with two of this paper's claims: Whitehead's *Principia Mathematica Volume 4* research on the logical texture of geometry formed his pathway to relativity; and, in 1916, Cunningham was one of the most prominent British mathematicians engaged in the quest of bringing gravitation within the scope of the principle of relativity. At the same time, the quote also links Whitehead with Eddington's research on general relativity. It must be said, however, that in September 1916, Cunningham and Eddington's research on general relativity was still premature,

[34] Cf. Douglas 1957: 5 & 9–10, and Warwick 2003: 449–451.

[35] Cf. Sanchez-Ron 1992: 76.

[36] "The Organization of Thought," to which I already referred, because Whitehead also read it at the Aristotelian Society in December 1916.

and according to Andrew Warwick: "It is a measure of Cunningham and Eddington's ignorance of Einstein's work at this time that the official account of the session (published in 1917) made no mention of their presentations, but referred the reader directly to de Sitter's first two papers in the *Monthly Notices*." (Warwick 2003: 462–463)

This Warwick quote does not only point at the British ignorance of Einstein's general theory of relativity in 1916. It also points at the fact that the official account of Section A, presided by Whitehead, referred to two of de Sitter's general relativity papers, which immediately establishes a link between Whitehead and de Sitter's general relativity output. Actually, both elements — the British ignorance with regard to general relativity in 1916, and the fact that the ignorant British readers, including Whitehead, were referred to De Sitter's papers instead of Einstein's papers — are closely related. A common cause was World War I.

Germany and Britain being at war, the German publications of Einstein did not easily reach the British, partially explaining their ignorance.[37] However, the Netherlands was neutral, and the news of Einstein's completion of the general theory of relativity in November 1915 reached Britain in the form of a letter from the Netherlands.[38] Indeed, on the one hand, the Dutch astronomer de Sitter was one of the three Leiden University physicists who acted as Einstein's sounding board during the development of his general theory of relativity — the other two were Hendrik Antoon Lorentz and Paul Ehrenfest. So, de Sitter was well informed on Einstein's struggle with, and completion of, a new theory of gravitation. On the other hand, when Einstein published a general summary of his new theory in May 1916, including some discussion of its cosmological consequences,[39] de Sitter realized its importance for astronomers in the English-speaking world, while at the same time

[37] Einstein's struggle to formulate a relativistic theory of gravitation started not long after the 1905 publication of his special theory of relativity, and hence, prior to World War I. However, his pre-war attempts, and his corresponding publications, were undervalued in Britain. This undervaluation has to be included in order to fully explain the British ignorance in 1916, and is exemplified by both Cunningham and Eddington. In 1914, Cunningham wrote: "No attempt has been made to present the highly speculative attempt of Einstein at a generalization of the principle [of relativity] in connection with a physical theory of gravitation." (Cunningham 1914: vi) And whereas Cunningham dismissed Einstein's pre-war attempts as too speculative, Eddington — who knew Einstein's 1911 paper "On the Influence of Gravitation on the Propagation of Light" — mainly focused on Einstein's empirical predictions, and undervalued the fact that these predictions were based upon a new hypothesis concerning the physical nature of gravity (Einstein's equivalence principle). Cf. Warwick 2003: 455–457.

[38] For more complete accounts than the one I can give here, cf. Stachel 2002: 455–456, Warwick 2003: 457–462, and Crelinsten 2006: 94–98.

[39] Cf. Einstein 1916.

realizing that one could not very well reprint the work of a German in a British journal during the wartime. So, in June 1916, de Sitter wrote a letter to inform Eddington, then Secretary of the Royal Astronomical Society, and he offered to submit a paper of his own on the subject. In his reply to de Sitter, Eddington confirmed that he was immensely interested, and he encouraged de Sitter to submit the promised paper. In the event, de Sitter published a paper in *The Observatory* magazine, and a series of three papers in the *Monthly Notices of the Royal Astronomical Society*.

In a letter of July 4th, 1916, Eddington informed de Sitter:

> We are having a discussion at the British Association on Gravitation — at Newcastle, Dec. 5–8. I wish we could have invited you to come over to take part; but we are not inviting any foreign guests this year because Newcastle is a "restricted area" and aliens are not allowed in it. [. . .] I feel sure you will allow me to make use of the papers you send, in making my contribution to the discussion. So far as I can make out, no one in England has yet been able to see Einstein's paper and many are very curious to know the new theory. So I propose to give an account of it at the Meeting.[40]

Whitehead listened at Eddington's account at Newcastle, but this account was not included in the official report. However, a good idea of what Whitehead heard can be formed by reading Eddington's first published paper devoted to the general theory of relativity, "Gravitation and the Principle of Relativity," published in the December 28th, 1916, issue of *Nature*. The reason is offered by John Stachel's remark that "it is presumably based on his talk on the same subject to the British Association for the Advancement of Science." (Stachel 2002: 457) Eddington refers his readers — and hence, presumably referred his audience — to the following three paper on the subject: Einstein's May 1916 paper "Die Grundlage der algemeinen Relativitätstheorie," de Sitter's October 1916 *Observatory* paper, and de Sitter's first 1916 *Monthly Notices* paper. This means that, most likely, Whitehead was referred to de Sitter's writings on general relativity prior to the appearance of the official report of the Newcastle meeting in 1917.[41]

Of course, *The Observatory*, the *Monthly Notices*, and *Nature*, were readily available to Whitehead, but we do not know whether Eddington offered

[40] This quote is taken from Stachel 2002: 456. The British Association meeting was held in September.

[41] Further research might provide an answer to the following question: Was Whitehead himself, having been the Section A president in 1916, responsible for the official report on that section or not?

his former Cambridge lecturer the opportunity to read the paper that led to all the excitement in the first place — Einstein's summary paper, which Eddington got from de Sitter. Likewise, we do not know whether, half a year later, Silberstein offered his former University College colleague a reprint of Einstein's summary paper. Silberstein, by then, had also received reprints via a neutral country, namely via Michele Besso in Switzerland. In fact, on May 7, 1917, Einstein wrote to his close friend: "Lieber Michele! Ich sende Dir einige Abhandlungen mit der Bitte, Sie an Herrn Dr. L. Silberstein, 4 Anson Road Cricklewood London N.W.2. weiterzusenden, der mich darum gebeten hat."[42] But what we do know is that Whitehead had both the appropriate personal contacts, and the references to all 1916–1917 English articles on the topic, to get acquainted with Einstein's general theory of relativity.

Despite his remarkable speed to master new mathematical theories, and despite his prior knowledge on differential geometry, acquired a decade earlier in the lectures of his coach, Herman, at Trinity College, it took Eddington almost two years to master Einstein's general theory of relativity, and even then, upon completion of his 1918 official *Report on the Relativity Theory of Gravitation* for the Physical Society of London, he asked for de Sitter's "general criticism and detection of blunders."[43] Nonetheless, as Andrew Warwick puts it, "for many British mathematicians and physicists the *Report* represented the definitive English-language account of general relativity and further established Eddington's emergent reputation as the theory's master and champion in Britain." (Warwick 2002: 468).

One cannot imagine that Whitehead — whose research dealt with the question of how to derive, by means of the *Principia Mathematica* logic, the space-time geometry of physics from the spatio-temporal texture of our experience; who repeatedly discussed relativity with his Aristotelian Society friends, e.g., with Alexander on July 5$^{\text{th}}$, 1918, following Alexander's address on "Space-Time";[44] whose 1918–1919 lecture courses on applied mathematics at the Imperial College of Science and Technology included his postgraduate lecture "Relativity and the nature of space";[45] and who started Herbert Dingle's lifelong interest in the theory of relativity, and encouraged

[42] "Dear Michele! I'm sending you some reprints, asking you to forward them to Dr. L. Silberstein, 4 Anson Road Cricklewood London N.W.2., who has requested them." The German quote in the main text is a quote from Document 335 in The Collected Papers of Albert Einstein, Volume 8, Part A, p. 446. For the English translation, and more details on the Silberstein-Einstein correspondence, cf. Duerbeck & Flin 2006: 1089.

[43] Letter of Eddington to de Sitter, dated August 16$^{\text{th}}$, 1918. Cf. Warwick 2003: 467–468.

[44] Cf. *Proceedings of the Aristotelian Society*, New Series, Vol. 18, p. 640, and *CN* viii.

[45] Imperial College's annual *Calendar* lists the 'Special advanced courses' at the postgraduate level that Whitehead gave from 1915 to 1924. With regard to relativity, the list includes:

him to write *Relativity for All*[46] — ignored the definitive English-language account of general relativity, written by his former student, and meanwhile famous astronomer, Eddington.

Whitehead's Enquiry

After all that has been said and done in this paper to link Whitehead with Eddington and de Sitter, a surprise awaits the reader when turning to Whitehead's first book on relativity, his 1919 *Enquiry Concerning the Principles of Natural Knowledge*. The book shows no trace of any Eddington or de Sitter impact! However, the explanation is straightforward.

Whitehead's *Enquiry* is the apex of his research to find — in terms of the logic of relations — an answer to the question: "How is space rooted in experience?" The first output of this research, written in 1905, and published in 1906, was Whitehead's Royal Society memoir "On Mathematical Concepts of the Material World," in which 'space' still meant 'Euclidean space,' and in which 'points' were logically defined by Whitehead in terms of 'linear objective reals' — entities closely resembling Faraday and Maxwell's spatial lines of force. However, Einstein and Minkowski's unification of space and time prompted Whitehead to replace 'space' with 'space-time', 'points' with 'event-particles', and 'linear objective reals' with Minkowski's world lines. In other words, special relativity caused an update of Whitehead's research question into: "How can Minkowski's space-time geometry be logically abstracted from our experience of spatio-temporal events?"

By the time he wrote his *Enquiry*, Whitehead had developed a method — the method of extensive abstraction — to do just that; a method which harmonized the world of physics with the world of everyday experience — Whitehead's main philosophical motivation; and hence, a method he was not willing to put aside because Einstein's general relativity invited us to give up Minkowski's non-curved space-time geometry in favor of a variably curved

"Relativity and the nature of space," given in 1918–1919 and 1919–1920; "Relativity, gravitation and electromagnetism," given in 1920–1921; and "The tensor theory and its applications to mathematical physics," given in 1921–1922. For the complete list of Whitehead's advanced lecture courses, which reveals his major interest in electromagnetism and relativity, see Ivor Grattan-Guinness, "A.N. Whitehead on Mathematics Education in the 1910s," published in Desmet & Weber 2010: 249–268. Cf. also Lowe 1990: 64–65.

[46] In the July 1921 Preface of *Relativity for All* Dingle wrote: "The author is glad to acknowledge his deep indebtedness to Professor Whitehead for invaluable help and unwearying kindness in unveiling the mysteries of a difficult subject." (Dingle 1922: vi) Cf. also Lowe 1990: 65.

space-time geometry. In the April 20th, 1919, Preface of his *Enquiry*, Whitehead expresses this concern as follows:

> The whole investigation is based on the principle that the scientific concepts of space and time are the first outcome of the simplest generalizations from experience, and that they are not to be looked for at the tail end of a welter of differential equations. This position does not mean that Einstein's recent theory of general relativity and of gravitation is to be rejected. The divergence is purely a matter of interpretation. [...] It has certainly resulted from Einstein's investigations that a modification of the gravitational law [...] will account for the more striking outstanding difficulties otherwise unexplained by the law of gravitation. This is a remarkable discovery for which the utmost credit is due to the author. Now that the fact is known, it is easy to see that it is the sort of modification which on the simple electromagnetic theory of relativity is likely to be required for this law. I have however been anxious to disentangle the considerations of the main positions in this enquiry from theories designed to explain special laws of nature. Also at the date of writing the evidence for some of the consequences of Einstein's theory is ambiguous and even adverse. In connection with the theory of relativity I have received suggestive stimulus from Dr L Silberstein's *Theory of Relativity* [...]. (*PNK* vi–vii)

The first sentence of this quote confirms Whitehead's main philosophical challenge — to avoid the bifurcation of nature into the mathematical world discovered by Einstein (and his predecessors) "at the tail end of a welter of differential equations," and the common word of our day-to-day experience — and it confirms Whitehead's main answer at the time, both to this challenge, and to his research question: a method of "the simplest generalizations" — the method of extensive abstraction.

Clearly, Whitehead was well aware of the general theory of relativity when writing his Preface, and he did not reject its new law of gravitation. On the contrary, he credited Einstein for solving the outstanding difficulties of Newton's law of gravitation, such as the difficulty of accounting for the observed precession of the perihelion of Mercury, a difficulty Poincaré, Minkowski, de Sitter, and Cunningham, were unable to solve. At the same time, Whitehead distanced himself from Einstein's general relativistic interpretation of his new law of gravitation, and already gave two hints on how to reinterpret it: (1) by learning from how Einstein modified Newton's law, and by performing a similar modification while adhering more closely to the special theory of relativity, that is, while respecting the main position of his research, that the Minkowskian space-time structure was at one with the spatio-temporal texture of our experience; and (2) by disentangling the problem of discovering the general structure of space-time from the problem of discovering the particular character of physical laws, or, in other words, by separating again what Einstein had unified, space-time geometry and physics.

To summarize, the Preface of *An Enquiry Concerning the Principles of Natural Knowledge* clearly confirms our claim that Whitehead was familiar with Einstein's general theory of relativity on April 20, 1919, while at the same time providing an explanation of why no trace leading to Eddington or de Sitter can be found in the book. Whitehead's research aimed at thinking together the geometrical world of Minkowski, and the spatio-temporal world of the events we experience. And with his *Enquiry*, Whitehead wanted to communicate how he had managed to do so, without explicitly addressing and answering the next question on his research agenda: "How to interpret Einstein's new law of gravitation in terms of a gravitational field against the background of Minkowski's space-time, instead of accepting Einstein's interpretation of his new law in terms of the identification of the field of gravitation with a variably curved space-time?"

Silberstein's 1918 paper

Early in 1919, Whitehead had a good reason for not yet addressing his new research question. Its relevance was dependent on "the evidence for some of the consequences of Einstein's theory," and when writing the Preface of his *Enquiry*, this evidence was still "ambiguous and even adverse," even though a month later it was going to be strengthened thanks to some relevant observations by British astronomers, including Eddington, at the occasion of the May 29$^{\text{th}}$, 1919, solar eclipse.

It is no coincidence that Whitehead indicated to have received "suggestive stimulus from Dr L. Silberstein's *Theory of Relativity*." I claim that Whitehead's referral to Silberstein in the context of reinterpreting Einstein's new law of gravitation, disentangling space-time geometry and physics, and highlighting adverse evidence with regard to empirical consequences, points to the fact that Silberstein was a source of inspiration to help Whitehead answer his new, but still private, research question of reinterpreting general relativity. I will now first add an element to substantiate the latter claim, and then return to the solar eclipse.

In 1923, George Temple — a mathematician who had taken his first degree as an evening student, and at the time was working as a research assistant at Birkbeck College, London — gave a lecture on "A Generalization of Professor Whitehead's Theory of Relativity" at the Physical Society of London. The importance of mentioning Temple's lecture at this point is formed by the following facts. Temple treated Silberstein's 1918 paper, "General Relativity without the Equivalence Hypothesis," as a precursor of Whitehead's alternative theory of gravitation. Whitehead was present, and responded to Temple's paper. His extensive response is registered in the Proceedings of the Physical Society of London, and shows that he was very pleased with this paper "from the pen of a young scientist whose work augurs a very distinguished career."[47] At no point did Whitehead object to treating Silberstein's 1918 paper as a precursor of his own 1920–1922 theory. The opposite is true. Whitehead emphasized that at the heart of his alternative theory of gravitation lies the distinction "between space-time relations as universally valid and physical relations as contingent."[48] In other words, Whitehead stressed the importance of separating the general space-time structure from the more particular physical structures, a separation that is central in Silberstein's 1918 paper, in which Einstein's equivalence principle — Einstein's identification of inertial and gravitational descriptions to the point of identifying space-time geometry and gravitational physics — was rejected. This adds to what has been said before on the Whitehead-Silberstein link, and thus helps to substantiate the claim that Silberstein was a source of inspiration for Whitehead.

Silberstein, who had a kind of love-hate relationship with Einstein's general theory of relativity, willing to accept it wholeheartedly, and yet, relentlessly criticizing it, has been called Einstein's *"advocatus diaboli"* (Pais 1983: 305), as well as "Einstein's antagonist" (Duerbeck & Flin 2005: 186). This might suggest that Whitehead mainly derived his critical attitude towards Einstein from Silberstein. However, in order not to overestimate the influence of Silberstein on Whitehead, an important comment is due. Whitehead's critique of Einstein's approach has many more sources, too many to list here. Some date from his Cambridge period, others from his London period. Some are to be found in the domain of mathematical physics, others in the domain of philosophy.

Also, one must not forget that from its introduction in Britain, Einstein's general theory of relativity was exposed to critique. Most importantly, the 1916–1917 *Monthly Notices* papers of de Sitter reflect an at that time ongoing Einstein-de Sitter debate on Einstein's Machian explanation of the cosmic structure of space-time in terms of the cosmic distribution of matter,

[47] Temple 1923: 192.

[48] Temple 1923: 193.

and hence, on the priority of matter over space-time.[49] So there never was a "pure" or "uncritical" transmission of Einstein's general theory of relativity from the Continent to Britain to start with. When Eddington, Silberstein, and Whitehead learned about it, Einstein's theory was already wrapped in de Sitter's anti-Machian critique, elements of which became part of their critiques. I deliberately include Eddington, because in his 1918 *Report*, he clearly sided with de Sitter in the debate with Einstein on the various cosmological hypotheses at the time.

No wonder that Whitehead, inspired by de Sitter, Eddington, and Silberstein, repeatedly takes his distance from Machian interpretations in his 1920–1922 writings on relativity, and that he replaced Einstein's theory, in which matter has priority over space-time, and in which space-time is constantly curved at the local and at the cosmological scale (respectively zero curved and non-zero curved), while being variably curved at the intermediate scale (e.g., of the solar system), with an alternative in which space-time has priority over matter, and in which the universe is Minkowskian (and hence at one with our common experience) at all scales.

The 1919 solar eclipse

One of the most important consequences of the general theory of relativity was Einstein's prediction concerning the deflection of rays of starlight passing near the limb of the sun. If the starry sky is photographed twice, once by night, and once during a solar eclipse, all other things being equal, then, upon comparison of the two pictures, we will observe exactly calculated deflections of rays of starlight (that is, shifts of starlight spots on the pictures) near the solar corona. The pictures taken by English astronomers during the solar eclipse on May 29th, 1919, seemed to confirm Einstein's prediction, and when Eddington made this confirmation public on November 6th, 1919, at a joint meeting of the Royal Society and the Royal Astronomical Society, it immediately launched Einstein's career to superstar-heights, despite Silberstein's unease with Eddington's way of handling the solar eclipse data, and his warning to await confirmation of Einstein's red shift prediction.[50]

[49] For an account of the 1916–1917 Einstein-De Sitter dialogue, see Janssen 1998: 351–357 (can also be found on http://www.tc.umn.edu/ janss011/) & Crelinsten 2006: 103–108 & Matteo Realdi's 2007 lecture "The Universe of Willem de Sitter" (to be found on http://www.phil-inst.hu/∼szekely/PIRT_BUDAPEST/).

[50] There is a longstanding debate about Eddington's way of handeling the solar eclipse data, which has recently been set straight quite nicely by D. Kennefick, "Not Only Because of Theory: Dyson, Eddington and the Competing Myths of the 1919 Eclipse Expedition"

As said before, Whitehead was present at this meeting, and in an account published years later in *Science and the Modern World*, Whitehead wrote:

> The whole atmosphere of tense interest was exactly that of the Greek drama: we were the chorus commenting on the decree of destiny as disclosed in the development of a supreme incident. There was dramatic quality in the very staging: — the traditional ceremonial, and in the background the picture of Newton to remind us that the greatest of scientific generalizations was now, after more than two centuries, to receive its first modification. Nor was the personal interest wanting: a great adventure of thought had at length come safe to shore. (*SMW* 10)

Not only was Whitehead present at this memorable meeting, by November 1919, the British considered Whitehead as an authority on the subject of general relativity. On November 15th, 1919, his first article on the momentous confirmation of Einstein's revolutionary theory appeared in *The Nation* under the title "A Revolution in Science." Also, Wildon Carr (already mentioned in this paper as one of Whitehead's Aristotelian Society friends), Frederick Lindemann (a famous Oxford physicst), and Whitehead, were asked to write a contribution on "Einstein's Theory" for the readers of the Educational Supplement of *The Times*. Carr's article was published on January 22nd, 1920, Lindemann's on January 29th, and Whitehead's on February 12th.[51]

The opening of Whitehead's 1920 article reads: "The articles on this subject, which appeared on January 22 and 29, summarized the general philosophical theory of relativity and the physical ideas involved in Einstein's researches. The purpose of the present article is in some respects critical, with the object of suggesting an alternative explanation of Einstein's great achievement." (*ESP* 332) Whereas Whitehead's 1919 article, "A Revolution in Science," does not give away Whitehead's critical attitude, and is as orthodox as Lindemann's *Times* article, his 1920 article, "Einstein's theory," not only reveals Whitehead's critique of Einstein, but also gives a first outline of his alternative theory of gravitation.

In it, Whitehead starts with the analysis of Einstein's work in three factors: "a principle, a procedure, and an explanation." (*ESP* 332) According

(to be found on http://arxiv.org/abs/0709.0685); see also "Testing relativity from the 1919 eclipse – a question of bias," *Physics Today*, March 2009.

[51] Whitehead's "Einstein's Theory" is reprinted in *ESP* (pp. 332–342) and in *IS* (pp. 125–135).

to Whitehead, Einstein's principle is the unification of space and time into space-time, and he writes: "What I call Einstein's principle is the connexion between time and space." (*ESP* 332) Whitehead does not give similarly clear definitions of Einstein's procedure and Einstein's explanation, but their meaning is rendered clear by the continuation of his article. According to Whitehead, Einstein's procedure is the procedure to formulate invariant tensor laws to describe physical phenomena in general, and gravitational phenomena in particular, and Einstein's explanation is the explanation in terms of Mach's principle (inertia is determined by matter), and the closely related equivalence principle (inertia and gravity are identical). So, when Whitehead writes: "Einstein's [...] discovery of the principle and the procedure constitute an epoch in science. I venture, however, to think that the explanation is faulty" (*ESP* 332), this 1920 way of expressing himself is completely in line with his 1922 way of putting things: "My whole course of thought presupposes the magnificent stroke of genius by which Einstein and Minkowski assimilated time and space. It also presupposes the general method of seeking tensor or invariant relations as general expressions for the laws of the physical field, a method due to Einstein. But the worst homage we can pay to genius is to accept uncritically formulations of truths which we owe to it." (*R* 88)

By rejecting Einstein's explanation, Whitehead rejects the principles that were already criticized by de Sitter in 1916 and 1917, and by Eddington and Silberstein in 1918. Whitehead was fully aware that he thus dropped two of the major principles that actually guided Einstein's search for a new law of gravitation. Indeed, Mach's principle and the principle of equivalence "formed the clue by which Einstein guided himself along the path from his principle to his procedure." However, as Whitehead immediately adds: "It is no novelty to the history of science that factors of thought which guided genius to its goal should be subsequently discarded. The names of Kepler and Maupertuis at once occur in illustration." (*ESP* 332) Of course, the rejection of Einstein's explanation implies the challenge to offer an alternative explanation. No wonder Whitehead ends his 1920 article with a brief outline of such an alternative.

Whitehead's alternative theory of gravitation

Whitehead writes that his alternative theory of gravitation starts from the general theory of time and space which is explained in his *Enquiry*, in other words, from Minkowski's space-time, and that it also starts "from Einstein's great discovery that the physical field in the neighborhood of an event-particle should be defined in terms of ten elements" (*ESP* 342), meaning that the gravitational field should be defined as a symmetrical second rank tensor.

Einstein's gravitational field tensor is called the fundamental tensor, and does not define a gravitational field apart from space-time, but space-time as being equal to the gravitational field. Contrary to Einstein, Whitehead calls his gravitational field tensor the impetus tensor, and he uses it to define the gravitational field against the background of Minkowski's space-time. This implies that Whithead keeps field physics and space-time geometry apart, as Silberstein did in 1918. Consequently, he writes: "According to Einstein such elements [the ten elements of the gravitational field tensor] merely define the properties of space and time in the neighborhood. I interpret them as defining in Euclidean space [or better: in Minkowskian space-time] a definite physical property of the field which I call the 'impetus."' (*ESP* 342)

Einstein's law of gravitation equates two tensors: the Einstein tensor, which results from second order differential operations on the elements of his fundamental tensor; and the energy-momentum-stress tensor, which represents the source of gravitation. However, Whitehead's treatment of gravitation focuses on point-masses (event-particles) as the sources of gravitation, and it does not take into account the more general case of a continuous mass-energy distribution. Hence, when comparing his law of gravitation with Einstein's, Whitehead is only taking into account the case in which the energy-momentum-stress tensor is the zero tensor — the case in which Einstein's law equates the Einstein tensor with the zero tensor, hence expressing the vanishing of this invariant tensor.

Consequently, differentiating his law from Einstein's law, Whitehead writes that "the essence of the divergence of the two methods lies in the fact that my law of gravitation is not expressed as the vanishing of an invariant expression, but in the more familiar way by the expression of the ten elements in terms of [...] what I call the 'associate potential."' (*ESP* 342) Whitehead means that the elements of Einstein's fundamental tensor are determined by solving the equation 'Einstein tensor = 0,' whereas the elements of his impetus tensor are determined by some kind of potential.

The gravitational potential Whitehead refers to, is a scalar potential satisfying the wave equation. In fact, it is the scalar and gravitational equivalent of the retarded Liénard-Wiechert four-vector potential of electrodynamics, as expressed by Cunningham in 1914. So, whereas in Einstein's procedure the ten elements of the fundamental tensor are solutions of a tensor equation, Whitehead's alternative defines the ten elements of the gravitational field tensor in terms of a single scalar — a Liénard-Wiechert-like retarded potential satisfying the familiar wave equation. In other words, Whitehead describes the gravitodynamic relation between point-masses in terms of an electrodynamic-like retarded potential, the way Minkowski pointed out in "Space and Time."

Enough has been said about Whitehead's 1920 article to claim that Whitehead's alternative theory of gravity was a Minkowkian theory, and that it was largely developed by February 1920. In his *Enquiry*, Whitehead had not explicitly addressed, let alone answered, the question: "How to reinterpret Einstein's new law of gravitation in terms of a gravitational field against the background of Minkowski's space-time?" However, the April Preface of his 1919 book *did* make it clear that this was the next question on his agenda. Hence we are led to the conclusion that Whitehead developed his alternative theory of gravitation during the academic year 1919–1920, most likely in conjunction with the development of his postgraduate lecture courses on relativity.[52]

Apart from his 1920 article, "Einstein's Theory," Whitehead also gave an outline of his Minkowskian theory of gravitation in his 1920 lecture for the students of the Chemical Society of the Imperial College of Science and Technology (Chapter VIII of *CN*). However, his most elaborated account was offered in 1922, in *The Principle of Relativity*. For further details on its content, I must refer the reader to the book itself, for the aim of this paper was not to present a fully-fledged account of Whitehead's theory of gravitation. My only aim was to put it in the relevant historical context of physics.

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[52] Cf. footnote 44.

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