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### A GRANULAR ACCOUNT FOR GRADABLE ADJECTIVES

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In this paper I consider one kind of vague linguistic expression: adjectives like *tall, big, expensive*. These are called *gradable adjectives*. The most well-known linguistic theories that account for them are the so-called degree-based theories. In this paper I present a formal model that accounts for vague gradable adjectives as an alternative to degree-based theories. The model is built on two basic ingredients: (i) comparison classes and (ii) granular partitions. (i) Comparison classes are introduced to account for the context-sensitivity of vague adjectives. The extension of the predicate *being tall* in the comparison class of men is different from its extension in the comparison class of precision, each of them corresponding to a granular level of observation. The finer the level is, the more differences between the individuals are detected. Granular partitions are used to represent indistinguishability relations between objects with respect to the properties expressed by vague adjectives.

The paper is divided into five sections: in section 1, I present the linguistic features of gradable adjectives; in section 2, I present the degree-based theories and, in section 3, I raise some objections to them. Section 4 is the central part of the paper: in it I sketch an alternative account for gradable adjectives. In the conclusion I try to make a brief comparison between degree-based theories and the model I have developed.

### 1. Gradable adjectives: Linguistic features

Consider adjectives such as *tall, long, expensive*. They are called *gradable* adjectives and are characterized by the following features:

- they can occur in a predicative position, that is, after verbs such as 'be', 'become', 'seem'
- they can be preceded by degree modifiers such as 'very', 'clearly'
- they can be made into comparatives and superlatives

Most of (or probably all) gradable adjectives have polar counterparts. Examples of polar pairs are: *tall/short*, *expensive/cheap*, *big/small*, *clever/stupid*.

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Each adjective belonging to a polar pair can be classified as positive or negative. Such a classification is based on some empirical features demonstrated by the adjectives themselves. Measure phrases can be associated with positive adjectives, but not with negative ones (you can say "John is 178 cm tall" but not "John is 178 cm short"). Negative adjectives allow downward entailments, while positive ones allow upward entailments. Consider, for instance, the pair *safe/dangerous: safe* is negative, *dangerous* positive. From "It is dangerous to drive in Paris" you infer "It is dangerous to drive fast in Paris" but not the reverse, and from "It is safe to drive fast in Des Moines" you infer "It is safe to drive in Des Moines", but not the reverse.

Consider, by contrast, non-gradable adjectives. Examples of non-gradable adjectives are *married*, *female*, *bachelor*. They seem to have a fixed meaning, given by a definition or stipulation to which every speaker can refer. For instance, the meaning of *bachelor* can be given by a definition such as 'man not-married'. Moreover, non-gradable adjectives cannot be modified by degree adverbs (unless your intention is to produce an emphatic effect) and cannot be made into comparatives nor superlatives.

Gradable adjectives can be distinguished between relative and absolute (Kennedy, 2007):

Absolute Adjectives like *wet*, *closed*, *flat* have positive forms that relate objects to maximal or minimal degrees, and are not affected by the Sorites paradox, nor do they have borderline cases. Consider, for instance, *wet*: *wet* requires its argument to have a minimal degree of the property it describes, therefore it is called a *minimum standard* absolute adjective. By contrast, its polar counterpart, *dry*, requires its argument to possess a maximal degree of the property in question: it is thus called a *maximum standard* absolute adjective.

Relative Adjectives like *tall*, *big*, *expensive* have the following features:

• Context-sensitivity: the extension of the predicates generated by relative adjectives changes from context to context. This also means that a sentence containing a relative gradable adjective can get a different truth value depending on the context of utterance. For example, sentence (1),

(1) John is tall,

can be true if John is compared within the class of men, but false if compared within the class of basketball players. Contextsensitivity can be thought of as a problem of the shifting standards from context to context: in each context or comparison class to which John belongs, the meaning of (1) can vary in that

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the standard of tallness can vary across contexts or comparison classes (Fara, 2000)

- Borderline cases: there are cases where it is difficult to determine whether the property expressed by an adjective can be attributed to some object. Moreover, there is no clear sharp boundary between a positive and a negative polar relative adjective
- Sorites-sensitivity: every relative gradable adjective can give rise to a Sorites paradox. Consider a certain number of men, such that each of them differs from any other by at most 0.5 millimeter. Suppose the intent is to order them in a series from the shortest to the tallest and suppose that the shortest is 160 cm, the tallest 190 cm tall. You look at the shortest man and you say that he is short. Then, you say the same for the second man: he is short as well because there is no relevant difference between him and the first man. If you go on in this way, comparing the *n* to the *n* + 1 man, towards the whole series, you will conclude that the tallest man is short as well

Following the characterisation of vagueness outlined by Keefe and Smith (1997), relative gradable adjectives turn out to have all the features of vague expressions: they do not have definite boundaries, they generate borderline cases and they are affected by the Sorites paradox. I will hereafter focus on this class of adjectives.

Because of the vagueness of relative gradable adjectives, sentences containing them cannot easily have a definite interpretation. Moreover, such sentences can have different truth values in different contexts, since the adjectives are context-sensitive. In order to determine the truth value of "x is  $\phi$ ", with  $\phi$  a relative gradable adjective, you have to determine the meaning of x and the features of the utterance context, and make a "judgment of whether x counts as  $\phi$  in that context"(Kennedy, 2001b, p. 34). A semantic analysis of relative gradable adjectives will then make such a judgment possible, giving to the sentence a definite interpretation and at the same time ensuring difference of interpretations across contexts.

### 2. Degree-based theories

In this section, I present the most well-known linguistic theories that account for gradable adjectives: the so-called degree-based theories (Seuren, 1973; Cresswell, 1976; Klein, 1991). According to them, gradable adjectives are analysed as relations or functions from objects to degrees on a scale. Such a scale is an abstract representation, that is a set of elements under a total ordering. Each of those elements is a *degree*. So, a sentence like "x is  $\phi$ "

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is true iff the degree to which x is  $\phi$  is at least as great as the degree on the same scale that represents the standard of  $\phi$ -ness.

Comparatives seem to get a simple treatment within a standard degreeapproach. Comparatives define ordering relations between degrees d on some scale S. A sentence like

(2) John is taller than Mary,

is analysed as follows:

(3)  $\exists d([d > \iota d'.tall(Mary, d')] \land [tall(John, d)]).$ 

Take an antonymous pair of adjectives, like *tall* and *short*: they both define (relations or) functions on the same scale, that is, the scale of height. The ordering relations they give rise to are reversed. Let  $\phi_{pos}$  and  $\phi_{neg}$  be functions representing respectively the positive and negative adjectives associated with a scale S.  $\phi_{pos}$  denotes a function from objects to degrees of S and orders the objects according to the relation  $<_{\delta}$  ('less than' on the scale of degrees).  $\phi_{neg}$  denotes a function from objects to degrees of S and gives rise to an ordering according to the relation  $>_{\delta}$  ('greater than' on the scale of degrees). Positive degrees, i.e. degrees associated with  $\phi_{pos}$ , are the same objects as negative degrees, i.e. degrees associated with  $\phi_{neg}$ . Therefore, (2) and (4) can be proved to be equivalent:

(4) Mary is shorter than John.

A degree-based account is not able, though, to explain the anomaly of the so-called cross-polar phenomenon. One instance of this phenomenon is (5):

(5) \* John is taller than Mary is short.

(5) is true whenever the degree of John's height exceeds the degree of Mary's height on a scale of height. (5) turns out to be interpretable and, even worse, logically equivalent to (2) and (4), which are not anomalous but perfectly acceptable.

# 2.1. Intervals

Kennedy (1997), (2001a) and (2001b) criticises the standard degree-based approach because of its incapacity to explain the cross-polar anomaly as in (5). He proposes another approach, based on degrees not taken as points, but as intervals, or extents, on a scale.

Kennedy considers it necessary to make a sortal distinction between positive and negative degrees, and at the same time he wants to assure the equivalence between (2) and (4). Antonymous pairs of adjectives convey the same kind of information about an object: for instance, *tall* and *short* both refer to an object's height. What differs is the perspective from which they consider the projection of any object on some scale. A positive adjective like *tall* has a 'down-up' perspective towards some object x, a negative adjective like *short* has an 'up-down' perspective towards the same object x.

Kennedy defines a scale S as a linearly ordered, infinite set of points. Each scale represents some type of measurement: height, length, weight, etc. A degree is not defined as a point on S, but as a convex, nonempty subset of S, and it is called 'extent' or 'interval'. Given a scale S, the set of positive degrees and the set of negative degrees are defined in such a way that they are disjointed. Moreover, given an antonymous pair, the maximal point of the interval identified by the positive adjective coincides with the minimal point of the interval identified by the negative adjective. So, the positive and negative projections of an object x on S are complementary intervals on S.

Gradable adjectives are thought of as functions from objects to intervals. More precisely, positive adjectives denote functions from objects to positive intervals and negative adjectives functions from objects to negative intervals. Two antonymous adjectives, then, have the same domain but different ranges; they map the same objects onto complementary regions of the same scale. The interval-based theory developed by Kennedy gets the same positive results as the degree-based ones. For instance, (2) and (4) turn out to be logically equivalent. Nevertheless, Kennedy's theory is also able to overcome the difficulties of the degree-based approach. Above all, cross-polar anomalies are not acceptable in the interval account. Consider (5): such a sentence is not acceptable because degrees associated with tall and degrees associated with short are not comparable, since they are different sorts of objects. (5) is true iff there is an extent that properly includes the extent of Mary's shortness, and John is tall to that extent. But the extent argument of a positive adjective must be a positive extent, and positive extents can include only positive extents. Similarly for negative adjectives: their extent arguments must be negative, and negative extents can include only negative extents. So, in order for (5) to be true, the argument of the positive adjective tall has to be a negative extent. That is not the case and (5) turns out to be anomalous.

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### 3. Objections to degree-based theories

There are some objections that can be raised against both the standard degreebased theory and Kennedy's theory of intervals. I list here three main objections. Objection (1) concerns the ontological commitments of the approach, objection (2) involves some cognitive aspects, and objection (3) is about the interpretation of the positive form of adjectives.

(1) What kind of objects are degrees? Does their use necessarily lead one to some ontological commitments? What is in doubt here is why we should assume a class of abstract objects. If we add a scale of degrees to our ontology, we have to justify it and say why we need to assume the existence of abstract objects. The ontology we get by adding scales of degrees is quite large, especially if we take a realvalued scale as the scale of degrees. The question is: is it necessary to have such a large ontology and admit infinite abstract objects to account for vague adjectives? The same question can also be raised for Kennedy's theory: what kind of objects are intervals? Kennedy is aware of this ontological question and claims to address a similar question in (Kennedy, 2001b). In that paper Kennedy's train of thoughts seems to be the following: if you want to give the right interpretation to gradable adjectives, you need to replace degrees with intervals. But Kennedy does not explain why you need a class of abstract objects in order to account for vague adjectives. He seems to recognise this problem, though, and in footnote 3 he refers to another article by himself (Kennedy, 1999), where he tries to show why approaches that do not make use of measure theories fail. Kennedy (1999) takes into account the theories that account for gradable adjectives by analysing them in terms of partial functions. According to Kennedy, those theories do a good job of explaining most of the semantic properties of gradable adjectives, but are not able to explain the behaviour of antonymous adjectives in comparatives (neither the anomalies, nor the normal uses). For this reason, that is, showing that the alternative theories that are on the market at the moment fail to grasp some phenomena, he argues for the necessity of an intervalbased approach. However, this kind of argument does not address the ontological problem directly. To the question of why we need abstract objects he replies: we need abstract entities because all the other alternatives given by now fail to grasp some phenomena that an interval-based theory does. But this is an *ad hoc* argument. That intervals work well is not a *sufficient* reason to make people believe in abstract things.

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- (2) When children learn to use relative gradable adjectives, they are taught that an individual is tall and another is short while comparing those individuals between them or within a class of individuals that differ from them. A child does not measure the difference between the individuals she sees, nor has she any clue about what a centimetre is, but despite all this she can learn and properly use *tall* and *short*. So, it seems that relative gradable adjectives can be used without the notion of measurement. On such a notion the degree-based theory is based. But if we do not need to refer to measures to use vague adjectives, why should we use measures to model our use of adjectives? Can we account for gradable adjectives without measures and degrees?
- (3) Consider the behaviour of positive and comparative forms of adjectives according to the degree and interval approaches: take, for instance, the predicate *tall*. First, the meaning of the expression 'tall to degree d' is determined. Then, the comparative 'taller than' is defined over 'tall to degree d'. Finally, the meaning of the positive form *tall* is defined over the meaning of the comparative. The dependence of positive form from the comparative form is controversial. Most linguists tend to prefer a different treatment of the positive form of adjectives, namely to take that as a primitive (function or relation) and define the comparative form on it.

In the degree and interval approaches considered above, all the properties are assumed to be measurable. My point is that such an assumption is not necessary and that it is possible to think of an alternative theory that explains how we use relative gradable adjectives without assuming degrees, nor intervals, and such that it does not run into objections (1)–(3). I have tried to develop an alternative theory that accounts for relative gradable adjectives, both in their positive and comparative forms, without committing to an infinite number of abstract entities as degrees, or taking measures as primitives. Moreover, the meaning of the comparative of the adjective will be defined over the meaning of the positive form. This means that first you determine the meaning of *tall*, then you define the meaning of 'taller than' by using comparison classes and constraints on the behaviour of the adjective functions in comparison classes. Such an attempt goes back to Kamp, 1975, and Klein, 1980, and takes van Rooij's suggestions as a reference point (van Rooij, 2008, 2009).

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### 4. A Model for Polar Adjectives

In this section I shall briefly sketch a model that tries to capture the intuitions of English speakers concerning their usage of relative gradable adjectives<sup>1</sup>.

I want to start with an observation that guided my research: not all the uses of vague adjectives are problematic. When there is a relevant difference between two objects with respect to some property, we describe them using a pair of polar adjectives without uncertainty. The idea is that if we can model the observations made for the unproblematic case, we can cast light on the problematic ones (e.g. Soritical series) and make clear the semantics of adjectives in those cases, too.

The model I shall introduce is built on two basic ingredients: comparison classes and granular partitions.

- Comparison classes: They are introduced to account for the context-sensitivity that vague adjectives show. Some constraints are put on the primitive functions standing for adjectives in order to make them behave in a different way in each comparison class. Such constraints also make it possible to define a weak ordering relation that represents the comparative relation (van Benthem, 1982; van Rooij, 2009).
- Granular partitions: You can look at the elements of a context under different standards of precision, each of them corresponding to a granular level of observation. The finer the level is, the more differences between the individuals are detected (Hobbs, 1985). For instance, consider a series of individuals who differ from each other by 0.5 mm with respect to their height. We cannot see any difference between the individuals with the naked eye: all of them are equally tall. However, making use of an instrument of measurement, we can detect some difference between them. On the basis of the similarities and differences observed, we can partition the domain into groups of objects, called granular partitions. To represent granular partitions in the formal model, a relation of similarity is defined over the comparative relation. Equivalence classes, then, are generated: all the elements belonging to an equivalence class are considered indistinguishable with respect to the property expressed by the adjective considered. Equivalence classes are thought of as granular partitions.

Consider now the formalisation of such ideas. Define first a language  $\mathcal{L}$  consisting of:

- individual constant symbols:  $\overline{j}, \overline{m}, \overline{s} \dots$
- individual variable symbols:  $x, y, z \dots$
- monadic predicates: A, B, C ...

<sup>1</sup> For a more detailed presentation of the model see Gaio, 2008.

- functions:  $P, Q, R \dots$
- usual logical connectives with identity, quantifiers

The set of terms consists of individual constant and individual variable symbols. Formulas are defined in a standard way.

Consider now the following interpretation of  $\mathcal{L}$ :

- Let  $\mathcal{D}$  be a domain of objects
- Let individual constant symbols represent proper names: *John, Mary, Sue ...*
- Let monadic predicates represent count nouns like *man*, *tree*, *cat*. They are interpreted in an extensional way: they select some objects of  $\mathcal{D}$ . Their extensions are called *comparison classes*, *c*. Let *CC* be the set of comparison classes *c*
- Each subset of a comparison class c is called *context*. Given a comparison class c, let  $O_c$  be the set of all subsets (contexts) o of c:  $O_c =_{def} \wp(c)$
- Each function such as P represents a gradable adjective (e.g. *tall*, *big*, *fat* ...). Fixed a comparison class c, each function maps individuals of a context  $o \in O_c$  to P(o). For instance, if the interpretation of P is *tall*, given some context  $o \in O_c$ , P(o) is the set of individuals that are tall in context o
- $\overline{P}$  is defined as the counterpart of P:  $\overline{P}(o) =_{def} \{x \in o : x \notin P(o)\}$ . For example, if P is interpreted as *tall*,  $\overline{P}$  is interpreted as *short*, that is, its polar counterpart<sup>2</sup>.

### 4.1. Across Comparison Classes and Contexts

Consider now how to account for the cross-contextual change of meaning of relative gradable adjectives. Some constraints can be put on functions P, Q, R in order to make them behave in a different way in each comparison class, and produce an ordering relation. Consider the cross-contextual constraints given by van Benthem (1982), based on the concept of difference pair (DP):

Definition 1: Two elements x, y form a difference pair in a context o iff x is in P(o) and y in  $\overline{P}(o)$ .

The constraints are the following:

Upward Difference (UD)

Let  $\langle e, e' \rangle$  be a difference pair in a context o. In each context o' such

<sup>2</sup>Here P and  $\overline{P}$  are considered contradictories. The model could be improved to treat P and  $\overline{P}$  as contraries.

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that  $o \subseteq o'$ , there exist different pairs.

Put otherwise, if in a context o one element, e, is tall, another, e', short, (UD) makes sure that all the supersets of o will contain at least one element that is tall and one that is short. Those elements are not necessarily e, e'.

Let  $\langle e, e' \rangle$  be a difference pair in a context *o*. There is no context *o'* such that  $\langle e', e \rangle \in DP(o')$ .

If in a context o one element e is tall and another e' short, in any other context o' the reverse cannot be the case. Maybe both e and e' are tall, or short, but it can never be the case that e' is tall and e short.

Downward Difference (DD)

Let  $\langle e, e' \rangle$  be a difference pair in a context o. In each context o' such that  $o' \subseteq o$  and that it includes e, e', there exist some difference pairs.

If e is tall and e' short in a large context o, in a smaller context o' containing e, e' there will be difference pairs, too.

Given such constraints, the comparative relation  $>_P$  (to read: 'more *P* than') can be defined as follows:

# Definition 2: $x >_P y$ iff $x \in P(\{x, y\}) \land y \notin P(\{x, y\})$ .

The relation  $>_P$  gives rise to a *weak order*, i.e. a structure  $\langle I, R \rangle$  with R a binary relation on a domain I that is irreflexive, transitive and almost-connected.

The relations 'being as P as' (i.e. the similarity relation  $\sim_P$ ) and 'being at least as P as' ( $\geq_P$ ) can also be defined as follows:

# Definition 3: $x \sim_P y$ iff it is not the case that $x >_P y$ nor $y >_P x$ .

Definition 4:  $x \ge_P y$  iff  $x >_P y$  or  $x \sim_P y$ .

### 4.2. Across Granular Levels

The conditions for comparatives do not uniquely determine the behaviour of function P. As mentioned before, an important aspect to take into account when modelling gradable adjectives is their meaning-shifting across granular levels of observation. As Luce (1956) highlights, the non-transitivity of indifference relations reflects human inability to discriminate with precision among things that do not differ much one from the other. Luce's consideration on this point perfectly fits our problem with vague adjectives. We cannot always make precise distinctions between two objects with respect to some

observable property. That is why we get into trouble with Sorites series. Nevertheless, if we have some more precise standard of precision or a better way of measurement, we can detect more differences between the elements we consider. According to different standards of precision, called *granular levels*, we can have various models that give rise to different orderings of the objects of our domain. Let us see how this works in a more detailed way.

Let M be a granular structure of type  $\langle \mathcal{D}, CC, P \rangle$ . A granular structure is a tuple containing a fixed domain, the interpretation of the monadic predicates (that is, the set of all comparison classes), and a function P. Different granular structures can give rise to different  $>_P$  orderings for the same set of contexts. That means that some granular structures detect more differences between the elements in the contexts than other granular structures.

Given a comparison class, c, take a context  $o \in O_c$ . Since any granular structure provides us with an equivalence relation  $\sim_P$ , equivalence classes partitioning the context are obtained. Equivalence classes are groups of objects, that according to that specific granular structure turn out to be indistinguishable:

Definition 5: Let  $a \in o$ . Define the equivalence class of e under  $\sim_P as$  follows:

$$[e]_{\sim_P} =_{def} \{ x \in o : x \sim_P e \}.$$

Different granular structures can give rise to different partitions, and therefore to different orderings between objects. Consider the following example.

Example1. Given some comparison class c, let  $o \in O_c$  be a context with three elements:  $o = \{a, b, c\}$ . Consider a function T, representing the adjective *tall*. We can have the following orderings for o:

- a ~<sub>T</sub> b ~<sub>T</sub> c. The granular structures modelling such an ordering give rise to only one equivalence class; all the elements are considered equal with respect to T, so we cannot have a distinction between objects that are T and objects that are T. The contexts structures giving such an ordering are the coarsest ones
- $a >_T b >_T c$ . The granular structures that model such an ordering give rise to three partitions:  $[a]_{\sim_T}$ ,  $[b]_{\sim_T}$ ,  $[c]_{\sim_T}$ . Each object is different from the others with respect to T. The granular structures giving such an ordering are the most fine-grained
- Between the coarsest and the finest orderings, there can be a third ordering: either  $a \sim_T b >_T c$  or  $a >_T b \sim_T c$ . If a granular structure gives rise to the first ordering, it gives two partitions:  $[a]_{\sim_T}$  and  $[c]_{\sim_T}$ . If it gives rise to the second ordering, then it again gives two partitions, but different ones:  $[a]_{\sim_T}$  and  $[b]_{\sim_T}$

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We can partially order the granular structures from the coarsest to the finest with respect to any context  $o \in O_c$ . I do not go into the question of how to obtain such an ordering. Rather, consider the following problem: For contexts with more than two equivalence classes, even if two granular structures give rise to the same  $>_P$  ordering, they might have different functions of type P and  $\overline{P}$ . Let me present an example to make the problem clear.

Example2. Consider three fellows, John, Bill and Marc. John and Bill are much taller than Marc. Suppose that John is 185 cm tall, Bill 182 cm, Marc 172 cm<sup>3</sup>. Looking only at those three fellows, you are quite sure about who is tall and who is short and you will naturally say that John and Bill are tall and Marc short. That is not a problematic case, like a Sorites-series or borderline cases. Let T stand for *tall* and T for *short*. Consider the comparison class of men and the following context in it: o = j, b, m, with j, b, m names for John, Bill, Marc, respectively. Granular structures can give the following orderings with respect to o:

(1)  $j \sim_T b \sim_T m$ 

- (2)  $j \sim_T b >_T m$ (3)  $j >_T b >_T m$

The granular structures that give rise to the coarsest ordering as in (1) are not considered. Ordering (2) can be generated by granular structures that agree with the extensions for T and  $\overline{T}$  in o:  $T(o) = \{j, b\}, \overline{T}(o) = \{m\}$ . Some problems arise with ordering (3). In fact, the ordering  $j >_T b >_T m$  can be generated by two sets of granular structures:

- Set 1: T(o) = {j,b}, T(o) = {m}
  Set 2: T(o) = {j}, T(o) = {b,m}

The granular structures belonging to Set 1 give us what we expect: John and Bill are tall, Marc is short. The granular structures of Set 2, instead, give a result that sounds wrong to us: John is tall, Bill and Marc are short. We would never say that Bill is short, probably because there is a lower difference in height between John and Bill than between Bill and Marc, and we are led to conclude that the boundary between *tall* and *short* cannot be between John and Bill, but rather between Bill and Marc. So, can we put some constraints to exclude granular structures with an adjectival function that behaves in an unnatural way, i.e. not according to our intuitions?

The suggestion is the following: given a comparison class c and a context  $o \in O_c$ , consider also the context  $o^c$ , defined as follows:

Definition 6:  $o^c =_{def} \{z \in c \mid \exists x, y \in o : x \geq_P z \geq_P y\}.$ 

<sup>3</sup> I refer to their height measures only in order to make it easy to visualize the scenario.

Any context o contains some elements of c. Fixed a set of granular structures that give rise to the same ordering for all contexts in  $O_c$ , and given some context  $o \in O_c$ , we consider  $o^c$ , that is, we consider the elements of o together with the elements in the domain of c that are 'in-between' the elements of o in the  $>_P$ -order raised by the set of granular structures considered.

But how can we guarantee that there are enough elements 'in-between'?

The intuition is the following. When you consider contexts, you look at real objects. Namely, when you use gradable adjectives you want to judge on the basis of some situation in the world. However, to correctly associate gradable adjectives with real objects, you can think to add possible objects that are 'in-between' the real objects according to the comparative relation. That means, if John and Bill are men who relevantly differ in their height, we say that John is tall and Bill short because there can be other men, whose height differs and is less than John's but more than Bill's. This intuition goes together with the fact that all vague relative adjectives suffer from the Sorites paradox: the crucial point of the Sorites paradox is that we have a series of objects, such that there are small differences between every two objects that are contiguous in the series. I want to model vague relative adjectives that give rise to Sorites series. So I need to assume that I can have a domain of individuals that are 'equally distributed' with respect to a property. Put otherwise, what I want is each set to have possible objects that form a Sorites series, and each real object to correspond to one of the 'possible' objects of the domain. So, a condition needs to be imposed to each comparison class:

Given a comparison class c, each element of c is observably indistinguishable from at least two others. It might happen only for two elements that each of them is indistinguishable from another element<sup>4</sup>.

Now, some constraints can be put on the extension of P and  $\overline{P}$  for any  $o^c$ . The first constraint is the following:

$$|\{[x]_{\sim_P} \in P(o^c)\}| = |\{[x]_{\sim_P} \overline{P}(o^c)\}| \pm 1.$$
(E)

*E* says that, given a *fulfilled* context  $o^c$ , half of the elements of  $o^c$  are *P* and half are  $\overline{P}$ .

Now, consider a set of granular structures that give rise to the same ordering, and a context  $o \in O_c$ . Let us accept only the granular structures that

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<sup>&</sup>lt;sup>4</sup> Those two elements are predicted to be the minimal and the maximal element of the set of individuals when ordered. For the formalisation of such a condition see Gaio, 2008, p. 261.

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make the following formula true:

$$\forall x \in o : x \in P(o) \text{ iff } x \in P(o^c). \tag{R}$$

*R* says that the elements in *o* are *P* in *o* if and only if they are *P* in the *fulfilled* context  $o^c$ . *R* is the constraint that restricts the set of granular structures, allowing exclusively the granular structures that correctly say which objects are *P* and which are  $\overline{P}$ .

Go back now to Example2. By the condition imposed on comparison classes, you get a context  $o^c$  containing fellows that are one-to-one indistinguishable in height. Among them there are j, b, m. Now, applying E you get that half of the individuals are tall, and half short. Look now at where j, b and m are in  $o^c$ : j and b are in the extension of T, and m is in the extension of  $\overline{T}$ . Applying then R, the elements in o that are tall in  $o^c$  must be tall in o too. So, also in o, j and b are in the extension of T, and m is in the extension of  $\overline{T}$ . Only granular structures of Set 1 are then accepted, while granular structures of Set 2 are ruled out.

### 5. Conclusion

In this paper, I presented a formal model to account for relative gradable adjectives, which differs from the degree-based accounts in the fact that it does not assume any scale of degrees and it does not run into objections (1)–(3). Nevertheless, an objection to the model could be the following: The model presents an account for gradable adjectives that is much more complex than the degree-based accounts. You have to take care of granular levels, comparison classes and contexts, while the degree-based theories need only a scale of degrees to explain how gradable adjectives are used. I would like to reply to such an objection with some observations. First of all, my goal is to respect our intuitions on the behaviour of gradable adjectives, and our intuitions can be not always formalised in a simple way. In fact, I want to take into account many aspects that characterise gradable adjectives, namely vagueness, context-sensitivity, granular-sensitivity, and not just gradability. Of course, taking into account more aspects than the degree-based theory makes the account more complex. Secondly, while degree-based theories add degrees to the ontology, I do not add granular levels, comparison classes and contexts as independent elements to the ontology: They are just ways to look at or to group the elements in the domain and not elements added to the domain. What I add to the ontology are what I called 'possible' objects. But I gave a justification of such an operation: I included those elements in the domain because they can build Sorites-series, and since I want to explain how

vague adjectives apply to Sorites-series, I need to have the possibility to arrive at Sorites-series in the domain. By contrast, degree-based theories add degrees as a tool to explain the behaviour of adjectives, without justifying such an operation<sup>5</sup>.

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