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BELIEVING AND ASSERTING CONTRADICTIONS

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Abstract

The debate around "strong" paraconsistency or dialetheism (the view that there *are* true contradictions) has — apart from metaphysical concerns — centred on the questions whether dialetheism itself can be definitely asserted or has a unique truth value, and what it should mean, if it is possible at all, to believe a contradiction one knows to be contradictory (i.e. an explicit contradiction). And what should it mean, if it is possible at all, to assert a sentence one knows to be contradictory?

The investigation of believing and asserting the two sides of a contradiction involves considering the semantic and pragmatic distinctions between asserting, believing, denying, rejecting, disbelieving a sentence, abstaining from an opinion and affirming the opposite. Standard logic with its treatment of negation and denial levels some important distinctions. Given a paraconsistent logic with bivalent truth operators and given an account when to assert a sentence there may be occasions on which it is rational not only to believe a sentence one knows to be contradictory, but also to assert it. Dialetheism turns out to be unambiguously affirmable and avoids sliding into trivialism (that anything is true and false and might be believed or asserted).

Ever since its arrival dialetheism (the thesis that there *are* true contradictions) has been met with the proverbial "incredulous stare", not only because of the inconsistent ontology of Routley's "noneism" (Routley 1979), but also with respect to the dialetheist's claim that one can knowingly believe and assert contradictions. Priest in the paper introducing his "logic of paradox" LP (Priest 1979) admits that the thesis of dialetheism is a dialetheia itself, and seems to be content with this. In his book *In Contradiction* (Priest 1987) he argues that one can avoid dialetheism being a dialetheia itself if one is prepared to give up contraposition for the conditional in Convention (T). Nevertheless he defends that one can believe and assert contradictions. Up to now (see some of the papers in (Priest/Beall/Armour-Garb 2004) or (Field 200x)) criticism of dialetheism has focused on the problems what the status of dialetheism itself is and how it may be possible to believe knowingly contradictions. In this paper it is argued that within dialetheism the resources are available to claim that dialetheism is true only (i.e. not false at the same time). Furthermore there may be occasions on which it is rational to believe and/or even assert contradictions, without thereby positioning oneself on a slippery slope towards an attitude of "anything goes".

1. *Some Levels of Commitment*

Concerning a sentence α there are several levels of commitment. Consider:

(1)	α is true.	[$T\ulcorner\alpha\urcorner$, "T" being here a truth *predicate*]
(2)	Believing α is true.	[$BT\ulcorner\alpha\urcorner$]
(3)	Rather concede α.	
(4)	Affirm α (assert that α is true).	[$A\alpha$]
(5)	Abstain from an opinion on α.	[$\neg B\alpha \wedge \neg B\neg\alpha$]
(6)	Disbelieve α.	[$\neg B\alpha$]
(7)	Reject α.	[$R\alpha$]
(8)	Believe the opposite of α.	[$B\neg\alpha$]
(9)	Assert the opposite of α.	[$A\neg\alpha$]
(10)	α is false.	[$F\ulcorner\alpha\urcorner$, "F" being a falsity *predicate*]
(11)	The opposite of α is true.	[$T\ulcorner\neg\alpha\urcorner$]

There seems to be a decrease in commitment to a along this scale:

Assert α

Believe α

Rather concede α

Abstain from α (and $\neg\alpha$)

Disbelieve α

Believe $\neg\alpha$

Assert $\neg\alpha$

Assertion as an speech act usually done in face of an audience commits one, at least *prima facie*, to provide reasons for one's beliefs, if challenged to do so, whereas mere believe need not.
There is a difference between abstaining from a judgement and disbelieving α if one seems to have reasons against believing α, but not against believing $\neg\alpha$.
There is a difference between disbelief and rejection if disbelief is based on seeming to have reasons against believing α, and rejection on positively endorsing some reasons against α. If these reasons are taken as sufficiently strong, one believes $\neg\alpha$.
Conceding α is a state of seeming to have reasons rather in favour of α than of $\neg\alpha$ (α is epistemically possible).
Given these distinctions at least the following attitudes can be distinguished (in a nicely symmetrical manner):

$\mathrm{A}\alpha$

$\mathrm{B}\alpha$

$\neg\mathrm{B}\neg\alpha$

$\neg\mathrm{B}\alpha \vee \neg\mathrm{B}\neg\alpha$

$\neg\mathrm{B}\alpha$

$\mathrm{B}\neg\alpha$

$\mathrm{A}\neg\alpha$

"$\neg\mathrm{B}\alpha$" may cover disbelief, $\mathrm{B}\neg\alpha$ then being "believing the opposite". $\mathrm{A}\neg\alpha$ is asserting the opposite. For this speech act the term "rejection" ($\mathrm{R}\alpha$) might be appropriate.[1]
Given a general theory of truth obeying the (T)-scheme

(T) $\mathrm{T}\ulcorner\alpha\urcorner \equiv \alpha$

a rational reasoner supposedly should satisfy:

(12) $\mathrm{A}\alpha \equiv \mathrm{AT}\ulcorner\alpha\urcorner$

(13) $\mathrm{B}\alpha \equiv \mathrm{BT}\ulcorner\alpha\urcorner$

(14) $\neg\mathrm{B}\neg\alpha \equiv \neg\mathrm{BT}\ulcorner\neg\alpha\urcorner$

[1] Henceforth, at least, "rejection" is used in that sense only.

(15) $\neg B\alpha \equiv \neg BT\ulcorner\alpha\urcorner$

(16) $B\neg\alpha \equiv BF\ulcorner\alpha\urcorner$

(17) $A\neg\alpha \equiv AT\ulcorner\neg\alpha\urcorner$ $[\equiv AF\ulcorner\alpha\urcorner]$

Consistency principles then might be:

(18) $\neg(A\alpha \wedge A\neg\alpha)$

(19) $\neg A(\alpha \wedge \neg\alpha)$

(20) $\neg B(\alpha \wedge \neg\alpha)$

(21) $\neg(B\alpha \wedge B\neg\alpha)$

These principles, of course, seem to forbid anything like dialetheism. What thus seems intuitively so may not be fine grained enough, however, given the occurrence of true contradictions. And standard logic in its treatment of negation may level some distinctions that should be kept.

2. *Standard Negation and Rejection*

In standard logic (PC) rejection is equivalent to assertion of the opposite, since there is no 3rd value. Affirming α is rejecting $\neg\alpha$, and vice versa. Semantically we have:

α	$\neg\alpha$
1	0
0	1

α being not true means α is false, "false" being a synonym for the truth of a negation, expressed with "$\neg$". The (T)-scheme is taken in its contrapositive form as well $[\neg\alpha \equiv \neg T\ulcorner\alpha\urcorner]$. Therefore the standard logician endorses (14)–(17).

With respect to logical truth even standard logic distinguishes $\nvdash \alpha$ from $\vdash (\neg\alpha)$. Having decisive reasons for $\neg\alpha$ (forcing $\neg\alpha$ as a logical truth) is something to be distinguished from not having decisive reasons for α (i.e. $\nvdash \alpha$).

In standard epistemic logic abstaining from a judgement on α is built in as an option by not having

(22) $B\alpha \vee B\neg\alpha$

So "B" ("Believe") does not distribute over "$\vee$". Even assuming $\alpha \vee \neg\alpha$ for truths (states of affairs) does not commit one to have a *tertium non datur* for belief. Typically consistency of belief is demanded:

(23) $\mathrm{B}\alpha \supset \neg\mathrm{B}\neg\alpha \qquad [\Leftrightarrow \neg\mathrm{B}\alpha \vee \neg\mathrm{B}\neg\alpha]$

At least one of a sentence and its negation has to be disbelieved. Expressed with a truth predicate one demands

(24) $\mathrm{BT}\ulcorner\alpha\urcorner \supset \neg\mathrm{BT}\ulcorner\neg\alpha\urcorner$

(25) $\mathrm{BT}\ulcorner\alpha\urcorner \supset \neg\mathrm{BF}\ulcorner\alpha\urcorner$

For the standard epistemic logician

(26) $\mathrm{R}\alpha \equiv \mathrm{AF}\ulcorner\alpha\urcorner$

may be taken as the very definition of "rejection". (26) yields duals like:

(27) $\neg\mathrm{B}\neg\alpha \supset \neg\mathrm{R}\alpha$

(28) $\mathrm{A}\neg\alpha \equiv \mathrm{R}\alpha$

(29) $\neg(\mathrm{R}\alpha \wedge \mathrm{A}\alpha)$

Given a *closed world* assumption (i.e. that all information to be had has arrived) one may even add:

(30) $\neg\mathrm{B}\alpha \supset \mathrm{B}\neg\alpha$

thus

(31) $\mathrm{B}\alpha \vee \mathrm{B}\neg\alpha$

On being asked then one may even have

(32) $\mathrm{A}\alpha \vee \mathrm{A}\neg\alpha$

3. *Conditions for Belief and Assertion*

Truth concerns what is the case whether we believe it or not. Belief concerns what we are willing to include in our inferring. What we believe we take into account in our reasoning (belief is cognitive).

Generally, being provided with reasons for α is seen as the basis for believing α, given that the reasons for some γ incompatible with α are not stronger. This *proviso* depends on the consistency requirement not to have $\mathrm{B}\alpha\wedge\mathrm{B}\gamma$ with $\vdash (\alpha \supset \neg\gamma)$ [respectively: $\vdash (\neg(\alpha \wedge \gamma))$].

On a gullible approach to (perceptual) belief one believes every α one has no reasons against. The best backing for a belief α is a proof of α. Having reasons is superior to mere belief in the truth of α. Having no independent access to the (ultimate) truth of α going with reasons is the rational way,

whatever the (ultimate) truth value of α is or turns out to be.
Given consistency and bivalence assumptions reasons against α may be reasons in favour of $\neg\alpha$, at least in non-empirical domains like semantics where a closed world assumption may be less idealized.
Typically it is taken to be rational to assent to [to affirm] what one believes. Assertion is to assent to or to affirm what one believes. If one has a belief α one also has the disposition to assert α. One does not need additional reasons to proceed from believing to asserting. On the other hand, asserting α is done by a speaker confronting an audience (assertion is pragmatic). Asserting α is done with a purpose in view of an audience, so that this purpose exceeds using α in one's processes of deliberation (this being one's self-satisfied belief that α). As an (speech) act with some purpose asserting α has to meet the basic conditions of successful action plans, like

I. the purpose is not achieved anyhow *without* my action

II. this specific action is *fit* to the purpose.

Asserting contradictions seems to fail both conditions, since there seems to be no specific commitment on the side of the speaker.
Thus the following principle may not be obviously true:

(33) $\neg \mathrm{A}\alpha \supset \neg \mathrm{B}\alpha$

Not having thought about antinomies some additional principles, once again several of them *prima facie* detrimental to dialetheism, however, might be considered obvious:

(34) $\mathrm{A}\alpha \supset \mathrm{B}\alpha$

(35) $\mathrm{B}\neg\alpha \supset \mathrm{A}\neg\alpha \qquad [\supset \neg \mathrm{A}\alpha]$

(36) $\mathrm{A}\alpha \supset \neg \mathrm{A}\neg\alpha$

At least in some versions of *strong belief* one may have:

(37) $\mathrm{B}\alpha \wedge \mathrm{B}\gamma \supset \mathrm{B}(\alpha \wedge \gamma)$

(38) $\mathrm{A}\alpha \wedge \mathrm{A}\gamma \supset \mathrm{A}(\alpha \wedge \gamma)$

(39) $\mathrm{BT}\ulcorner\alpha\urcorner \wedge \mathrm{BT}\ulcorner\gamma\urcorner \supset \mathrm{BT}\ulcorner\alpha \wedge \gamma\urcorner$

Leaving aside problems with the psychological reality of omniscience:

(40) $\vdash (\alpha \supset \gamma), \mathrm{B}\alpha \Rightarrow \mathrm{B}\gamma$

(41) $\vdash (\alpha \supset \gamma), \mathrm{B}\alpha \Rightarrow \mathrm{A}\gamma$

(42) $\vdash (\neg\alpha) \Rightarrow \neg \mathrm{B}\alpha$

(43) $\vdash (\neg\alpha) \Rightarrow \neg \mathrm{A}\alpha$

(44) $\vdash (\neg\alpha) \Rightarrow \mathrm{B}\neg\alpha$

(45) $\vdash (\neg\alpha) \Rightarrow \mathrm{A}\neg\alpha$

If rejection is understood as in (26) one may have

(46) $\mathrm{B}\neg\alpha \supset \mathrm{R}\alpha$ $[\neg\mathrm{R}\alpha \supset \neg\mathrm{B}\neg\alpha]$

If someone does not reject α he does not believe $\neg\alpha$, so α may be "an option". A strong opinionated (closed world) principle might be:

(32′) $\mathrm{A}\alpha \vee \mathrm{R}\alpha$

4. *The Paraconsistent Approach*

Given that standard logic runs into difficulties with antinomies also the principles supposedly governing belief, denial and asserting (the opposite) may need overhauling. Of special interest are now issues related to semantic closure and the formulation of the dialetheist position itself. Conditions to be met by dialetheism are:

I. Dialetheism as a thesis should be asserted as being *only/just true* (i.e. not being false at the same time).

II. One should be able to say, without saying something false, that a true sentence/statement is true.

III. One should be able to express the semantic properties of all sentences/statements (including the antinomies).

In meeting these conditions and — on the way — rejecting several of the above principles the dialetheist has to develop an understanding of denial and rejection which does not equate believing α with disbelieving $\neg\alpha$ and asserting α with rejecting $\neg\alpha$.

Reasons against α that are not reasons for $\neg\alpha$ may be reasons that undermine assumptions which usually support α. If α and $\neg\alpha$ are true at the same time reasons for $\neg\alpha$ are not — cannot be — reasons against α. If neither α nor $\neg\alpha$ has to be true, reasons against α are not *per se* reasons in favour of $\neg\alpha$. On the other hand, since following reasons is the rational way (cf. §2) having reasons for α may lead one to accept α and having reasons for $\neg\alpha$ may lead one to accept $\neg\alpha$ at the same time.

Dialetheism claims that some contradictions are true. So we have some sentence λ with λ, $\neg\lambda$, $\mathrm{T}\ulcorner\lambda\urcorner$, $\mathrm{T}\ulcorner\neg\lambda\urcorner$, $\mathrm{F}\ulcorner\lambda\urcorner$, $\mathrm{F}\ulcorner\neg\lambda\urcorner$ to start with. The reasons for this are that these contradictions are provable given some unassailable principles and structures in a semantically closed language. Now, these antinomies being *true* and being *justified* as true, by proving them, give all the reasons to *believe* that they are true and thus to believe them (themselves). So a dialetheist should *believe*

(i) The Liar is true.

thus

(ii) The Liar

and thus (by the definition of the Liar)

(iii) The Liar is false.

Giving up believing what one has proven seems to be a desperate and *ad hoc* manoeuvre. So a dialetheist has inconsistent beliefs. She reasons *using both* $\mathrm{T}\ulcorner\lambda\urcorner$ and $\mathrm{F}\ulcorner\lambda\urcorner$ if necessary.

Paraconsistent logics can level the distinction between object and meta-language. A semantically closed language not only is able to talk about its own expressions, but does contain at the same time its semantic expressions. These semantic expressions need not be taken as predicates (like a truth predicate applying to the quotation of a sentence), but can be taken as operators instead. One arrives at a paraconsistent language/logic which allows truth value talk without previously quoting the sentences which are evaluated.

To fulfil the condition of dialetheism being expressible we need bivalent truth operators working in the fashion of the following table:

α	$\neg\alpha$	$\mathrm{T}\alpha$	$\mathrm{F}\alpha$	$\Delta\alpha$	$\nabla\alpha$	$\circ\alpha$	$\bullet\alpha$
0	1	0	1	0	1	1	0
1	0	1	0	1	0	1	0
0,1	0,1	1	1	0	0	0	1

"$\Delta\alpha$" says that α is true only, "$\nabla\alpha$" that α is false only, "$\circ\alpha$" says that α is consistent (i.e. has only one truth value), "$\bullet\alpha$" says that α is contradictory. We can then say — and these being *just* true — that the Liar is true, false, not simply true, not consistent, and so on. "T" and "F" are *now* understood as *operators* applying to formulas/sentences not to quoted formulas/sentences. Thus dialetheism can fulfil the traditional condition on any decent theory: that it claims to be just true (and not only as true as its negation). Dialetheism is thus no form of trivialism (that everything is true). The trivialist proposes $(\forall\alpha)(T\alpha\wedge\mathrm{T}\neg\alpha)$ or $(\forall\alpha)(T\alpha\wedge\mathrm{F}\alpha)$. The dialetheist claims $(\exists\alpha)(\mathrm{T}\alpha\wedge\mathrm{F}\alpha)$, but also $(\exists\alpha)(\mathrm{T}\alpha\wedge\neg\mathrm{T}\neg\alpha)$, and $(\exists\alpha)\nabla\alpha$. And given some formal system some formulas can be exhibited having these properties (e.g., defining a bottom particle $\perp$ with $\nabla\perp$ being valid). $\top$ can be defined as the top particle with $\mathrm{T}(\alpha\vee\neg\alpha)$, being true only. The bottom particle $\perp$ can be defined as $\nabla(\alpha\vee\neg\alpha)$, being false only. Note that — in contrast to even the intuitionist negation rules — $\perp\equiv(\alpha\wedge\neg\alpha)$ need not hold if α is a dialetheia, since then

$\mathrm{T}(\alpha \wedge \neg\alpha)$, and ∇ is incompatible with T.
To have and use the (T)-scheme at the same time as these operators (be it for the operator "T" or "Δ") we need some revisions in the logic of the conditional, like giving up on the unrestricted validity of Contraposition. $\mathrm{T}\alpha \wedge \nabla\alpha$ is a well-formed formula, but false only. The language of this version of dialetheism thus contains formula that can be evaluated only as being simply false. These formulas, of course, cannot be derived.
We do not need the details of all these restrictions here. The reader has only to know the general idea of paraconsistent logics and the idea of "adaptive logics" (Batens 1989, 2000) to restrict some rules to consistent sentences (respectively to retract some supposed consequences if the rules to derive them employed, against the restrictions, some inconsistent sentences). A paraconsistent logic like Priest's LP can be developed into an adaptive logic with a restricted form of *Modus Ponens* and Contraposition (Priest 1991). Within paraconsistent logics "logics of formal inconsistency" (Marcos 2005) employ consistency operators in the object language. Truth operators can then be added. Blending these approaches one can have an adaptive paraconsistent logic which combines the extensional and intuitive truth conditions of LP with the use of truth and consistency operators (Bremer 2005). We suppose here that the dialetheist uses some such logic. Adaptive logics employ standard logic in consistent context and with respect to consistent objects and use a paraconsistent logic for the inconsistent cases. They are adaptive in that one proceeds on the assumption that one deals with a consistent case only on explicit information that the context is inconsistent some supposed consequences have to be retracted. Practically this works by adding to natural deduction style derivation a further column in which one notes the consistency or normality assumptions or presuppositions that have to be made when employing some critical rules of inference. For example, the paraconsistent logic LP makes — as do paraconsistent logics typically — Disjunctive Syllogism invalid; since LP, further on, uses the standard material conditional this means that *Modus Ponens* is not valid in general; but it is valid on the assumption that the antecedent φ of the conditional $\varphi \supset \phi$ used in an instance of *Modus Ponens* is a consistent statement. Thus noting the assumption $\circ\varphi$ in the extra column of a derivation one can employ *Modus Ponens*, but once it turns out by the internal dynamics of drawing further consequences that φ was not consistent after all, the derived line and all lines dependent on it have to be retracted. We have to deal also with the failure of substitution of identicals for inconsistent objects. Identity elimination, (=E), has to be restricted to consistent objects. We define a consistency predicate "K()" for objects (as a logical constant, of course) to do this:

(DK) $\mathrm{K(a)} \stackrel{\text{def}}{=} \neg(\exists \mathrm{P})(\mathrm{P(a)} \wedge \neg\mathrm{P(a)})$

Since we do not use a second order system here, we may employ (DK) in that way that we note $\neg K(a)$ in some line of a derivation if for the object named "a" we could have a line with an instance of the schema: $P(a) \wedge \neg P(a)$. Identity Elimination then takes the form:

n.<m>	$P(a)$	...	Γ
o.<k>	$a = e$	...	Λ
p.<m,k>	$P(e)$	$(= E)$ n,o	$\Gamma \cup \Lambda \cup \{K(e)\}$

where the column on the right takes down the sets of normality/consistency assumptions (or other presuppositions, cf. Bremer 2005: 224–36). The principal inconsistent object we are concerned with here is, of course, λ. An example derivation looks like this:

1.	<1>	p	Premise	
2.	<>	$p \supset \neg\neg p$	PC	
3.	<1>	$\neg\neg p$ $(\supset E)1,2$	$\{\circ p\}$	
4.	<2>	$\neg p \vee q$	Premise	
5.	<1,2>	q	$(\vee E)3,4$	$\{\circ p, \circ\neg\neg p\}$
6.	<2>	$p \supset q$	$(\supset I)1,5$	$\{\circ p, \circ\neg\neg p\}$
7.	<>	$(\neg p \vee q) \supset (p \supset q)$	$(\supset I)2,6$	$\{\circ p, \circ\neg\neg p\}$

To return to the truth operators: Saying $T\lambda$ is thus simply true: $\Delta T\lambda$. This does not exclude that $F\lambda$ is also simply true: $\Delta F\lambda$.
Now it *seems* that saying of the Liar that the Liar is false is just what the Liar is saying

(47) $F\lambda \equiv \lambda$

Then we might have

(48) $FF\lambda$

and this contradicts $\Delta F\lambda$! But to derive (48) we use either

(49) $F\lambda = \lambda$

taking the sentences as objects and expressing their identity, or

(50) $\vdash (F\lambda \equiv \lambda)$

which may be a *petitio* in the argument under consideration, and then substitution of identicals or substitution of equivalents.
The equivalence thesis (50) may be wrong. And substitution of identicals is one of those inferences restricted to consistent objects (to which λ does not belong). Even if (50) is not wrong deriving $FF\lambda$ supposedly has to use

some form of detachment, which again is restricted to consistent sentences (to which λ does not belong).[2]

Let us take it that Fλ can be believed and — being bivalent — can be asserted. Asserting Tλ or Fλ certainly fulfils some purpose, be it in explaining dialetheism or in arguing with opponents of dialetheism.

What about λ itself? What could be the purpose of asserting λ when one could assert $\neg\lambda$ as well? Can asserting an antinomic sentence have any purpose at all?

Believing λ — as the dialetheist does — is not enough (see §3).

Given that the dialetheist is engaged in discussions about dialetheism it may be important to affirm her position by giving an example of what is a true contradiction. This can be done by affirming the antinomy itself, since we and the dialetheist take assertion to involve being convinced of the affirmed sentence being true (being *at least* true in the dialetheist's case). So if asserting α can be taken as asserting Tα (not necessarily $\Delta\alpha$ in the dialetheist's case) and Tλ may be useful in a discussion about dialetheism, asserting λ has its place as well.

In memory of the distinction between object- and meta-language, dropped by the dialetheist, one may call this a *meta-assertion* of an antinomy. So there are occasions on which it is rational for a dialetheist to assert a contradiction.

Are there — apart from the just given purpose of uttering λ as a hidden/implied utterance of Tλ — other affirmative uses of λ?

It seems not, since it seems difficult to come up with a purpose for affirming

[2] It may even be that some of the restrictions used in the logics mentioned in the previous note block the well known proofs of the semantic antinomies — but that can hardly be held against dialetheism. Dialetheism can be weakened to the thesis that *if* given some basic principles of truth, denotation, membership and (semantic) closure we derive contradictions, these may be taken as being true. If some well known examples are lost that does not matter. The *purpose* of dialetheism is not to have true contradictions, but to have semantic closure or naïve set theory or ... even if this involves accepting some true contradictions. The controversy has centered on the dialetheist's claim that there are true contradictions, but the starting point has always been some other philosophical tenet. So in case there are no true contradictions, so the better for semantic closure or naïve set theory. I take the philosophical point of dialetheism to be that even if there are true contradictions this is the price to pay in some universal theory/logic. So we better model logic, reasoning, belief and assertion on the assumption that there are true contradictions. I just assume here that the Liar (still) can be proven and thus is taken as true by a dialetheist. It should also be mentioned that these systems of paraconsistent logic hinted at above have sentences that *look* like Strengthened Liars (e.g. sentences saying of themselves that they are *false only*). Switching to evaluation relations (cf. Priest 1996) and the restrictions on rules in proofs, however, avoids getting hyper-contradictions (cf. Bremer 2005: 193, 237). The only interesting observation with respect to (some of) these Strengthened Liars is that they seem to be incapable of achieving what they assert of themselves (i.e. being false only).

λ. Believing both $T\lambda$ and $F\lambda$ (respectively λ and $\neg\lambda$) one may — it seems — as well use/affirm λ as $\neg\lambda$. But if any (non-meta-)usage of λ corresponds to a usage of $\neg\lambda$, there is no point in asserting λ, it seems. There seems to be nothing *specific* to be *said* by using λ; even more so if a dialetheist accepts $\alpha \vee \neg\alpha$ as a tautology and rejects the use of disjunctive syllogism with antinomic sentences.

If there is no preference to affirm α in contrast to affirm $\neg\alpha$ why not affirm both? But again: Apart from conveying or displaying thus that α is taken as antinomic what is the supposed content of that assertion?

5. *Antinomies and Negative Facts*

The semantic account of some predicates may speak of some quality/structure that entities have to which this predicate applies. Once *tertium non datur* is accepted — as it is by standard dialetheists — one either has to assume that $\neg\alpha$ contains the absence of the qualities/structures contained in α, which would make it difficult indeed to understand $\alpha \wedge \neg\alpha$ in a mildly realistic manner, or α and $\neg\alpha$ are seen as exclusive and exhaustive in the sense that they *both* contain some quality/structure the absence of both being (metaphysically) impossible.

Given a *substantial* theory of truth $T\alpha$ may convey some quality of α like corresponding to a fact, being rationally justified ... A substantial theory of falsity should accompany this theory, so that $\neg\alpha$ conveys some quality like the presence of a *negative fact* (!), being rationally refutable ... These qualities may co-occur! The dialetheist has to postulate some appropriate epistemological or metaphysical axioms then.

If the positive and the negative fact tied to the Liar are situated not in space and time but somewhere in our linguistic representation of the world, there may be room for a realist dialetheism which sees a purpose in asserting both λ and $\neg\lambda$. Given a metaphysics of this sort one can commit oneself by *one side* of a dialetheia. One takes up the commitment to argue that a corresponding structure is given. (This position has not to assume that the goal of affirmation is truth only, it is rather something being *at least* true.)

If the felicity conditions of affirmation/assertion entail that the purpose of affirmation is to claim something as being only true, then (by this alone) dialetheias are not affirmable. But why should one assume this?

The first reason seems to be that one is eager to exclude at the beginning a metaphysical picture of negative facts. The second reason, however, may rest on pragmatic felicity conditions of assertions as speech acts. Assertion requires to be pragmatically relevant that there is a commitment to something which has to exclude something else. If nothing is excluded by what I assert, I should not have bothered the effort. Now, in the typical presentation of antinomies (for example by arguing by cases $T\alpha/T\neg\alpha$) an antinomy α implies/entails $\neg\alpha$, and $\neg\alpha$ implies/entails α. Thus by either of them I assert what the other *says* as well. Therefore an account that bases the informational content of a sentence on what this sentences entails (cf. Priest 1987: 118) is of no help in these cases. So far this may point to the *arbitrariness* of which side of an antinomy is asserted only. By this reasoning one has no sufficient reason to affirm one side, and thus seems to be in some limbo of assertion.

One may think choosing just one side of the antinomy gets oneself out of this problem, an ontology of negative facts doing the rest. Asserting α (or $\neg\alpha$), however, has a point only if the facts corresponding to α and $\neg\alpha$, which by the mutual entailment of α and $\neg\alpha$ are put forward by either of them, are not exhaustive, it seems; something logically or semantically exhaustive being usually taken as having no informational impact because involving nothing to be excluded by it. Negative facts have had a bad press in metaphysics.[3] So one better had not oneself committed to them. Again, however, it seems that a *general* commitment to negative facts is as superfluous as a general acceptance of any old contradiction being true. The dialetheist accepts only very special "true contradictions", namely those unavoidable given basic semantic or set theoretical concepts plus universality. The dialetheist, therefore, has to accept only very special negative facts. The failure of accounting for what the point of asserting a contradiction might be in terms of informational content or of what the two sides of the contradiction individually entail requires the more substantial account in terms of reference to distinct facts. In the case of Liar-like antinomies these facts consist in the negation of a sentence being as provable as the sentence itself. There is no further fact "behind" this. Since the proof is an existent *something* one may even speak of a positive fact here, like the intuitionist bases the claim for $\neg\alpha$ not on the absence of reasons for α, but on the (positive) proof of $\perp$ from the premise α (cf. Priest 1987: 87).

The case is different with the set-theoretical antinomies, since here we seem to have the negative fact of the Russell set not belonging to itself besides the (positive) fact of the Russell set belonging to itself. These negative facts — one may argue — have their residue in the realm of abstract objects, however; much may be going on there. (If one takes sets not as abstract entities dialetheism in set theory may be a problem for realists.)

Thus with respect to ordinary sentences (the truth of) $\neg\alpha$ may be the absence

[3] At least negative *first-order* facts; the absence of all instances of a predicate understood as a supervenient negative fact has had a better press. Whether all negative facts corresponding to one side of a dialetheia are supervenient (i.e. non first-order facts) is not clear and may go against the spirit of, say, set-theoretical antinomies. On negative facts see also (Beall 2000) and (Priest 2006: 53–54).

of (the truth of) α, but if α entails $\neg\alpha$ and vice versa, and both are of interest in as much as the fact corresponding to $\neg\alpha$ is not just the *absence* of the fact corresponding to α (as an "ordinary" supervenient negative fact would be) substantial metaphysical assumptions come to light:

I. Both facts are substantial (and interesting), and it is a further substantial *metaphysical* fact that although they do not stand to each other like contradictory sentences do in PC, not both can be false only [corresponding to the theorem $\perp(\neg(\nabla\alpha \wedge \nabla\neg\alpha))$].

II. The explication given above (the negative fact being the provability of $\neg\alpha$) seems metaphysically questionable then, since why should it not be possible that we do *not* have proofs of either of them. Expressed in terms of truth (and *tertium non datur*) the option of both being only false can be excluded, but leaving aside the *provability* of $\neg\alpha$ what should be the negative fact corresponding to $\neg\alpha$? One might settle for a metaphysical *tertium non datur* and simple negative facts (like the Liar *being* false, provably so or not).

Giving up *tertium non datur* (in logic or metaphysics) is no real option, since the argument can be repeated with strengthened Liars for multi-valued or gap semantics. *Tertium non datur* (or a $n+1$ *non datur* for some n-valued semantics) thus has to be understood as the *substantial* metaphysical thesis that — for some reason — one of two not exclusive facts has to obtain (since they are *metaphysically* — not just logically — exhaustive as well).
If there is a known metaphysical principle that excludes that both α and $\neg\alpha$ are only false, asserting one of them has no point in the sense of rejecting some modally available/accessible/possible fact. The same is true, however, with respect to any logical truth! They do not exclude anything either. Asserting dialetheias *simpliciter* (i.e., without semantic operators) thus has the same merit or futility like asserting logical laws *simpliciter*.

6. *Assertion, Denial and Rejection in Dialetheism*

Given that there is independent ground for $\neg\alpha$, accepting $\neg\alpha$ does not exclude accepting α. In contexts we know to be consistent we may reason to $\neg\alpha$ without independent grounds on the basis of $\neg\mathrm{T}\alpha$ and *tertium non datur* (or some version of this disjunctive syllogism like reasoning). Since in case of antinomies accepting $\neg\alpha$ does not exclude accepting α, accepting $\neg\alpha$ should not be the same as rejecting α.
Rejecting α cannot be understood by a dialetheist as affirming $\nabla\alpha$. Rejecting α would thus be incompatible with affirming α (i.e. affirming $\mathrm{T}\alpha$). One needs a distinction then between affirming $\nabla\alpha$ and affirming $\mathrm{F}\alpha$ [$\mathrm{T}\neg\alpha$].

Sticking with the usage employed in §2 and — arguably — standard logic let us take affirming Fα as *rejection* and affirming $\nabla\alpha$ as *denial* of α [Dα]. Whereas there are situations in which a dialetheist accepts both α and $\neg\alpha$, there are no situations in which a dialetheist accepts and denies α at the same time. Dialetheism does not accept just any contradiction. This is one reason — prejudices and puns to the side — why rational argument with a dialetheist is possible. As the foregoing distinction shows there is, furthermore, one kind of contradiction that (even) a dialetheist cannot support:

(51) $\neg(\mathrm{A}\alpha \wedge \mathrm{D}\alpha)$

since Tα and $\nabla\alpha$ are semantically incompatible.
Another simple point is that no-one (including the dialetheist) can have pragmatic contradictions: Speech acts being bodily movements that either occur or do not, there is no pragmatic parallel to having it both ways, i.e.

(51) $\neg(\mathrm{A}\alpha \wedge \neg\mathrm{A}\alpha)$

This instance of the accepted tautology $\neg(\alpha \wedge \neg\alpha)$ expresses not only a semantic exclusion the dialetheist accepts (and sometimes nevertheless supersedes), but the absence of the mysterious feat of asserting something and not doing it at the same time. There is no pragmatic dialetheism (without a verbal manoeuvre of redefining "not asserting" on the lines of "asserting $\neg\alpha$").
Having in mind these distinctions and the truth operators intuitively dialetheism allows for the truth (not necessarily the validity) of several sentences excluded by the principles in §2:

(52) $\mathrm{B}\alpha \wedge \mathrm{B}\neg\alpha$ [believing a contradiction]

(53) $\mathrm{B}(\alpha \wedge \neg\alpha)$

(54) $\mathrm{BT}\lambda \wedge \mathrm{BT}\neg\lambda$ [being semantically explicit about λ]

(55) $\mathrm{BF}\alpha \wedge \mathrm{BF}\neg\alpha$

There is some spreading of believed or asserted contradictions in those paraconsistent logics in which $\neg(\alpha \wedge \neg\alpha)$ is a theorem or which are extended with the usual principle (44) of closure of belief. Then we may have, for example:

(56) $\mathrm{B}(\lambda \wedge \neg\lambda) \wedge \mathrm{B}(\neg(\lambda \wedge \neg\lambda))$

More controversial may be corresponding principles of assertion:

(57) $\mathrm{A}\alpha \wedge \mathrm{A}\neg\alpha$

(58) $\mathrm{AT}\alpha \wedge \mathrm{AT}\neg\alpha$

(59) $\mathrm{AF}\alpha \wedge \mathrm{AF}\neg\alpha$

(60) $\mathrm{A}(\alpha \wedge \neg\alpha)$

(61) $\mathrm{A}\alpha \wedge \mathrm{R}\alpha$ [invalidating $\mathrm{A}\alpha \supset \neg\mathrm{R}\alpha$ and $\mathrm{R}\alpha \supset \neg\mathrm{A}\alpha$]

Given the truth operators some new principles (and their duals), however, are in force now:

(62) $\mathrm{B}\Delta\alpha \supset \neg\mathrm{BF}\alpha$

(63) $\mathrm{A}\Delta\alpha \supset \neg\mathrm{AF}\alpha$

(64) $\mathrm{R}\Delta\alpha \supset \mathrm{BF}\alpha$

(65) $\neg\mathrm{A}(\Delta\alpha \wedge \mathrm{F}\alpha)$

(66) $\neg\mathrm{B}(\nabla\alpha \wedge \mathrm{T}\alpha)$

7. *The Problem of Ideal Reasoners*

Ideal reasoners seem to pose a problem if an ideal reasoner is defined as an agent who asserts/accepts α iff α is true. For such an ideal reasoner there is an "assertion"-version of the Liar:

(aλ) (aλ) is not asserted (by an ideal reasoner).

If (aλ) is not asserted, it is true and thus accepted by an ideal reasoner (by definition of "ideal reasoner"), but then she asserts (aλ) ... (The antinomical reasoning breaks down, of course, for non-ideal agents.)

Now, believing Tα and Fα is one thing — even asserting Tα and Fα — but assertion is a speech act you either produce/enact or do not. It occurs in the world or it does not. Even if something is *evaluated* as being true and false at the same time, this appears to be less bizarre that the idea that someone can in one and the same act assert and not assert α (see §6). So either

I. The antinomical reasoning for something like (aλ) does not really go through (e.g., because the use of *Modus Ponens* with an inconsistent antecedent is not available), something difficult to establish, since there may be some other version of an "assertion"-Liar

or

II. One denies that there can be ideal reasoners in the sense given! This would contain the profound insight that "asserting" and "holding true" respectively "being true" have to be kept apart for conceptual reasons.

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