Logique & Analyse **179–180** (2002), 249–268

A LAKATOSIAN APPROACH TO THE QUINE-MADDY DEBATE

LIEVEN DECOCK

1. *Introduction*

In the last decade, Penelope Maddy has criticised Quine in a series of books and articles. The main target of her criticism was Quine's defence of the set-theoretic axiom "$V = L$". This axiom of constructibility has first been proposed by Gödel in 1938 in order to show that the Zermelo-Fraenkel set theory (ZFC) was compatible with the continuum hypothesis (CH). Gödel, however, soon rejected the axiom. Also[1] Maddy thinks that in view of contemporary research in set theory, this axiom is untenable, and should be rejected. The major problem with the axiom is that it restricts the set-theoretic hierarchy, and blocks off the flight to higher cardinals.

Maddy has not always disagreed with Quine. In *Realism in Mathematics*, Maddy endorsed a realistic position in the philosophy of mathematics. She believed that there is a fact of the matter whether a given set-theoretic axiom is true or not. The argumentation drew on Quine's indispensability argument[2] and Gödel's mathematical 'intuition'. The indispensability argument was needed to guarantee a secure basis for the set-theoretic universe, and Gödel's intuition to get to the higher reaches of the set-theoretic universe. In *Naturalism in Mathematics*, Maddy no longer defends realism, and advocates naturalism instead.[3] The reason is that she thinks that

[1] Maddy writes that also Gödel's arguments in favour of '$V = L$' are no longer plausible, see Maddy 1997, 83.

[2] The argument is often called the Quine-Putnam indispensability argument. In a section "Indispensability arguments" in *Philosophy of Logic* (Putnam 1979, 347), Putnam writes: "So far I have been developing an argument for realism along roughly the following lines: quantification over mathematical entities is indispensable for science, both formal and physical; therefore we should accept such quantification; but this commits us to accepting the existence of the mathematical entities in question. This type of argument stems, of course, from Quine, who has for years stressed both the indispensability of quantification over mathematical entities and the intellectual dishonesty of denying the existence of what one daily presupposes."

[3] See Maddy 1996, 1997, 1998, 2001.

Quine's indispensability argument leads to wrong conclusions, i.e. it limits the set-theoretic universe. In Decock 2002b, I argue that the indispensability argument itself is unproblematic. In this paper I want to analyse the real divergence between Quine and Maddy's points of view.

Maddy and Quine dissent over mathematical methodology. Quine's work in set theory took place in the thirties, while Maddy's views are related to recent work. In Lakatos's terminology, one could say that they belong to different research programmes in set theory. This will become clear when their set-theoretic background, and the concomitant epistemic normative principles employed, so-called epistemic virtues, are scrutinised. In the next three sections, I will respectively sketch Quine's methodological maxims, his set-theoretic background, and his wavering over constructivism. In the fifth and sixth section I will present Maddy's intrinsic and extrinsic evidence. In the penultimate section I will discuss the views of Maddy and Quine from a historical and sociological point of view. I will conclude, along a Lakatosian line, that instant adjudication between the two programmes is unwarranted.

2. *Quine's empiricism*

According to Maddy, Quine's defence of '$V = L$' is a straightforward consequence of Quine's empiricist epistemology. In *Naturalism in Mathematics* Maddy quotes Quine:

> Further sentences such as the continuum hypothesis and the axiom of choice, which are independent of those axioms, can still be submitted to the considerations of simplicity, economy, and naturalness that contribute to the molding of scientific theories generally. Such considerations support Gödel's axiom of constructibility, $V = L$. It inactivates the more gratuitous flights of higher set theory, and incidentally, it implies the axiom of choice and the continuum hypothesis.[4]

The epistemological basis for rejecting the flights of higher set theory are thus considerations like 'simplicity', 'economy', and 'naturalness'. These are normative principles that guide the scientist in his research. Such principles are often called epistemic virtues. The best account of Quine's epistemic virtues can be found in *The Web of Belief*. This is a booklet he wrote together with Joe Ullian, and is an introduction to philosophy. Quine introduces five virtues, namely conservatism, modesty, simplicity, generality, and refutability. In *The Roots of Reference* Quine only presents the virtues of

[4] Quine 1990, 95. For similar quotes see Quine 1986, 400, or Quine 1991, 220.

simplicity and conservatism, and says they are related dialectically. Subsequently, he relates conservatism to empiricism and proposes the maxim of relative empiricism:

> Don't venture farther from sensory evidence than you need to. . . . We recognize that between the globally learned observation sentences and the recognizably articulate talk of bodies there are irreducible leaps, but we can still be glad to minimize them, and to minimize such further leaps as may be required for further reaches of ontology.[5]

This maxim is at the core of Quine's epistemology, and is in fact one of the epistemic virtues that have drastic repercussions for ontology. The maxim demands that in constructing scientific theories, one should abstain from positing further reaches in ontology if this can be avoided.

The maxim of relative empiricism is of course of crucial importance in a discussion of higher cardinals. The maxim may be regarded as the demand that no parts of mathematics should be accepted in our global scientific theory if they do not simplify or generalise our theories, and if some parts of mathematics do this, then the gains in simplicity should be weighed against the epistemological costs. The maxim leaves room for deliberation over certain cases, but at any rate parts of mathematics that are not readily applicable should be rejected as mathematical recreation, and should be banished from decent science. The implication for ontology is that one should not posit mathematical entities beyond what is strictly necessary for empirical science, and thus that the higher cardinals should get a "resounding negative".[6]

In the discussion of the higher reaches of set theory, Quine heavily leans on his virtues in defending his philosophical position. If one thinks that this is Quine's only motivation, as Maddy does, Quine undoubtedly overplays his hand. Quine's position is based on global epistemic considerations and idiosyncratic epistemic virtues, which are not farther defended or justified. These epistemological views for the realm of mathematics differ from what mathematicians in the field are doing. Quine's global epistemological views differ from the specific epistemological views in contemporary mathematics. Quine's naturalism states that the scientist should get the final verdict, but his overall epistemological views and epistemic virtues seem to overrule the verdict of the mathematician in the field. As Maddy ironically points out, Quine's plea for the axiom of constructibility seems to contradict his naturalism.[7]

[5] Quine 1974, 138.

[6] Quine 1991, 220.

[7] Maddy 1997, 106.

Of course, Quine's specific use of his epistemic virtues of conservatism and simplicity, and his relative empiricism, are well entrenched in his overall philosophy, and are related to his overall tenet of empiricism. The normative strength of empiricism however, is extremely difficult to assess. Though it is in general a praiseworthy philosophical and scientific attitude, it remains an open question what the precise implications should be. One readily accepts that astrology, telepathy and the ilk should be dispelled because they blatantly contradict the empiricist thesis. On the other hand, it remains hard to believe that the empiricist thesis really implies that inaccessible cardinals should get a resounding no. Moreover, assuming that the empiricist thesis does imply this, it still remains an open question whether this precise and far-reaching empiricist thesis has to be accepted. After all, empiricism itself is not really an empirical doctrine that can be tested, and it had better not be some metaphysical position. The tenet is hard to characterise, and a more or less felicitous attempt is Van Fraassen's notion of 'stance'[8]. This notion immediately hints at the normative weakness of empiricism. However, one should not conclude that the stance is not normative at all. Empiricism has been a great success in science, and it would be wrong to abandon a successful practice.

The normative role of empiricism is directly related to the success it had in the past and the promise of future success. It would be an enormous labour to substantiate the empiricist thesis by spelling out very precisely the role of empiricism in the past, and it would be even more difficult to relate this to future success. The credibility of restrictions and revisions in the scientific practice depends on the amount of labour done in sociology and history of science. Quine's account of empiricism is certainly not precise and elaborate enough to impose harsh restrictions on set theory.

3. *Quine's set-theoretic background*

Quine's plea for the axiom of constructibility seems to contradict his naturalism. Whereas for most scientific areas, Quine simply believes, sometimes very uncritically, what scientists in the field are saying, for mathematics this is not the case. The reason for this is quite simple. Quine thinks of himself as a mathematician in the field. He has graduated in mathematics at Oberlin College, and been a leading set theorist in the thirties. Therefore, his philosophy of mathematics is both inspired by general epistemological considerations, and by his own work in set theory. His views are far more intricate

[8] Van Fraassen 1995, 83.

than the simple sketch Maddy presents, and, avowedly, not always coherent. Quine's work in set theory is nowadays less known or almost forgotten.

Quine's real breakthrough in the philosophical scene were his articles "On what there is" (1948) and "Two dogmas of empiricism" (1951).[9] But during the two decades before, he was primarily known as a logician. In 1932 he wrote a Ph.D. thesis, *The Logic of Sequences* under the supervision of Whitehead. The thesis is a generalisation of Whitehead and Russell's *Principia Mathematica*. Quine presented an elegant way of deriving the theorems of PM for polyadic predicates, while in PM these were only derived for monadic and dyadic predicates. With hindsight, the result is "scarcely surprising"[10], but at the time it was considered a real improvement. Quine was offered the opportunity to publish his thesis, which resulted in *A System of Logistic*, and he got the opportunity to visit Europe. In Europe, Quine met the Vienna Circle, and the Polish school of logicians. After discussions with Leśniewski, he started a nominalistic enterprise. In several articles he tried to construct logistics in which the use of universals, i.e. classes or sets, could be avoided. One attempt (1936a) foundered on the truth predicate. In other articles,[11] his source of inspiration was Schönfinkel's combinatorial logic. Eventually, Quine gave up his nominalistic programme and had to accept that classes can be the values of variables.

In 1937, Quine published his most famous contribution to set theory, "New foundations for mathematical logic" (NF). Although, the system is based on the idea of a type hierarchy, it is a radical departure from PM. In PM, Russell's paradox is blocked by means of a type hierarchy. In NF, the quantifiers are not restricted to a single type, but are general, and thus there is a single universe of objects (and classes). Russell's paradox is avoided by means of a restriction on the abstraction of classes. Not every well-formed formula determines a class. The abstraction axiom of NF, called the "stratification axiom", states that only expressions whose variables can be numbered in such a way that they can be read as formulas in PM, determine a class. Abstraction is based on a grammatical rule, and this leads to an unintuitive set theory. Various difficulties arise. It is still an open question whether NF is consistent. The axiom of choice is not universally valid, but should be restricted to 'small' non-Cantorian classes. There is a universal class, i.e. a class of all classes, that is an element of itself. NF has attracted a lot of

[9] See Quine 1953, 1–19 and ibid., 20–46.

[10] Quine 1932, iii.

[11] Quine 1936b, 1936c, 1936d.

interest, but most mathematicians find it very hard to get insight in the structure of NF. But the system is still studied today.[12] In 1940, Quine published *Mathematical Logic*, and it became the standard handbook in mathematical logic. Quine reworked the set-theoretic part to the system ML. The major innovation was the use of proper classes.

During the Second World War, Quine served three years in the American army, and afterwards there has been a steady decline in his logical and set-theoretic output. His vehement opposition to the rise of modal or other intensional logics is still notorious. But gradually his interests gradually shifted toward philosophy of language, and later to epistemology. His last important work on logic and set theory is the handbook *Set Theory and its Logic*. The book is both an introduction to set theory and a comparison of various set-theoretic systems. Quine does not opt for one set theory, but discusses the merits of type theory, cumulative type theory, ML, ZFC, and the system von Neumann-Bernays, and highlights the relations between the systems.

Quine's philosophical ideas have been thoroughly influenced by his own work in logic and set theory.[13] Practical problems in mathematics often forced Quine to change his philosophical views drastically. It is a pity that Quine has never written a book in which this relation is clearly exhibited. Nevertheless, *Set Theory and its Logic* can be read as a philosophical work, and one readily sees how Quine's philosophical ideas are grounded in mathematical insights. Also, throughout the book, it becomes apparent that the set-theoretic tradition in which Quine has worked differs drastically from contemporary research in set theory.

I have already mentioned nominalism as a case in point. Maddy points out that in view of Quine's empiricism, the best course for Quine to follow would have been a Millian nominalism.[14] After discussions with Leśniewski, Quine has sincerely tried this. The discussion concentrated on the role of quantification, and whether it is necessary to quantify over universals. According to Quine's criterion of ontological commitment, to be is to be the value of a variable, so it was essential to avoid quantification over universals (rather than merely using them in a non-committal way). Quine has tried to do this in the thirties, and again in a joint paper with Nelson Goodman (1947), but difficulties remained. But even after Quine accepted mathematical entities, he liked to keep the set-theoretic universe small. One of the most striking

[12] For a brief overview see Forster 1997, for technical details see Forster 1992.

[13] The analysis of the relation between Quine's work in logic and set theory and his philosophical views is the main subject of Decock 2002a. For Quine's work in set theory, see especially chapters 3.2 and 4.

[14] Maddy 1997, 100.

features of *Set Theory and its Logic* is the extreme attention that is paid to the ontological commitments that are made when accepting some set-theoretic axiom.

Another striking feature is the intricate interplay between logic and set theory. Though basically *Set Theory and its Logic* deals with set theory, the part "and its logic" expresses an important bias of the book. It is generally held that set theory has grown from two roots, namely logic and mathematical analysis. The logical tradition started with Frege, and the discovery of Russell's paradox led to the type hierarchy in *Principia Mathematica*. The other root starts with Cantor's work on discontinuities of functions, and led to the Zermelo-Fraenkel set theory. Quine's work however, remains almost entirely within the logical tradition. Quine has never been able to take a radical departure from *Principia Mathematica*. He started his career reworking *Principia Mathematica*, and at the end of the sixties, Russell's paradox was still the central problem in *Set Theory and its Logic*.

Quine has never been actively involved in mathematical set theory. Whereas Quine has elaborated or responded to most developments in logic, many developments in set theory are completely absent. Cantor's work is seldom referred to. Also, important results of contemporaries, like Luzin, Suslin, or Novikov, are never mentioned. ZFC, the basis of all modern set theory, does play a role in Quine's work, though not a central one. The first paper in which he introduces the quantifier as a logical primitive notion (1936e) is based on ZFC, but already a year later he presents "New foundations for mathematical logic" as an improvement. Even in *Set Theory and its Logic* Quine is not really in favour of ZFC. The system is not discussed as a whole, but the pro and cons of each of its axioms are discussed one by one. The resulting system is presented as just one set theory among others.

This brings us to another major point in Quine's views on set theory, namely his pluralism in set theory. In 1932, when Quine started working on logic and set theory, there was a clear standard in logic, namely *Principia Mathematica*.[15] This logical system was based on Frege's logistic, which was clearly motivated by mathematical intuitions. Also *Principia Mathematica* starts by embedding the system in common intuitions, though in some instances, especially for the axiom of reducibility, this proves very hard. Over the years, various logistics and set theories were developed, with divergent intuitive motivations, or just as unmotivated technical frameworks. Quine came to conclude that the initial bond between logic and set theory

[15] In hindsight, there is some irony in the fact that in Quine 1936e, Quine tried to reconcile his new logistic, based on quantification, negation, implication and membership with "standard logic", by which Quine meant the logic of *Principia Mathematica*, see Quine 1936e, 85.

was cut.[16] He firmly believed that first order logic is 'obvious', and he settled for 'standard' logic, based on quantification, negation and implication. On the other hand, he no longer believed that there are good intuitions for set theory. As he put it: "Intuition here [in set theory] *is* at a loss."[17] There are only technical, mathematical reasons for preferring one set-theoretic system over another.

Most of Quine's views, which are the views of an earlier generation of set-theorists, are no longer shared by contemporary set-theorists. At the time the second printing of *Set Theory and its Logic* appeared, Quine had entirely lost contact with what was going on. The best illustration is Donald Martin's trenchant review of *Set Theory and its Logic* (1970). Martin harshly comments on the logical style, and claims that only ZFC is the only set-theoretic system that is in a natural way compatible with the theorems one would like. In an (unusual) reply to this review (1970), Quine points out that Martin's intuitions are definitely different from his own.

In this respect, it is interesting to recall Quine's assessment of the type theory of *Principia Mathematica* in "Whitehead and the rise of modern logic":

> But a striking circumstance is that none of these proposals (logistics), type theory included, has an intuitive foundation. None has the backing of common sense. Common sense is bankrupt, for it wound up in contradiction. Deprived of his tradition, the logician has had to resort to mythmaking. That myth will be best that engenders a form of logic most convenient for mathematics and the sciences; and perhaps it will become the common sense of another generation.[18]

Quine predicted a new common sense for set theory in 1941, but failed to appreciate the new 'myth' when it was proposed.[19] The new intuitive foundation was the iterative conception of set.[20] It became in vogue at the beginning of the seventies, and is the typical defence of the axioms of ZFC.

[16] See Decock 2002a, 117.

[17] Quine 1969, x.

[18] Quine 1995, 27.

[19] Quine did not claim that set theory should be based on the iterative conception, but did agree that it was the most natural intuition, see Hahn & Schilpp 1986, 646: "The most he [Wang] can do is plump for the primacy of the iterative concept, but he recognizes that I have plumped for it too, in *Set Theory and Its Logic* and *Roots of Reference*."

[20] See Benacerraf & Putnam 1983 for a discussion on the iterative conception of set.

4. *Quine on constructivism*

It is remarkable that Quine's sympathy for the axiom '$V = L$' was only expressed in his latest writings. It can be found in the reply to Parsons in the Schilpp volume on Quine (1986), and later in *Pursuit of Truth* (1990), and in a text "Immanence and validity" (1991). Apart from a few papers on predicate functors and the role of the variable, Quine has written no articles on logic or set theory after the publication of *Set Theory and its Logic*. Moreover, Quine does not offer an elaborate argumentation why one should accept the axiom of constructibility. The passages on constructibility in these later texts are very brief. Quine claims that for reasons of economy and simplicity one should accept the axiom, but as Maddy rightly complains, this is scarcely convincing.

The axiom of constructibility is related to the constructivist or predicativist tradition in set theory. It states that the set-theoretic universe contains all and only the constructible sets, i.e sets that can be constructed by means of first order definitions from previously formed sets.[21] The existence of a set is not presupposed in its definition, and so there are no impredicative sets. However, Quine has not always looked askance at impredicative sets. Of course, he started with a predicative set theory, namely the type theory in his generalisation of *Principia Mathematica*. But in his own set theories, he abandoned this requirement. In *Set Theory and its Logic*, he explicitly rejects the suggestion that Russell's paradox is the result of an impredicative definition of set:

> For we are not to view classes literally as created through being specified - hence as dated one by one, and as increasing in number with the passage of time. Poincaré proposed no temporal implementation of class theory. The doctrine of classes is rather that they are there form the start. This being so, there is no evident fallacy in impredicative specification. It is reasonable to single out a desired class by citing any trait of it, even though we chance thereby to quantify over it along with everything else in the universe. Impredicate specification is not visibly more vicious than singling out an individual as the most typical Yale man on the basis of averages of Yale scores including his own. So the ban urged by Russell and Poincaré is not to be hailed as the exposure of some hidden but (once exposed) palpable fallacy that underlay the paradoxes.[22]

[21] For formal details, see Devlin 1977, 455.

[22] Quine 1969, 243.

But already in *The Roots of Reference*, Quine was aware of another disadvantage. He tried to present an intuitive genetic account of set theory. He ventured that classes were first conceived through substitutional variables for general terms. But since classical set theory demands the impredicative line, one has to go beyond substitutional quantification.[23] Without the need of impredicative set theory for classical analysis, Quine would have settled for substitutional quantification.

The first passage, which is the one Maddy always quotes, where Quine explicitly endorses constructivism can be found in the reply to Parsons (Hahn & Schilpp, 1986, 400). In the same volume, there is an interesting passage in Quine's reply to Wang. He repeats his earlier remarks on substitutional quantification, and its implications for predicative set theory. For him and Charles Parsons substitutional quantification over abstract objects is attractive, but the one obstacle is the need of impredicative classes. In his article Wang had offered a solution, and Quine writes:

> In his present essay Wang urges that this obstacle can be lifted without prejudice to the uses of mathematics in natural science. ... Wang rightly suggests that a predilection for predicativity would have been in keeping with my philosophical temper. Though never tempted to embrace constructivism at the cost of trading our crystalline bivalent logic for the fog of intuitionism, I have indeed wished that I could see my way to following Hermann Weyl in settling for classical logic and predicative set theory without compromising the needs of science. The idea appealed to me on grounds of economy and clarity ... I looked hopefully into Lorenzen, but had trouble sorting out sign from object and seeing what was achieved. Bishop's ponderous work appeared only after my concerns had shifted to less austere domains. ... Wang now offers a gratifyingly sharp and conservative cut-off point for the hierarchy and a reassuring expression of confidence that the needs of science are met. Hope shines forth of a firm and down-to-earth ontology.[24]

Quine admits that he always has liked predicative set theory, but for various contingent reasons did never really analyse the results of constructivists. When predicative set theory got off the ground, in the work of Bishop, and even more so in the work of Feferman, Quine was already doing other things.

[23] Quine 1974, 112. The third chapter is a long elaboration of the possibility of having substitutional quantification. After initial qualms, Quine came to regard substitutional quantification as more natural from a psychogenetic point of view than objectual quantification.

[24] Hahn & Schilpp 1986, 648.

In conclusion, Quine is more of a naturalist in set theory than Maddy assumes. Through his career, his philosophical position has always been tuned to the state of the art in set theory. His assessment of constructivism is based both on his dislike of inapplicable mathematics, which he calls 'mathematical recreation', and on the contemporary work in set theory. After his nominalist projects failed, predicativism would have been a natural 'step back from disaster', but since predicativism faced too many difficulties at times he really considered it, he never really countenanced it. Only after he became convinced that predicative set theory is feasible without damaging classical mathematics, he strongly rejected the flight to the higher infinities, and came to see '$V = L$' as the lowest convenient cut-off point.

5. *Maddy's rejection of the axiom of constructibility*

Through her whole philosophy, Maddy's major aim has been to defend the research programme in the higher reaches of set theory. She wants to defend the positing of inaccessible cardinals, and more specifically the large inaccessibles such as measurable cardinals and supercompact cardinals. The central axiom in her work is the axiom '$V = L$'. She finds it unpalatable, because it restricts[25] the set-theoretic universe. It is for example incompatible with the existence of measurable cardinals. Maddy's wish to maximise the set-theoretic universe is of course incompatible with Quine's demand for a small set-theoretic universe. The second chapter in *Naturalism in Mathematics* may be regarded as the elaboration of her qualms. She argues that the notion of existence scientists and mathematicians use, does not coincide with the notion of existence that results from combining holism, the criterion of ontological commitment and Quine's "particular account of the theoretical virtues and scientific confirmation".[26] Maddy's critique on the indispensability argument is in the first place a critique on Quine's account of epistemic virtues and scientific confirmation.

Maddy proposes epistemic virtues for the realm of mathematics that are diametrically opposed to Quine's. Instead of restricting the set-theoretic universe on the basis of epistemic virtues that are entrenched in the empiricist tradition, Maddy wants to maximise this universe on the basis of epistemic virtues that are more expedient to mathematics. In her latest book *Naturalism in Mathematics*, and in older articles, especially in "Believing the axioms" (1988), Maddy has justified the research in higher set theory by looking at

[25] For a technical elaboration, see Maddy 1997, 216–232.

[26] See Maddy 1997, 134.

the epistemological principles that guide the mathematician. In *Naturalism in Mathematics* she presents two maxims, namely "Unify" and "Maximize". The principle "Maximize" is of special importance here, because it instigates the mathematician to seek for bigger set-theoretic universes. The maxims "Unify" and "Maximize" are not really elaborated, but Maddy relies on earlier work.

In "Believing the axioms" Maddy has presented a whole inventory of rules of thumb, which are in fact epistemic virtues, of set-theorists. In the article(s) Maddy presents the intrinsic justification mathematicians give for using various set-theoretic axioms, starting from the axioms of ZFC until the large cardinal axioms. The justification is based on rules of thumb, which are "vague intuitions about the nature of sets, intuitions too vague to be expressed directly as axioms, but which can be used in plausibility arguments for more precise statements."[27] The first introduced rules are 'limitation of size', and the 'iterative conception', and they are used to justify the axioms of Pairing and Union. Limitation of size is the rule to avoid sets that are too large to be sets, and so run into paradox. The iterative conception is motivated by the common intuition that sets are ordered in superposed layers. A next rule, used to justify Separation is 'one step back from disaster', and it says that one has to weaken principles just enough to block contradiction. The axiom of infinity is justified by means of 'Cantorian finitism', the idea that infinite sets should be treated as resembling finite sets. A very important rule for the debate between Maddy and Quine is the rule 'maximize'. It stipulates that the collection of ordinals is very 'long', and that the power set is very 'thick'. Further rules are 'realism', 'whimsical identity', 'inexhaustibility', 'uniformity', 'reflection', 'generalization', 'richness', and 'resemblance'.[28] In addition to these intrinsic rules of thumb, Maddy also justifies axioms on the basis of extrinsic evidence, i.e. the indispensability of the axioms in desired mathematical results.[29] Maddy's paper is a good survey of the epistemic rules set-theorists in the field use and it is very well documented with textual evidence.

In contrast with Quine, Maddy only gives epistemic virtues for the field of mathematics, and not general epistemic virtues. She rejects Quine's holism, and wants to widen the gap between mathematics and physics so to preserve

[27] Maddy 1988, 484.

[28] See Maddy 1988, 484: 'limation of size'; 485: 'iterative conception'; 485: 'one step back from disaster'; 492: 'maximize'; 497: 'realism'; 499: 'whimsical idendity'; 502: 'inexhaustibility'; 502: 'uniformity'; 503: 'reflection'; 749: 'generalization'; 750: 'richness'; 752: 'resemblance'.

[29] See Maddy 1988, 488.

special privileges for mathematics. The mathematical research should be done without reference to physics:

> My guess is that the practice of set theory, the methods set theorists actually use to pursue the independent questions would be unaffected, no matter how these issues in natural science might turn out. In other words, the vicissitudes of applied mathematics do not seem to affect the methodology of mathematics in the way that they would if applications were in fact the arbiters of mathematical ontology.[30]

According to Maddy, mathematics has its own methodology, and the heuristics of the working mathematician are what count.

In fact, Maddy does not even describe the methodology of mathematics in general, but only the heuristic tools of modern set theory. She has carefully specified the rules of thumb of the set-theoretic community, but she still has to establish the relevance of these specific rules for mathematics in general. A large part of the mathematical community thinks set theory is irrelevant for mathematics, and thinks it is a rapidly growing abscess rather than a fruitful research programme. In many mathematics departments, there is even no regular course in set theory. It is especially disconcerting that research in the higher reaches of set theory has yielded very few results that have repercussions within traditional mathematics, and even more so that few set-theoretic techniques can be transposed to other domains.

It is important to consider Maddy's background from a sociological point of view. She is the ideological spokeswoman of the CABAL network of set theorists. This is a rather small group of leading American East Coast set-theorists, who further develop descriptive set theory. For them, there is no doubt at all about the basis for set theory, this is simply ZFC.[31] The line of research is also clear, namely to thicken the cardinals and to lengthen the ordinals. Maddy's major argument against '$V = L$' is a 'naturalistic' argument,[32] which means that it is in accordance with what these set-theorists believe. This is intrinsic evidence that '$V = L$' should be outright rejected. Of course, one could question whether this research programme in higher set theory is valuable, or, as Maddy must claim, the only sensible line of research.

[30] Maddy 1997, 159.

[31] In Maddy 1997, I.3, the axioms of ZFC are discussed, and the title of the chapter is "The standard axioms".

[32] Maddy 1997, 216–232.

6. *Maddy's extrinsic evidence*

Apart from the intrinsic evidence, Maddy also offers extrinsic evidence for endorsing the results in higher set theory. Even if for Maddy this extrinsic motivation can at most have an ancillary role, the argumentation is worth analysing. Maddy gives two extrinsic reasons. First, she offers a 'realistic' argument against '$V = L$', based on the extrapolation of a historical evolution. Though she no longer regards this as a compelling argument against '$V = L$', it still "has considerable merits".[33] Second, she considers whether set theory can be regarded as the practice of a social group, or in other words, whether set theory is more than a social construct, and argues that set theory is not arbitrary. But both arguments are unconvincing.

As for the first strategy, Maddy has made an interesting case study of the decline of Definabilism,[34] i.e. the thesis that functions should be definable by means of a mathematical expression. This strict notion of function gave way to a combinatorial definition; a function now is an arbitrary set of pairs of objects. From the contemporary use and historical case-studies, Maddy draws far-reaching conclusions about the prospects of set theory. Maddy thinks that the decline of Definabilism clearly shows that one can better give up a restrictive attitude in mathematics, and that sets should not be definable (i.e. that one should reject '$V = L$'), since this has proven wrong for the concept of function. One may wonder whether this is entirely justified. One could interpret Maddy's clearly depicted example in a far more banal way.

It is clear from Maddy's account that Definabilism was both respectable and successful in Descartes' time. Later it gave way to Combinatoralism. Before presenting this decline, Maddy had presented a similar decline in the history of physics, namely the decline of Mechanism in favour of the Field Conception[35]. In Newton's time Mechanism was very successful, but yet it has become obsolete. One can easily take these examples as a warning that extrapolations of a scientific or mathematical practice are dangerous, rather than as an instigation to pursue Combinatorialism or the Field Conception (without much further reflection). One could easily add examples of scientific communities who entirely misjudged the relevance of their work. Even the famous Gauss failed to see the relevance of Galois's work, and the whole contemporary French mathematical community did not believe that Fourier's work on the decomposition of functions into series of sines and cosines would lead anywhere. Lie's work in group theory at the end of

[33] Maddy 1997, 130.

[34] Maddy 1997, 116–128.

[35] Maddy 1997, 111–116.

the 19th century was regarded as mathematical recreation, but no one could foresee that it would be really indispensable in contemporary physics. Knot theory is at first glance a game rather than a mathematical discipline, but Edward Witten has been rewarded with a Field Medal for work in this area. Knot theory has become indispensable in superstring theories[36]. Examples of successful research programmes in mathematics that have been entirely abandoned later are even more abundant.

Maddy's second argument is a defence against a predictable objection.[37] If mathematics should not be judged by means of extra-mathematical standards,[38] why not take a similar naturalistic attitude towards astrology? If intrinsic justification is all that is needed, what can the difference between the astrological and the mathematical practice be? First, Maddy points out that astrology makes predictions in the physical world, and therefore should be judged by scientific criteria. It then will soon be dismissed as pseudoscience. Since mathematics does not deal with the real world, mathematics can survive. Of course, this does not solve the problem. There are other disciplines that do not deal with the physical world, theology for one. Also, as Maddy points out, astrology could be reformulated as a theory of supernatural phenomena that do not interfere with physical processes. Subsequently, Maddy points out that mathematics is different, since "mathematics is staggeringly useful, seemingly indispensable, to the practice of natural science".[39] This rejoinder has enormous consequences, because, as a last resort, Maddy has to rely on external justification for her defence of higher set theory. It de facto means that after all, mathematics cannot be judged by it own standards. The relevance of the work of set-theorists can and should be considered from the broader perspective. And it has already been pointed out that even within the realm of mathematics, set theory is not staggeringly useful, let alone in a broader context.

[36] Of course, a considerable number of physicists doubt the relevance of these theories.

[37] Maddy 1997, 203–205.

[38] Maddy 1997, 201: "respect for the distinctive mathematical methods requires the naturalistic philosopher of mathematics to refrain from criticizing or defending those methods from an extra-mathematical standpoint."

[39] Maddy 1997, 204–205.

7. *Competing research programmes*

In some respects, Quine and Maddy defend similar views. They both give a rather holistic account of science. In "Two dogmas of empiricism" Quine wrote that in experiments the whole of science is tested "as a corporate body"[40]. And although Maddy draws a very sharp distinction between empirical science and mathematics, nevertheless, empirical science and mathematics are regarded as two monolithic blocks. Maddy contrasts the mathematical methodology to the 'scientific' methodology, but not to the methodologies of different sciences. Neither Quine nor Maddy draw any distinctions within empirical science. Also mathematics is not further split up in subdisciplines with an individual subject matter and methodology. Futhermore, neither Maddy nor Quine consider divergences between research groups. Put roughly, one could say that for Quine there is a single scientific community, and for Maddy, there is a scientific and a mathematical community. No other sociological ruptures seem to exist.

Equally, neither Maddy nor Quine believe in sharp historical ruptures within the scientific framework. Quine often mentions Neurath's metaphor; science is regarded as a ship that is continuously rebuilt while at sea. In other words, for Quine, scientific changes are necessary, but one does never radically abandon a scientific practice. The continuity of science is also taken for granted by Maddy. In her historical accounts, she assumes that scientific change has a uniform direction. In the case of mathematics, this goes from the definabilist conception towards a combinatorial conception of function. In the case of physics, there is a natural line from a mechanical view towards a field conception. Maddy's argumentation heavily relies on the fact that scientific evolution will remain on this track.

These views are certainly not mainstream within contemporary philosophy of science. Since the sociological and historical turn within philosophy of science, especially since Kuhn's *The Structure of Scientific Revolutions*, most philosophers of science no longer believe that historical or sociological ruptures can be neglected. Though various philosophers have so been led to relativism, one can perfectly remain rationalist. Lakatos has given interesting historical and sociological accounts of scientific evolutions and revolutions, especially in the field of mathematics.[41] Science is seen as a competition between various research programmes. Each research programme has a hard core, some central claims it will defend at any cost, and a protective belt, i.e. additional assumptions and a set of heuristic tools. Reformulating

[40] Quine 1953, 41.

[41] See Lakatos 1977, 1–103. Also his unpublished Ph.D. Thesis was on the philosophy and methodology of mathematics, see Lakatos 1961.

the Quine-Maddy discussion hitherto presented in Lakatosian terms immediately destroys the force both of Maddy and Quine's arguments for or against '$V = L$'.

Especially Quine's defence of '$V = L$' is very weak. Quine uses very general epistemic virtues, such as simplicity and economy. These virtues are however much too vague to be of any use in very specific cases such as '$V = L$'. Only within an elaborate research programme in set theory, with its specific heuristic tools, one can make this kind of judgments. If one then looks at the research programme Quine has worked in, one only finds a multitude of research programmes. For decades, anticipating Lakatos's views, Quine has even professed pluralism in set theory. There are also considerable changes in his thought over the years. Only in the mid-eighties, he has chosen side with the predicative research programme. His motivation is twofold. He has always sympathised with the predicative tradition, but thought that there were unsolved technical problems. Only after these anomalies were removed, and after he became convinced that predicative set theory is not a degenerating but a progressive research programme, he wholeheartedly chose for the predicative research programme.

Maddy's project can easily been translated into an ideological or political defence of the work of a specific contemporary research programme within set theory. It concerns a quite specific group, namely the CABAL network. It has a clear research programme. It central axioms are ZFC, and one tries to get very large sets. Maddy's has given an excellent survey of the heuristic tools of the community. Within this tradition, one can give intrinsic evidence that '$V = L$' should be dismissed.

On the other hand, this research programme is one among many. In the discussion of Quine's views, it has become apparent that in the last 120 years many research programmes in logic and set theory have stood up and have vanished. The basic intuitions behind the contemporary prevalent set theory have only been formulated at the end of the sixties. There is no reason why the contemporary views should be everlasting. Also, there are serious competitors, though Maddy scarcely mentions them.[42] Some set theorists still want a minimal instead of a maximal set theory. Solomon Feferman has kept predicative set theory alive. Ronald Jensen favours an ordinal rather than a cardinal approach to sets (i.e. counting is a better paradigm than drawing lines), and hence still defends '$V = L$'. Other set theories still exist, and as has been mentioned, even Quine's NF is still claimed to be intuitive by Forster. Moreover, category theory is a competitor to the very project of set theory. And if set theory does not manage to provide new insights that are also relevant for mainstream mathematicians, even the set-theoretic project

[42] See e.g. Feferman et al. 2000.

can degenerate. In Lakatos's words, "the idea of instant rationality can be seen to be utopian".[43]

8. *Conclusion*

Maddy has forcefully attacked Quine for endorsing '$V = L$'. An analysis of Maddy's argumentation against '$V = L$', of Quine's argumentation in favour of it, of Maddy and Quine's epistemic virtues, of Maddy and Quine's set-theoretic background, bears out that neither of them has convincing arguments. Quine's views on the matter have changed considerably over the years, and after finally confessing himself a predicativist, he did not really elaborate his case for '$V = L$'. Maddy gives a very elaborate intrinsic evidence within her research tradition, but, as can be expected, fails to give decisive arguments in favour of this tradition. In view of the ongoing debate within the set-theoretic community, there is little hope of deciding the matter soon, and little merit in trying to settle the matter prematurely.

Faculty of Philosophy, Tilburg University
Postbox 90153, 5000 LE Tilburg
The Netherlands
E-mail: `l.decock@uvt.nl`

REFERENCES

Benacerraf, P. & Putnam, H. (eds.) 1983, *Philosophy of Mathematics*, 2nd revised edition, Cambridge University Press, Cambridge.

Decock, L. 2002a, *Trading Ontology for Ideology. The Interplay of Logic, Set Theory and Semantics in Quine's Philosophy* (Synthese Library 313), Kluwer, Dordrecht.

Decock, L. 2002b, "Quine's weak and strong indispensability argument", *Journal for the General Philosophy of Science*, *33*, 231–250.

Devlin, K. 1977, "Constructibility", in J. Barwise, *Handbook of Mathematical Logic*, North-Holland, Amsterdam, 453–489.

Feferman, S. et al. 2000, "Does naturalism need new axioms?", *Bulletin of Symbolic Logic*, *6*, 401–446.

Forster, T. 1992, *Set Theory with a Universal Set*, Clarendon, Oxford.

Forster, T. 1997, "Quine's NF – 60 years on", *American Mathematical Monthly*, *104*, 838–843.

[43] Lakatos & Musgrave 1970, 174.

Hahn, L. & Schilpp, P. (eds.) 1986, *The Philosophy of W.V. Quine*, Open Court, La Salle.

Lakatos, I. 1961. *Essays in the Logic of Mathematical Discovery*, Unpublished Ph.D. thesis, Cambridge.

Lakatos, I. 1977, *Mathematics, Science and Epistemology: Philosophical Papers*, volume 2, edited by J. Worrall & E. Zahar, Cambridge University Press, Cambridge.

Lakatos, I. & Musgrave, A. (eds.) 1970, *Criticism and the Growth of Knowledge*, Cambridge University Press, Cambridge.

Maddy, P. 1988, "Believing the Axioms", *Journal of Symbolic Logic*, *53*, 481–511; 736–764.

Maddy, P. 1990, *Realism in Mathematics*, Oxford University Press, Oxford.

Maddy, P. 1996, "Set-theoretic naturalism", *Journal of Symbolic Logic*, *61*, 490–514.

Maddy, P. 1997, *Naturalism in Mathematics*, Oxford University Press, Oxford.

Maddy, P. 1998, "How to be a naturalist about mathematics", in G. Olivieri (ed.), *Truth in Mathematics*, Oxford University Press, Oxford.

Maddy, P. 2001, "Naturalism: Friends and foes", *Nous Supplement*, *15*, 37–67.

Martin, D. 1970, "Review of *Set Theory and its Logic*", *Journal of Philosophy*, *67*, 111–114.

Putnam, H. 1979, *Mathematics, Matter and Method. Philosophical Papers*, Vol. 1, Cambridge University Press, Cambridge.

Quine, W.V.O. 1932, *The Logic of Sequences*, Ph.D. thesis Harvard, reprint Garland, New York, 1995.

Quine, W.V.O. 1934, *A System of Logistic*, Harvard University Press, Cambridge Mass.

Quine, W.V.O. 1936a, "A theory of classes presupposing no canons of type", *Proceedings of the National Academy of Sciences*, *40*, 753–761.

Quine, W.V.O. 1936b, "A reinterpretation of Schönfinkel's logical operators", *Bulletin of the American Mathematical Society*, *42*, 87–89.

Quine, W.V.O. 1936c, "Toward a calculus of concepts", *Journal of Symbolic Logic*, *1*, 2–25.

Quine, W.V.O. 1936d, "Concepts of negative degree", *Proceedings of the National Academy of Sciences*, *40*, 320–326.

Quine, W.V.O. 1936e, "Set-theoretic foundations for logic", reprinted in Quine 1995, 83–99.

Quine, W.V.O. 1937, "New foundations for mathematical logic", reprinted in Quine 1953, 80–101.

Quine, W.V.O. 1953, *From a Logical Point of View*, Harvard University Press, Cambridge Mass.

Quine, W.V.O. 1969, *Set Theory and its Logic*, 2nd ed., Harvard University Press, Cambridge Mass.

Quine, W.V.O. 1970, “Reply to D.M. Martin”, *Journal of Philosophy*, *67*, 247–248.

Quine W.V.O. 1974, *The Roots of Reference*, Open Court, La Salle.

Quine, W.V.O. 1986, “Reply to Parsons”, in: L. Hahn & P. Schilpp (eds.), *The Philosophy of W.V. Quine*, Open Court, La Salle, 396–403.

Quine, W.V.O. 1990, *Pursuit of Truth*, Harvard University Press, Cambridge Mass.

Quine, W.V.O. 1991, “Immanence and validity”, *Dialectica*, *45*, 219–230.

Quine, W.V.O. 1995, *Selected Logic Papers*, 2nd enlarged ed., Harvard University Press, Cambridge Mass.

Quine, W.V.O. & Goodman, N. 1947, “Steps toward a constructive nominalism”, reprinted in N. Goodman, *Problems and Projects*, Indianapolis/New York, Bobbs-Merrill, 1972, 173–198.

Quine, W.V.O. & Ullian, J. 1978, *The Web of Belief*, Random House, New York.

Van Fraassen, B. 1995, “Against naturalized epistemology”, in P. Leonardi & M. Santambrogio, *On Quine*, Cambridge University Press, Cambridge, 68–88.