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# QUESTIONS AND INFERENCES

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*Abstract*
Central concepts of inferential erotetic logic are discussed. Search scenarios and search rules are defined in terms of the logic of questions.

## 1. *Introduction*

The aim of this paper is to present the basic ideas of a certain approach to the logic of questions. We call it here *inferential erotetic logic* (IEL for short). IEL focusses the attention of the logic of questions on erotetic inferences and thus forms an alternative to the received view, that is, roughly, to the approach which puts the structure of questions and the problem of question-answer relationship at the center of attention of erotetic logic.[1] At the same time IEL presents an alternative to the "Interrogative Model of Inquiry" developed by Hintikka and his associates.[2]

We shall concentrate here on basic concepts, underlying intuitions and examples, leaving aside proofs and more technical results (these can be found in the papers and books listed in the "Bibliography"). The last sections contain some new developments.

## 2. *Erotetic Inferences and Validity*

IEL starts with a trivial observation that before a question is asked or posed, a questioner must arrive at it. In many cases arriving at questions resembles coming to conclusions: there are premises involved and some inferential thought processes take place. If we admit that a conclusion need not be

[1] The logic of questions is also called erotetic logic (from Greek "erotema" = question).

[2] Hintikka's book *Inquiry as Inquiry: A Logic of Scientific Discovery* (Kluwer, Dordrecht /Boston /London 1999) includes most of his papers devoted to the subject.

"conclusive", we can say that sometimes questions play the role of conclusions. But questions can also perform the role of premises: it often happens that we arrive at a question when we are looking for an answer to another question. Thus the concept of an *erotetic inference* can be introduced. As a first approximation an erotetic inference may be defined as a thought process in which we arrive at a question on the basis of some previously accepted declarative sentence or sentences and/or a previously posed question. There are, then, erotetic inferences of (at least) two kinds: the key difference between them lies in the type of premises involved. In the case of erotetic inferences of the first kind the premises are declarative sentences. The premises of an erotetic inference of the second kind consist of a question and possibly some declarative sentence(s); erotetic inferences which do not involve any declarative premises can be regarded as a special case of erotetic inferences of the second kind. From the syntactical point of view, an erotetic inference of the first kind can be identified with an ordered pair $<X, Q>$, where $X$ is a finite and non-empty set of declarative sentences (the premises) and $Q$ is a question (the conclusion). Similarly, an erotetic inference of the second kind can be identified with an ordered triple $<Q, X, Q_1>$, where $Q$, $Q_1$ are questions and $X$ is a finite (possibly empty) set of declarative sentences; the question $Q$ is the interrogative premise (we shall call it an initial question), the elements of $X$ are declarative premises and $Q_1$ is the conclusion. Erotetic inferences construed syntactically are also called erotetic arguments.

IEL proposes some conditions of validity for erotetic inferences.

As long as we are concerned with "standard" inferences, validity amounts to the transmission of truth: if the premises are all true, the conclusion must be true as well. But it is doubtful whether it makes any sense to assign truth or falsity to questions. On the other hand, a question usually has many possible answers, and each of them may be true or false. Moreover, there are questions which have well-defined sets of direct (i.e. possible and just-sufficient) answers. Assume that $Q$ is a question which has a well-defined set of direct answers. We say that $Q$ is *sound* if and only if at least one direct answer to $Q$ is true, and unsound otherwise. It seems natural to impose the following necessary condition of validity on erotetic inferences of the first kind:

($\mathsf{C}_1$) (*transmission of truth into soundness*) if the premises are all true, then the question which is the conclusion must be sound.

Is this sufficient? Certainly not. For if ($\mathsf{C}_1$) were sufficient, the following inferences would be valid:

(I) *Andrew is rich.*
*Andrew is happy.*
―――――――――
*Is Andrew happy?*

(II) *If Andrew is rich, then he is happy.*
*Andrew is rich.*
―――――――――
*Is Andrew happy?*

What is wrong with the above inferences? The question which is the conclusion has a (direct) answer which provides us with information which is already present (directly or indirectly) in the premises. In other words, the question which is the conclusion is logically redundant and thus not informative. So IEL imposes the following additional necessary condition of validity on erotetic inferences of the first kind:

($\mathsf{C}_2$) (*informativeness*) a question which is the conclusion must be informative with respect to the premises.

Informativeness is then explicated as the lack of entailment of any direct answer from the premises; the applied concept of entailment need not be classical (see below).

Let us now turn to erotetic inferences of the second kind. The natural generalization of the standard condition of validity is:

($\mathsf{C}_3$) (*transmission of soundness/truth into soundness*) if the initial question is sound and all the declarative premises are true, then the question which is the conclusion must be sound.

Again, ($\mathsf{C}_3$) is only a necessary condition. If ($\mathsf{C}_3$) were sufficient, the following would be valid inferences:

(III) *Is Andrew a logician?*
―――――――――
*Is Andrew married?*

(IV) *Where did Andrew leave for: Chicago or New York?*
―――――――――
*Did Andrew leave for the US?*

The problem here is that the questions which are conclusions have (direct) answers which are cognitively useless: these answers, if accepted, do not contribute in any way to the process of finding answers to initial questions.

IEL imposes the following additional condition of validity on erotetic inferences of the second kind:

($\mathsf{C}_4$) (*open-minded cognitive usefulness*) each direct answer to the question which is the conclusion is potentially useful, on the basis of the declarative premises, for finding an answer to the initial question.

Condition ($\mathsf{C}_4$) is then clarified by requiring that each direct answer to the question which is the conclusion should, together with the declarative premises, *narrow down* the class of possibilities offered by the initial question. In the majority of cases this amounts to entailment of a direct or partial answer to the initial question (see below). Note that we do not require that an answer to the question which is the conclusion should, together with the declarative premises, yield a single answer to the initial question: we only require that the class of possibilities should be narrowed down. Yet, when the class is narrowed down to a singleton class, a single answer to the initial question is forthcoming.

Conditions ($\mathsf{C}_1$)–($\mathsf{C}_4$) are expressed in rather vague terms. Of course, IEL does not stop here. The following semantical concepts are introduced: (i) *evocation* of questions by sets of declarative sentences, and (ii) (erotetic) *implication* of questions by questions and possibly declarative sentences. Validity of erotetic inferences of the first kind is then defined in terms of evocation, whereas validity of erotetic inferences of the second kind is defined by means of erotetic implication. The proposed definitions of evocation and erotetic implication are explications of the relevant notions of the arising of questions (cf. Wiśniewski 1995a, Chapter 1). By defining the semantical concept "a question $Q$ is evoked by a set of declarative sentences $X$" we explicate the concept "a question $Q$ arises from a set of declarative sentences $X$", whereas the definition of "a question $Q_1$ is implied by a question $Q$ on the basis of a set of declarative sentences $X$" gives us an explication of the notion "a question $Q_1$ arises from a question $Q$ and a set of declarative sentences $X$." Thus, although conditions ($\mathsf{C}_1$), ($\mathsf{C}_2$) on the one hand, and conditions ($\mathsf{C}_3$), ($\mathsf{C}_4$) on the other seem diverse at first sight, the analysis of validity of erotetic inferences proposed by IEL is based on a certain general idea.

## 3. *Questions and Answers*

IEL presupposes a certain analysis of questions and answers.

The approaches to questions proposed by various logicians and formal linguists can be divided into *reductionist* and *non-reductionist* positions.

Within the reductionist approach, in turn, *radical* and *moderate* standpoints can be distinguished.[3]

According to the radical view, questions are not linguistic entities. Questions are identified with: sets of sufficient answers, or sets of possible answers, or sets of true answers, or functions defined on possible worlds, or functions from categorial answers to propositions, or speech acts of a special kind, etc.

The moderate reductionist view considers questions as linguistic entities which, however, can be reduced to expressions of some other categories. It is claimed that each question can be adequately paraphrased as an expression belonging to some other syntactic category and then formalized within a certain logic which, although not primarily designed as the logic of questions, can thus be regarded as providing us with the foundations of erotetic logic. Sometimes questions are identified with declarative sentences of a strictly defined form (Harrah). More often questions are identified with imperatives of a special kind. But the most popular approach is the imperative-epistemic approach, with Lennart Åqvist and Jaakko Hintikka as its most eminent representatives.[4]

According to the non-reductionist approach, questions are specific expressions of a strictly defined form; they are not reducible to expressions of other syntactic categories. The most widespread proposal here is to regard a question as an expression which consists of an interrogative operator and a sentential function (declarative formula with free variable(s)). This idea is most widely and thoroughly elaborated on in the works of Tadeusz Kubiński.[5] But Nuel D. Belnap's theory of questions and answers is the most well known.[6] Belnap distinguishes between: (a) natural language questions,

[3] For a general overview, see Wiśniewski 1995a, Chapter 2. See also the survey paper of David Harrah 'The logic of questions', included in the second edition of *Handbook of Philosophical Logic* (Kluwer, Dordrecht/Boston/London 2002, Volume 8, pp. 1–60), or (for more linguistically oriented approaches) the paper of Jeroen Groenendijk and Martin Stokhof 'Questions', published in *Handbook of Logic and Language* (Elsevier, Amsterdam 1996, pp. 1055–1125).

[4] Cf. Lennart Åqvist *A New Approach to the Logical Theory of Interrogatives* (Almqvist & Wiksell, Uppsala 1965); Jaakko Hintikka *The Semantics of Questions and the Questions of Semantics* (North-Holland, Amsterdam 1976).

[5] Cf. e.g. Tadeusz Kubiński *An Outline of the Logical Theory of Questions* (Akademie-Verlag, Berlin 1980) or a survey paper: A. Wiśniewski 'Kubiński's theory of questions', in: A. Wiśniewski and J. Zygmunt (eds.), *Erotetic Logic, Deontic Logic and Other Logical Matters. Essays in Memory of Tadeusz Kubiński* (Wydawnictwo Uniwersytetu Wrocławskiego, Wrocław 1997, pp. 9–50).

[6] Cf. e.g. Nuel D. Belnap and Thomas B. Steel *The Logic of Questions and Answers* (Yale University Press, New Haven 1976).

(b) interrogatives and (c) questions understood as abstract (set-theoretical) entities. Interrogatives are expressions of some formalized languages. They are not only formal counterparts of natural language questions, but they also express questions understood as abstract entities. The basic idea of Belnap's approach is that an interrogative "presents" a set of alternatives together with some suggestions or indications as to what kind of choice or selection among them should be made. A simple interrogative consists of the question mark ?, the lexical subject and the lexical request. The function of a lexical subject is to offer the relevant (nominal) alternatives, whereas the role of a lexical request is to characterize the required kind of selection. A direct answer to an interrogative is then, roughly, a declarative formula made up of the relevant alternative(s) which "satisfies" the conditions imposed by the lexical request.

IEL accepts the non-reductionist approach to questions, but does not forejudge its particular form. What we need is a formalized language which has both declarative formulas and questions (the distinction between interrogatives and questions is disregarded) as meaningful expressions, where questions are not declarative formulas. Languages of this kind can be defined in various ways, and questions can be introduced, e.g. according to Kubiński's approach or Belnap's approach (to mention only the richest sources of ideas). Yet, in the majority of published papers on IEL a specific attitude has been adopted. But before this is presented, let us say a few words about answers to questions.

Most logical theories of questions pay at least as much attention to answers to questions as to the questions themselves. It is usually assumed that a question may have many answers and thus the phrase "answer to a question" is not tantamount to "the true answer to a question." In other words, the analyzed answers are usually *possible* answers; a possible answer may be true or false (or even have no logical value at all). But it is not the case that all possible answers are equally interesting to erotetic logicians. The standard way of proceeding is to define some *basic category* of possible answers. They are called direct answers (Åqvist, Belnap, Harrah, Kubiński), sufficient answers (Stahl), conclusive answers (Hintikka), etc. Those "principal" possible answers (let us use this general term here) are supposed to satisfy some general conditions, usually expressed in pragmatic (in the traditional sense of the word) terms. For example, direct answers in Belnap's sense are the answers which "are directly and precisely responsive to the question, giving neither more nor less information than what is called for."[7] In the light of Hintikka's theory a reply is called a conclusive answer if it completely satisfies

[7] N.D. Belnap 'Åqvist's corrections-accumulating question sequences', in: J.W. Davis *et al.* (eds.), *Philosophical Logic* (D. Reidel, Dordrecht 1969), p. 124.

the epistemic request of the questioner. Although the above conditions are formulated in pragmatic terms, logical theories of questions usually define the principal possible answers to questions or interrogatives of formalized languages in terms of syntax and/or semantics. For example, Kubiński as well as Belnap (in the case of elementary interrogatives) proceed syntactically, whereas Hintikka's approach is twofold: possible replies and so-called conclusiveness-conditions are characterized in syntactic terms, and a possible reply is a conclusive answer if this reply together with the description of the questioner's state of knowledge entails (in the sense of some underlying epistemic logic) the desideratum of the question, that is, an epistemic formula which describes the cognitive state of affairs the questioner wants to be brought about.

IEL proceeds syntactically: it is assumed that the *syntax* of a considered formalized language assigns to a question a set of sentences, which are then called *directs answers* to the question. As long as we are pursuing general considerations, the details of this assignment need not be decided on; we only assume that the following conditions are fulfilled:

($\mathsf{C}_5$) direct answers are sentences, i.e. declarative formulas with no individual or higher-order free variables (since sentential functions are not definite enough in order to answer anything);

($\mathsf{C}_6$) each question has at least two direct answers (since a necessary condition of being a question is to present at least two "alternatives" or conceptual possibilities among which some selection can be made);

($\mathsf{C}_7$) each finite and at least two-element set of sentences is the set of direct answers to some question.

Direct answers to a question of a formalized language are then viewed as the *possible and just-sufficient answers*. Yet, the following have to be carefully distinguished: (a) direct answers to a question of a formalized language, and (b) possible and just-sufficient (i.e. direct) answers to a natural-language question. As many philosophers and linguists have pointed out, there are cases in which it is strongly context-dependent what sentences may be counted as the possible and just-sufficient answers to a natural-language question. Moreover, there are natural-language questions for which it makes no sense at all to speak about sets of possible and just-sufficient answers; why-questions and how-questions are often recalled in this context. Yet, we can still claim that questions of a formalized language in which the question-answer relationship is defined syntactically represent questions of a natural language. The relation of representation we have in mind can be characterized as follows: a question $Q$ of a formalized language represents a question

$Q^*$ of a natural language *construed in such a way* that the possible and just-sufficient answers to $Q^*$ have the *logical form* of direct answers to $Q$. If a natural-language question has many readings, it has many representations. The richer the underlying formalized language, the more we can represent within it.

Since there is no (and in many cases cannot be) one-to-one correspondence between natural-language questions and questions of a formalized language, we can go further and define questions of a formalized language as expressions whose form does not resemble their natural-language counterparts. One possibility is to define a question (of an object-level formalized language) as an expression which consists of the question mark ? and an expression of the object-language which is equiform to the expression of the metalanguage which designates the set of direct answers to the question. For example, when we add the question mark ? and the brackets $\{\}$ to the vocabulary of a first-order language (or a propositional language), we can supplement the language with questions of the form:

$$(\#)\,?\,\{A_1, ..., A_n\}$$

where $n > 1$ and $A_1, ..., A_n$ are syntactically distinct sentences (i.e. closed well-formed formulas); these sentences are direct answers to the question. A question of the form $(\#)$ represents those natural-language questions (or their clarified counterparts) whose sets of possible and just-sufficient answers consist of sentences which have the logical form of $A_1, ..., A_n$, exclusively. Note that an expression of the form $(\#)$ still belongs to the object-language, whereas the expression $\{A_1, ..., A_n\}$ is equiform to a metalanguage-expression which designates the set of direct answers to the question. One advantage of this solution is that it is now extremely easy to say what counts as a direct answer to the question, and what natural-language questions are represented by it. This procedure can also be applied to (possible representations of) some wh-questions (cf. Wiśniewski 1995a, Chapter 3). When we go in this direction, we arrive at a level of abstraction at which the so-called conclusiveness conditions in Hintikka's sense are no longer needed. Yet, let us stress that the above approach to questions is not essential to IEL: if only conditions ($\mathsf{C}_5$), ($\mathsf{C}_6$) and ($\mathsf{C}_7$) specified in the previous section are fulfilled, questions can be introduced in the way proposed by Belnap, or by Kubiński, or by other logicians who adopt the non-reductionist standpoint.

In what follows we assume the von Neumann-Gödel-Bernays version of set theory. We shall use the standard set-theoretical terminology and notation. The expression "iff" abbreviates "if and only if."

4. *Minimal erotetic semantics*

Let $L$ be a formalized language of the required kind. Thus well-formed formulas (wffs for short) of $L$ split into two disjoint categories: declarative well-formed formulas (d-wffs), defined in a standard way, and erotetic well-formed formulas (e-wffs), that is, questions of $L$ defined according to some pattern, but in such a way that the conditions ($\mathsf{C}_5$), ($\mathsf{C}_6$) and ($\mathsf{C}_7$) of the previous section are fulfilled. In order to go further we now have to supplement $L$ with a semantics. But, surprisingly enough, in order to pursue general investigations within IEL, the semantics need not be very specific with respect to the erotetic part of $L$.

As a matter of fact, it suffices to suppose that the declarative part of $L$ is supplemented with a semantics which is rich enough to define a certain concept of truth for d-wffs of $L$. Depending on the particular form of the semantics, truth can be defined in terms of models, or algebraic structures, or Boolean valuations, or games, etc. Let $\mathsf{D}_L$ stand for the set of all d-wffs of $L$, and $\mathsf{E}_L$ for the set of all e-wffs of $L$. Of course, $\mathsf{D}_L \cap \mathsf{E}_L = \emptyset$. By a *partition* of $L$ we mean a partition of $\mathsf{D}_L$, that is, an ordered pair $<\mathsf{T}, \mathsf{U}>$, where $\mathsf{T} \cap \mathsf{U} = \emptyset$ and $\mathsf{T} \cup \mathsf{U} = \mathsf{D}_L$. Intuitively, $\mathsf{T}$ consists of all the wffs which are true, and $\mathsf{U}$ is made up of all the d-wffs which are untrue. Then it is assumed that in the class of all partitions a subclass of *admissible partitions* is distinguished. In the general case it is only required that admissible partitions should comply with the underlying semantics and that the intersection of the first elements of all admissible partitions forms a non-empty set (elements of this intersection may be thought of as valid formulas). Yet, for the purposes of this paper we will also assume that admissible partitions are directly determined by the accepted semantics of the declarative part of $L$. The following examples should clarify this:

*Example 1:*

Assume that $L$ results from the language of Classical Propositional Calculus (CPC) by enriching it with questions. Let $\mathsf{D}_L$ be the set of all CPC-formulas. Assume that the semantics for $\mathsf{D}_L$ is based on the notion of a Boolean valuation. The class of admissible partitions of $L$ is determined by Boolean valuations of $\mathsf{D}_L$: a partition $<\mathsf{T}, \mathsf{U}>$ of $L$ is admissible iff there is a Boolean valuation $\mathsf{v}$ such that $\mathsf{T} = \{A \in \mathsf{D}_L : \mathsf{v}(A) = 1\}$.

*Example 2:*

Assume that $L$ is a first-order language enriched with questions. Assume that the concept of truth of a d-wff $A$ of $L$ in an interpretation $\mathsf{M} = <M, f>$ of the declarative part of $L$ (in symbols: $\mathsf{M} \models A$) is defined in the standard way. A partition $<\mathsf{T}, \mathsf{U}>$ of $L$ is admissible iff there is an interpretation $\mathsf{M}$ of the declarative part of $L$ such that $\mathsf{T} = \{A \in \mathsf{D}_L : \mathsf{M} \models A\}$.

*Example 3:*

Assume again that $L$ is a first-order language enriched with questions, and that the declarative part of $L$ is supplemented with a standard model-theoretical semantics. Assume also that in the class of all interpretations of the declarative part of $L$ a proper subclass of normal interpretations was distinguished (for instance, one can stipulate that an interpretation $<M, f>$ is normal iff each element of $M$ is the value of some closed term of $L$; of course, there are also other options). A partition $<\mathsf{T}, \mathsf{U}>$ of $L$ is said to be admissible iff there is a normal interpretation $\mathsf{M}$ of the declarative part of $L$ such that $\mathsf{T} = \{A \in \mathsf{D}_L : \mathsf{M} \models A\}$.

Let $\mathsf{d}Q$ denote the set of direct answers to a question $Q$. The concept of soundness of a question in a partition is defined as follows:

*Definition 1: A question $Q$ is* sound *in a partition $<\mathsf{T}, \mathsf{U}>$ iff $\mathsf{d}Q \cap \mathsf{T} \neq \emptyset$.*

When the semantics of the declarative part of $L$ becomes more specific, soundness in a partition can be replaced by soundness in an interpretation, or soundness with respect to a Boolean valuation, etc. The underlying idea is always the same: a sound question is a question which has at least one true direct answer.

We need two concepts of entailment: the standard concept of single-conclusion entailment and the concept of multiple-conclusion entailment.[8] The latter is a semantical relation between sets of d-wffs.

*Definition 2: A set of d-wffs $X$* entails *a d-wff $A$ (in symbols: $X \models A$) iff for each admissible partition $<\mathsf{T}, \mathsf{U}>$:*
*(*) if $X \subset \mathsf{T}$, then $A \in \mathsf{T}$.*

*Definition 3: A set of d-wffs $X$* multiple-conclusion entails *a set of d-wffs $Y$ (symbolically: $X \Vdash Y$) iff for each admissible partition $<\mathsf{T}, \mathsf{U}>$:*
*(**) if $X \subset \mathsf{T}$, then $Y \cap \mathsf{T} \neq \emptyset$.*

[8] Cf. D.J. Shoesmith and T.J. Smiley *Multiple-conclusion logic* (Cambridge University Press, Cambridge 1978).

Note that both entailment and multiple-conclusion entailment (mc-entailment for short) are defined by means of admissible partitions, and these, in turn, have sets of all true d-wffs (true in the sense of underlying semantics) as the first elements. The idea which lies behind the concept of multiple-conclusion entailment is: if all the d-wffs in $X$ are true, then at least one d-wff in $Y$ must be true. Or, to put it differently, the truth of all the $X$-es guarantees the existence of at least one truth in $Y$.

If $L$ fulfills the conditions of Example 2, we get:

(i) $X \models A$ iff the following condition holds:
(*′) for each interpretation $\mathsf{M}$ of the declarative part of $L$: if all the d-wffs in $X$ are true in $\mathsf{M}$, then $A$ is true in $\mathsf{M}$.

(ii) $X \Vdash Y$ iff the following holds:
(**′) for each interpretation $\mathsf{M}$ of the declarative part of $L$: if all the d-wffs in $X$ are true in $\mathsf{M}$, then at least one d-wff in $Y$ is true in $\mathsf{M}$.

When admissible partitions are defined in terms of Boolean valuations or normal interpretations (cf. examples 1 and 3 above), the relevant conditions refer to Boolean valuations and normal interpretations, respectively. This is as it should be.

Note that entailment can be defined in terms of mc-entailment. But the converse does not hold: it can happen that $X$ mc-entails $Y$ although no element of $Y$ is (single-conclusion) entailed by $X$.

The concept of multiple-conclusion entailment has proved its usefulness in the logic of questions in many ways. In particular, both evocation and erotetic implication can be defined in terms of mc-entailment (cf. sections 4 and 6).

For conciseness, we write $A \Vdash Y$ instead of $\{A\} \Vdash Y$, and $X \Vdash A$ instead of $X \Vdash \{A\}$.

Following Belnap, we shall now introduce the concept of a presupposition of a question.

*Definition 4*: *A d-wff $A$ is a* presupposition *of a question $Q$ iff $A$ is entailed by each direct answer to $Q$.*

The set of presuppositions of a question $Q$ will be referred to as Pres$Q$.

Some questions, but not all, have prospective presuppositions.

*Definition 5*: *A d-wff $A$ is a* prospective presupposition *of a question $Q$ iff (i) $A$ is a presupposition of $Q$, and (ii) $A \Vdash \mathsf{d}Q$.*

In general, it is possible that all the presuppositions of a question are true, but the question is not sound, that is, has no true direct answer. But if a question has a prospective presupposition, then the question is sound given that a prospective presupposition of it is true.

By means of the concepts introduced above further concepts of erotetic semantics can be defined (cf. Wiśniewski 1995a, 1996, 1997). This, however, would go beyond the scope of the present paper. We only introduce the following:

*Definition 6*: *A question $Q$ is:*
*(a)* normal *iff* Pres$Q \Vvdash \mathsf{d}Q$;
*(b)* regular *iff for some* $A \in$ Pres$Q$: $A \Vvdash \mathsf{d}Q$.

For intuitions, see Wiśniewski (1995a, 1997). Let us stress that the fact that a question has any of the above characteristics is dependent upon both the form of the question and the properties of an underlying semantics. In particular, it may happen that a given question is normal in the light of one semantics, but not normal from the standpoint of another. Note also that since mc-entailment need not be compact, normality is not tantamount to regularity.

So far we have introduced only one category of answers. Yet, the conceptual apparatus characterized above allows us to define further categories. For example, we can say that a d-wff $A$ is a *just-complete answer* to a question $Q$ if and only if there is a direct answer $B$ to $Q$ such that $B$ entails $A$ and $A$ entails $B$. Roughly, just-complete answers are equivalent to direct answers; this concept is not superfluous since direct answers are defined syntactically. Partial answers are defined by:

*Definition 7*: *A sentence $A$ is a* partial answer *to a question $Q$ iff $A$ is not a just-complete answer to $Q$ and for some non-empty proper subset $Y$ of the set of direct answers to $Q$ we have:*
*(i)* $A \Vvdash Y$*, and*
*(ii)* $B \models A$ *for each* $B \in Y$.

Thus a partial answer to a question is a sentence which is not equivalent to any direct answer to the question, but which is true if and only if a true direct answer belongs to some specified proper subset of the set of all the direct answers to the question.[9]

Since we have assumed that each question has at least two direct answers, Definition 7 yields that a question which has exactly two direct answers has

[9] For a discussion, see Wiśniewski (1995a, pp. 114–115).

no partial answers. This is not to say, however, that binary questions have only direct and just-complete answers; these questions, as well as other questions, have also incomplete answers, corrective answers, etc.

## 5. *Definition of Evocation*

Looked at a formal point of view, evocation of questions is a rather simple notion. As we have said, it can happen that no d-wff in a set of d-wffs $Y$ is entailed by a set of d-wffs $X$, but still $X$ mc-entails $Y$. Now think of $Y$ as of the set of direct answers to a question. Direct answers are the "just-sufficient possibilities" offered by a question. It may happen that none of them holds. Yet, suppose that $X$ mc-entails the set of direct answers to a question $Q$, but does not entail any particular direct answer to $Q$. Thus if $X$ consists of truths, we have a guarantee that a true direct answer to $Q$ exists. On the other hand, since no direct answer to $Q$ is entailed by $X$, the problem as to which direct answer to $Q$ is true cannot be resolved in a logically legitimate way by means of $X$ only. In other words, $Q$ is sound relative to $X$ and informative with respect to $X$. In such a situation we say that $X$ evokes $Q$. More formally, we introduce:

*Definition 8*: *A set of d-wffs $X$* evokes *a question $Q$ (in symbols: $\mathsf{E}(X, Q)$) iff*

*(i) $X \Vdash \mathsf{d}Q$, and*
*(ii) for each $A \in \mathsf{d}Q$: $X non\Vdash A$.*[10]

Yet, it is neither assumed nor denied that an evoking set consists of truths. Let $\mathsf{P} = <\mathsf{T}, \mathsf{U}>$ be an arbitrary but fixed admissible partition of (the declarative part of) the language. Assume that $\mathsf{E}(X, Q)$. The following tables describe possible connections:

| $X$ | $Q$ |
|---|---|
| $X \subset \mathsf{T}$ | sound in $\mathsf{P}$ |
| $X non\subset \mathsf{T}$ | sound in $\mathsf{P}$ or unsound in $\mathsf{P}$ |

Table 1.

[10] Of course, clause *(ii)* is equivalent to:

*(ii′) for each $A \in \mathsf{d}Q$: $X non\models A$.*

| $Q$ | $X$ |
|---|---|
| sound in $\mathsf{P}$ | $X \subset \mathsf{T}$ or $X non\!\subset \mathsf{T}$ |
| unsound in $\mathsf{P}$ | $X non\!\subset \mathsf{T}$ |

Table 2.

For properties of evocation, see Wiśniewski (1991a, 1995a, 1996).[11] Let us only mention the following:

*Fact 1*: If $X$ evokes $Q$, then $X$ entails each presupposition of $Q$.

*Fact 2*: Assume that $Q$ is normal. Then $X$ evokes $Q$ iff $X$ entails each presupposition of $Q$, but does not entail any direct answer to $Q$.

*Fact 3*: Assume that $Q$ is regular. Then $X$ evokes $Q$ iff $X$ entails some prospective presupposition of $Q$, but does not entail any direct answer to $Q$.

We say that an erotetic inference of the first kind $<X, Q>$ is *valid* iff $\mathsf{E}(X, Q)$.

*Generation* of questions is a special case of evocation: we say that a set of d-wffs $X$ generates a question $Q$ iff $X$ evokes $Q$ and $\emptyset\ non\!\Vdash \mathsf{d}Q$. The underlying intuition is: a generated question is made sound by the generating set[12] and is informative with respect to this set. For generation see Wiśniewski (1989, 1991a, 1995a).[13]

[11] In Wiśniewski (1991a) evocation is called "weak generation".
Chapter 5 of Wiśniewski (1995a) is devoted to properties of evocation in first-order languages enriched with questions and supplemented with a standard model-theoretic semantics. A more general approach (in terms of the minimal erotetic semantics) is adopted in Wiśniewski (1996). Meheus (cf. Meheus 1999) proposes some generalizations which also cover the case of evocation from inconsistent sets of premises.

[12] If $\emptyset \Vdash \mathsf{d}Q$, then $Q$ is sound in each admissible partition of the language and thus clause *(ii)* of Definition 8 is fulfilled for trivial reasons. Generation excludes this.

[13] See also Meheus (1999) for the inconsistent case.

## 6. *Examples of Evocation*

In order to present examples let us consider two languages of the kind analyzed above.

### 6.1. *Language $L_1$*

Language $L_1$ results from the language of Classical Propositional Calculus (CPC) by enriching it with questions of the form $(\#)$. To be more precise, the declarative part of $L_1$ is the language of CPC; we shall use the letters $p$, $q$, $r$, $s$, $t$, $u$, $p_1$, .... for propositional variables and the symbols $\neg$, $\supset$, $\wedge$, $\vee$, $\equiv$ for negation, implication, conjunction, disjunction and biconditional, respectively. By d-wffs of $L_1$ we shall mean the CPC-formulas. The vocabulary of the erotetic part of $L_1$ consists of the following signs: $?$, $\{$, $\}$. Questions of $L_1$ are of the form:

$$(\#)\ ?\,\{A_1, ..., A_n\}$$

where $n > 1$ and $A_1, ..., A_n$ are syntactically distinct d-wffs of $L_1$; these d-wffs are called direct answers to the question.

A question of the form $(\#)$ can be read: "Is it the case that $A_1$, ... , or is it the case that $A_n$?". Yet, sometimes other readings can be recommended. For example, questions of the form (called *simple yes-no questions*):

(6.1) $\quad ?\,\{A, \neg A\}$

can be read "Is it the case that $A$?" and abbreviated as:

(6.2) $\quad ?\,A$

Questions of the form:

(6.3) $\quad ?\,\{A \wedge B, A \wedge \neg B, \neg A \wedge B, \neg A \wedge \neg B\}$

are called *binary conjunctive questions* and abbreviated as:

(6.4) $\quad ? \pm \mid A, B \mid$

The semantics for the declarative part of $L_1$ is based on the concept of Boolean valuation; the concepts of Truth and Falsehood of a d-wff under a Boolean valuation are understood in the standard way. Boolean valuations determine admissible partitions of $L_1$ (cf. Example 1 in Section 4); the remaining semantical concepts are defined according to the pattern presented in Section 4.

The following present simple examples of evocation in $L_1$; for transparency, we write $A_1, ..., A_n \vdash_{\mathsf{E}} Q$ instead of $\mathsf{E}(\{A_1, ..., A_n\}, Q)$:

(6.5) $p \vee \neg p \vdash_{\mathsf{E}} ?\, p,$

(6.6) $p \vee q \vdash_{\mathsf{E}} ?\, p,$

(6.7) $p \vee q \vdash_{\mathsf{E}} ?\, q,$

(6.8) $p \supset q \vdash_{\mathsf{E}} ?\, p,$

(6.9) $p \supset q \vdash_{\mathsf{E}} ?\, q,$

(6.10) $p \equiv q \vdash_{\mathsf{E}} ?\, \{p \wedge q, \neg p \wedge \neg q\},$

(6.11) $p \vee q \vdash_{\mathsf{E}} ?\, \{p, q\},$

(6.12) $p \vee q \vdash_{\mathsf{E}} ?\, \{p \wedge q, p \wedge \neg q, \neg p \wedge q\},$

(6.13) $p \supset q \vee r, p \vdash_{\mathsf{E}} ?\, \{q, r\},$

(6.14) $p \wedge q \supset r, \neg r \vdash_{\mathsf{E}} ?\, \{\neg p, \neg q\},$

(6.15) $(p \vee q) \vee r \vdash_{\mathsf{E}} ?\, \{p, q \vee r\}.$

6.2. *Language* $L_2$

Language $L_2$ results from the language of Monadic Predicate Calculus with identity (MPC$_=$) by enriching it with questions of the form $(\#)$ as well as with existential and general wh-questions. We assume that the vocabulary of the declarative part of $L_2$ contains individual constants, but does not contain function symbols. The letters $P$, $R$, $T$ will be used as metalinguistic variables for predicates, and the letters $c$, $c_1$, ... will be metalinguistic variables for individual constants. By d-wffs of $L_2$ we shall mean the well-formed formulas of the language of MPC$_=$, defined in the standard manner. A *sentence* is a d-wff with no free variable(s); freedom and bondage of variables are defined as usual. By terms of $L_2$ we shall mean the individual variables and individual constants. An expression of the form $Ax$ refers to d-wffs of $L_2$ which have $x$ as the only free variable. The result of the substitution of an individual constant $c$ for a variable $x$ in a d-wff $Ax$ will be designated by $A(x/c)$; needless to say, $A(x/c)$ is always a sentence.

The vocabulary of the erotetic part of $L_2$ consists of the signs: ?, {, }, $\mathsf{S}$, $\mathsf{U}$. First, $L_2$ contains questions of the form:

$$(\#)\,?\,\{A_1, ..., A_n\}$$

where $n > 1$ and $A_1, ..., A_n$ are nonequiform (i.e. syntactically distinct) sentences of $L_2$. Direct answers to questions of the form $(\#)$ are defined as above; we accept here analogous notational conventions as in the case of $L_1$. Second, $L_2$ contains questions falling under the schemata:

$$(\#\#)\,?\,\mathsf{S}(Ax),$$

$$(\#\#\#)\,?\,\mathsf{U}(Ax),$$

where $x$ stands for an individual variable and $Ax$ is a d-wff of $L_2$ which has $x$ as the only free variable. A direct answer to a question of the form $(\#\#)$ is a sentence of the form $A(x/c)$, where $c$ is an individual constant. Direct answers to questions of the form $(\#\#\#)$ fall under the schema:

$$(!)\; A(x/c_1) \wedge ... \wedge A(x/c_n) \wedge \forall x(Ax \supset x = c_1 \vee ... \vee x = c_n)$$

where $n \geq 1$ and $c_1, ...., c_n$ are distinct individual constants.

Questions of the form ? $\mathsf{S}(Ax)$ can be read "Which $x$ is such that $Ax$?". We shall call them existential wh-questions. Questions of the form ? $\mathsf{U}(Ax)$ can be read "What are all of the $x$'s such that $Ax$?" and thus we call them general wh-questions.

Wh-questions of $L_2$ are counterparts of some natural-language wh-questions. Yet, there are natural-language wh-questions which can be formalized by means of questions of the form (#). Note that a question of the form $?\,\{Pc_1, ..., Pc_n\}$ can be read "Which of the $c_1, ..., c_n$ has the property $P$?".

The symbols $\mathsf{S}$ and $\mathsf{U}$ belong to the vocabulary of the object-level language $L_2$. Yet, we can introduce them to the metalanguage as well (but with different meanings). We assume that on the metalanguage level an expression $\mathsf{S}(Ax)$ designates the set of all the sentences of the form $A(x/c)$, whereas $\mathsf{U}(Ax)$ designates the set of all the sentences of the form (!). We adopt this solution for two reasons. First, we will be dealing with sets of direct answers to questions of $L_2$ and it is convenient to have a specific notation for them. Second, we can now say that each question of $L_2$ consists of the sign ? followed by an (object-language) expression which is equiform to a metalanguage expression that designates the set of direct answers to the question.

The semantics for the declarative part of $L_2$ is a model-theoretic one. By an *interpretation* of the declarative part of $L_2$ we mean an ordered pair $<M, f>$, where $M$ is a non-empty set (the domain) and $f$ is the interpretation function defined on individual constants and predicates of $L_2$ in the standard way. The concept of truth of a d-wff in an interpretation is defined in the standard manner.

Admissible partitions of $L_2$ are determined by the class of all interpretations (cf. Section 4, Example 2) and the remaining semantical concepts are defined accordingly (cf. Section 4).

We are now ready to give some examples of evocation in $L_2$. In what follows we assume that distinct metalinguistic variables represent distinct object-language entities.

(6.16) $Pc_1, ..., Pc_n \vdash_{\mathsf{E}} ?\forall xPx,$

(6.17) $Pc_1 \wedge Rc_1, ..., Pc_n \wedge Rc_n \vdash_{\mathsf{E}} ?\forall x(Px \supset Rx),$

(6.18) $Pc_1 \wedge Rc_1, ..., Pc_n \wedge Rc_n, Pc_{n+1} \vdash_{\mathsf{E}} ?\, Rc_{n+1},$

(6.19) $Pc_1, Pc_2 \vdash_{\mathsf{E}} ?\, c_1 = c_2,$

(6.20) $\exists xPx \vdash_{\mathsf{E}} ?\, \{Pc, \exists x(Px \wedge x \neq c)\},$

(6.21) $\exists x(Px \wedge (x = c_1 \vee ... \vee x = c_n)) \vdash_{\mathsf{E}} ?\, \{Pc_1, ..., Pc_n\},$

where $n > 1$,

(6.22) $\neg\forall x(x = c_1 \vee ... \vee x = c_n \supset Px) \vdash_{\mathsf{E}} ?\, \{\neg Pc_1, ..., \neg Pc_n\},$

where $n > 1$,

(6.23) $\exists x(Px \wedge (x = c_1 \vee ... \vee x = c_n)) \vdash_{\mathsf{E}} ?\, \mathsf{S}(Px),$

where $n > 1$,

(6.24) $\neg\forall x(x = c_1 \vee ... \vee x = c_n \supset Px) \vdash_{\mathsf{E}} ?\, \mathsf{S}(\neg Px),$

where $n > 1$,

(6.25) $\forall x(Px \equiv x = c_1 \vee ... \vee x = c_n) \vee ... \vee$
$$\forall x(Px \equiv x = c'_1 \vee ... \vee x = c'_k) \vdash_{\mathsf{E}} ?\,\mathsf{U}(Px),$$

where $n > 1$ and $k > 1$.

We have considered only relatively simple languages enriched with rather simple questions. For further examples of evocation, in particular evocation of complex questions in more sophisticated languages see Wiśniewski (1995a), Chapter 5.

## 7. *Erotetic Implication*

Unlike evocation, erotetic implication is a complex notion.

Assume again that $L$ is a language which fulfills the conditions specified in Sections 3 and 4. Erotetic implication is characterized by:

*Definition 9*: *A question $Q$* implies *a question $Q_1$* on the basis of *a set of d-wffs $X$ (in symbols: $\mathsf{Im}(Q, X, Q_1)$) iff*
*(i) for each $A \in \mathsf{d}Q$: $X \cup \{A\} \models \mathsf{d}Q_1$, and*
*(ii) for each $B \in \mathsf{d}Q_1$ there exists a non-empty proper subset $Y$ of $\mathsf{d}Q$ such that $X \cup \{B\} \models Y$.*

If $\mathsf{Im}(Q, X, Q_1)$, then $Q_1$ is said to be the *implied question*, $Q$ is the *implying question* and the elements of $X$ are called *auxiliary d-wffs*.

Let us now analyze the intuitive content of the proposed definition.

According to clause (i), the set of direct answers to an implied question is multiple-conclusion entailed by each set made up of the auxiliary d-wffs and a direct answer to the implying question (recall that a question has at least two direct answers). Thus erotetic implication warrants the transmission of soundness/truth into soundness: if an implying question is sound (i.e. has at least one true direct answer) and all the auxiliary d-wffs are true, then the implied question is sound as well, and this condition is fulfilled for each admissible partition of the language. Yet, clause (ii) yields (but does not amount to!) the reverse: if an implied question is sound and all the auxiliary d-wffs are true, then the implying question is sound (again, this condition is fulfilled for each admissible partition). Of course, it is neither assumed nor denied that $Q$ and/or $Q_1$ are sound, and that $X$ consists of truths. The following table presents possible connections; we assume that $\mathsf{Im}(Q, X, Q_1)$

and that $P =<T, U>$ is an arbitrary but fixed admissible partition of the language:

| $Q$ | $X$ | $Q_1$ |
|---|---|---|
| sound in $P$ | $X \subset T$ | sound in $P$ |
| unsound in $P$ | $X \subset T$ | unsound in $P$ |
| sound in $P$ | $X non\subset T$ | sound in $P$ or unsound in $P$ |
| unsound in $P$ | $X non\subset T$ | sound in $P$ or unsound in $P$ |

Table 3.

| $Q_1$ | $X$ | $Q$ |
|---|---|---|
| sound in $P$ | $X \subset T$ | sound in $P$ |
| unsound in $P$ | $X \subset T$ | unsound in $P$ |
| sound in $P$ | $X non\subset T$ | sound in $P$ or unsound in $P$ |
| unsound in $P$ | $X non\subset T$ | sound in $P$ or unsound in $P$ |

Table 4.

Thus erotetic implication is an "almost equivalence" with respect to soundness. Yet, one cannot reduce the intuitive content of the analyzed concept to such an equivalence. This is due to the second clause of the proposed definition.

According to clause (ii) of Definition 9, *each* set made up of a direct answer to an implied question and the auxiliary d-wffs *multiple-conclusion* entails a certain non-empty *proper* subset of the set of direct answers to the implying question. The words italicized above are crucial. Let us start from the last. The set of direct answers to a question can be viewed as the class of possibilities offered by the question. Thus when we have a proper subset of a set of direct answers, the class of possibilities is narrowed down. The clause (ii) requires that for each direct answer $B$ to the implied question $Q_1$ there exists a certain non-empty proper subset $Y$ of the set of direct answers to the implying question $Q$ such that $Y$ is multiple-conclusion entailed by $X \cup \{B\}$. Thus if $B$ is true and all the auxiliary d-wffs in $X$ are true, we have a guarantee that a true direct answer to the implied question $Q$ is among those in the restricted class $Y$. Now observe that clause (ii) speaks about *each* direct answer to the implied question. Consequently, each couple $<B, X>$, where $B \in \mathsf{d}Q_1$, fixes a certain restricted class of possibilities, and, given that $X$ consists of truths, a couple which contains a true direct answer to $Q_1$

(if there is any) determines the real possibilities (that is, a class of answers to $Q$ which must contain at least one true answer). Thus an implied question is potentially *cognitively useful* with respect to the implying question and, since clause (ii) speaks about each direct answer to the implied question, the usefulness of an implied question can be called "open-minded". Using the philosophical jargon, erotetic implication defined above has a "teleological feature" and this feature distinguishes it from other proposals.

Of course, different answers to an implied question usually point at different sets of possibilities. Moreover, it may happen that the appropriate proper subsets are simply singleton sets, as happens in the example presented below:

*Example 4:*

$Q$: *How old is Andrew?*
$X$: *Andrew is as old as Peter.*
$Q_1$: *How old is Peter?*

It may also happen that no single direct answer to the implying question is forthcoming (but still subclasses are fixed), viz.:

*Example 5:*

$Q$: *Where does Andrew live?*
$X$: *Andrew lives in a university town in Western Poland.*
$Q_1$: *Which towns in Western Poland are university towns?*

The implied question is construed here as a "complete list" question: a direct answer to it provides us with a list of towns together with the clause that they are the only university towns in Western Poland.

A mixed situation is also possible. Let us consider:

*Example 6:*

$Q$: *Where did Andrew leave for: Paris, London, or Moscow?*
$X$: *If Andrew left for Paris, London or Moscow, then he departed in the morning or in the evening.*
*If Andrew departed in the morning, then he left for Paris or London.*
*If Andrew departed in the evening, then he left for Moscow.*
$Q_1$: *When did Andrew depart: in the morning, or in the evening?*

| / | \ |
|---|---|
| [*morning*] | [*evening*] |
| [*Paris, London*] | [*Moscow*] |

Yet, irrespective of the possible practical benefits which may be gained, erotetic implication is still a well-defined semantical concept. We say that an erotetic inference of the second kind $<Q, X, Q_1>$ is *valid* iff $\mathsf{Im}(Q, X, Q_1)$.

The properties of erotetic implication have been characterized elsewhere (cf. Wiśniewski 1990a, 1994a, 1995a, 1996). Let us only mention here a few facts. First, in the general case erotetic implication is not “transitive”, i.e. if $\mathsf{Im}(Q, X, Q_1)$ and $\mathsf{Im}(Q_1, X, Q_2)$, then not always $\mathsf{Im}(Q, X, Q_2)$. But if $Q_1$ regularly implies $Q_2$ on the basis of $X$, then transitivity holds. Regular erotetic implication is a special case of erotetic implication. We have:

*Definition 10*: *A question $Q$* regularly implies *a question $Q_1$* on the basis of *a set of d-wffs $X$ iff*
*(i) for each $A \in \mathsf{d}Q$: $X \cup \{A\} \models \mathsf{d}Q_1$, and*
*(ii) for each $B \in \mathsf{d}Q_1$ there exists $A \in \mathsf{d}Q$ such that $X \cup \{B\}$ entails $A$.*

The difference lies in the fact that *single* direct answers to an implying question are pointed at (i.e. entailed) by couples made up of $X$ and a direct answer to the implied question (cf. Example 4).

Second, observe that when the following condition is fulfilled:

(*) for each $B \in \mathsf{d}Q_1$: $X \cup \{B\}$ entails some direct or partial answer to $Q$

then clause (ii) of Definition 9 is fulfilled as well. Moreover, condition (*) is equivalent to clause (ii) of Definition 9 given that mc-entailment in a language is compact.

Third, observe that if $\mathsf{Im}(Q, X, Q_1)$, then each presupposition of the implied question $Q_1$ is entailed by any set made up of $X$ and a direct answer to the implying question $Q$.

Fourth, if $Q$ and $Q_1$ are normal questions (cf. Section 4), then clause (i) of Definition 9 is equivalent to the following:

(**) for each $C \in \text{Pres}Q_1$: $\text{Pres}Q \cup X$ entails $C$.

Moreover, if $Q$ and $Q_1$ are regular questions, then clause (i) of Definition 9 is equivalent to:

(***) there exist: a prospective presupposition $D$ of $Q$, and a prospective presupposition $C$ of $Q_1$ such that $X \cup \{D\}$ entails $C$.

Recall, however, that erotetic implication is defined by means of two clauses.

## 8. *Erotetic Implication: Examples*

In what follows we shall use the letters $A$, $B$, $C$, $D$, $E$, $F$, with subscripts if needed, as metalinguistic variables for d-wffs of $L_1$ and $L_2$ (cf. sections 6.1 and 6.2). When no quantifiers or symbols referring to individual variables or constants occur, the formulas presented below refer both to erotetic implication in $L_1$ *and* in $L_2$; otherwise only $L_2$ is taken into consideration. We shall supplement the examples with comments included in square brackets, which will indicate the type of erotetic implication involved.

### 8.1. *Pure erotetic implication*

Definition 9 neither assumes nor denies that $X$ is non-empty. If $\mathsf{Im}(Q, \emptyset, Q_1)$, then we say that $Q$ implies $Q_1$ and we write $\mathsf{Im}(Q, Q_1)$. In this case we have *pure erotetic implication*. Analytic erotetic implication is a special case of pure erotetic implication.

*Definition 11*: *A question* $Q$ analytically implies *a question* $Q_1$ *iff* $\mathsf{Im}(Q, Q_1)$ *and each immediate subformula of a direct answer to* $Q_1$ *is a subformula*[14] *of a direct answer to* $Q$ *or a negation of a subformula of a direct answer to* $Q$.

For transparency, we write $Q \vdash_{\mathsf{Im}} Q_1$ instead of $\mathsf{Im}(Q, Q_1)$. The following hold:

(8.1) $?\neg A \vdash_{\mathsf{Im}} ?A$, [*regular*, *analytic*]

(8.2) $?A \vdash_{\mathsf{Im}} ?\neg A$, [*regular*, *analytic*]

(8.3) $?\{A, B\} \vdash_{\mathsf{Im}} ?\{B, A\}$, [*regular*, *analytic*]

(8.4) $?\{A, B \vee C\} \vdash_{\mathsf{Im}} ?\{A, B, C\}$, [*regular*, *analytic*]

(8.5) $?\{A, B, C\} \vdash_{\mathsf{Im}} ?\{A, B \vee C\}$, [*analytic*]

(8.6) $?(A \otimes B) \vdash_{\mathsf{Im}} ?\pm \mid A, B \mid$, [*regular*, *analytic*]

where $\otimes$ is any of the connectives: $\wedge$, $\vee$, $\supset$, $\equiv$.

[14] We assume that subformulas are defined in the standard manner; in particular, a d-wff $A$ is a subformula of $A$. Moreover, we assume that negation is present in a language.

(8.7) $? \pm \mid A, B \mid \vdash_{\mathsf{Im}} ?\,(A \otimes B)$, [*not analytic if* $\otimes \neq \wedge$]

where $\otimes$ is any of the connectives: $\wedge$, $\vee$, $\supset$, $\equiv$.

(8.8) $? \pm \mid A, B \mid \vdash_{\mathsf{Im}} ?\,A$, [*analytic*]

(8.9) $? \pm \mid A, B \mid \vdash_{\mathsf{Im}} ?\,B$, [*analytic*]

(8.10) $?\,A \vdash_{\mathsf{Im}} ? \pm \mid A, B \mid$, [*regular*, *not analytic*]

(8.11) $?\,B \vdash_{\mathsf{Im}} ? \pm \mid A, B \mid$, [*regular*, *not analytic*]

8.2. *Erotetic implication on the basis of non-empty sets of d-wffs*

We write $Q, A_1, ..., A_n \vdash_{\mathsf{Im}} Q_1$ instead of $\mathsf{Im}(Q, A_1, ..., A_n, Q_1)$. The following hold:

(8.12) $?\,(A \otimes B), \alpha A \vdash_{\mathsf{Im}} ?\,B$, [*regular*]

where $\otimes$ is any of the connectives: $\wedge$, $\vee$, $\supset$, $\equiv$, and $\alpha A$ is equal to $\neg A$ or to $A$,

(8.13) $?\,(A \otimes B), \alpha B \vdash_{\mathsf{Im}} ?\,A$, [*regular*]

where $\otimes$ is any of the connectives: $\wedge$, $\vee$, $\supset$, $\equiv$, and $\alpha B$ is equal to $\neg B$ or to $B$,

(8.14) $?\,\{\neg A, A \wedge B, A \wedge \neg B\} \vdash_{\mathsf{Im}} ?\,A$,

(8.15) $?\,\{A_1, ..., A_n\}, A_1 \vee ... \vee A_n \vdash_{\mathsf{Im}} ?\,A_i$, [*regular if* $n = 2$]

where $1 \leq i \leq n$.

(8.16) $?\,\{A_1, ..., A_n\}, A_1 \vee ... \vee A_n, \neg(A_1 \wedge ... \wedge A_n) \vdash_{\mathsf{Im}} ?\,\{\neg A_1, ..., \neg A_n\}$, [*regular if* $n = 2$]

(8.17) $?\,\{A_1, ..., A_n\}, A_1 \equiv B_1, ..., A_n \equiv B_n \vdash_{\mathsf{Im}} ?\,\{B_1, ..., B_n\}$, [*regular*]

(8.18) $?\{A_1, ..., A_n\}, B \supset A_1 \vee ... \vee A_i, \neg B \supset A_{i+1} \vee ... \vee A_n \vdash_{\mathsf{Im}} ?B,$
[*regular if* $n = 2$]
where $1 \leq i < n$.

(8.19) $?\{A_1, ..., A_n\}, B \supset A_1 \vee ... \vee A_i, C \supset A_{i+1} \vee ... \vee A_n, B \vee C \vdash_{\mathsf{Im}}$ $?\{B, C\},$
[*regular if* $n = 2$]
where $1 \leq i < n$.

(8.20) $?\{A_1, ..., A_n\}, A_1 \vee ... \vee A_i \equiv B, A_{i+1} \vee ... \vee A_n \equiv C \vdash_{\mathsf{Im}} ?\{B, C\},$
[*regular if* $n = 2$]
where $1 \leq i < n$.

(8.21) $?\{A_1, ..., A_n\}, A_1 \vee ... \vee A_i, \supset B, A_{i+1} \vee ... \vee A_n \supset \neg B, A_1 \vee ... \vee$ $A_n \vdash_{\mathsf{Im}} ?B,$
[*regular if* $n = 2$]
where $1 \leq i < n$.

(8.22) $?A(x/c), \forall x(Ax \equiv Bx \wedge Cx) \vdash_{\mathsf{Im}} ? \pm \mid B(x/c), C(x/c) \mid,$
[*regular*]

(8.23) $?\mathsf{S}(Ax), \forall x(Ax \equiv Bx) \vdash_{\mathsf{Im}} ?\mathsf{S}(Bx),$ [*regular*]

(8.24) $?\mathsf{U}(Ax), \forall x(Ax \equiv Bx) \vdash_{\mathsf{Im}} ?\mathsf{U}(Bx),$ [*regular*]

(8.25) $?\mathsf{S}(Ax), \forall x(Ax \supset x = c_1 \vee ... \vee c_n) \vdash_{\mathsf{Im}} ?\{A(x/c_1), ..., A(x/c_n)\},$
[*regular*]
where $c_1, ..., c_n (n > 1)$ are distinct individual constants,

(8.26) $?\mathsf{S}(Ax), \exists x Ax, \forall x(Ax \supset x = c_1 \vee ... \vee c_n) \vdash_{\mathsf{Im}} ?A(x/c_i),$

where $c_1, ..., c_n (n > 1)$ are distinct individual constants and $1 \leq i \leq n$.

It is no accident that wh-questions of $L_2$ occur in the above examples only a few times. It may be argued that their proper analysis requires a semantics in which all the elements of a domain have names (i.e. are values of some closed terms). Moreover, in order to analyze erotetic implication of general wh-questions of $L_2$ and other "complete-list" questions one needs the quantifier "there exist finitely many" (cf. Wiśniewski 1990a) or at least numerical quantifiers (cf. Wiśniewski 1995a). For further examples of erotetic implication, in particular in more sophisticated languages and pertaining to more complex questions, see Wiśniewski (1990a) and Wiśniewski (1995a), Chapter 7.

## 9. *Rules and Scenarios*

### 9.1. $\mathsf{E}$-*rules and* $\mathsf{Im}$-*rules*

Erotetic inferences occur in almost every process of reasoning. Yet, the common attitude has been to regard them as belonging to the "pragmatics" of reasoning, in the pejorative sense of pragmatics as referring to something that is not subjected to any objective rules. No doubt, there are erotetic inferences of this kind. But there are also erotetic inferences which have a well-established structure due to the fact that evocation or erotetic implication hold between their premises and conclusions. IEL regards them as valid. Of course, validity is a normative concept and the appropriate concept of validity is neither given by God nor by Tradition (even Jaakko Hintikka is silent here). But even if one has other intuitions concerning validity of erotetic inferences, the semantic warrants connected with IEL-valid erotetic inferences still remain. Moreover, if we have a formula which says that a question of a given form is evoked by a finite set of d-wffs made up of d-wffs of strictly defined forms (cf. examples in Section 6), we can formulate a rule each application of which leads to the question which is both sound and informative with respect to the premises. Similarly, when a formula of the form $Q, A_1, ..., A_n \vdash_{\mathsf{Im}} Q_1$ (or of the form $Q \vdash_{\mathsf{Im}} Q_1$) holds (cf. examples in Section 8), we can formulate a rule each application of which leads to the question which is both sound with respect to the premises and cognitively useful with respect to the initial question. These rules are called $\mathsf{E}$-rules and $\mathsf{Im}$-rules, respectively.

Yet, IEL does not stop here. The logical tools elaborated within IEL make an analysis of *search scenarios* possible. As a result of the analysis, *search rules* are characterized.

### 9.2. *Erotetic search scenarios*

What are erotetic search scenarios (e-scenarios for short)? A few examples will help to clarify matters here.

*Example 7:*

Let us imagine that we are looking for an answer to the question:

(9.1) Did John meet Mary and talk to her?

Question (9.1) is construed here in such a way that the following are direct answers to it:

(9.2) John didn't meet Mary.
(9.3) John met Mary and talked to her.
(9.4) John met Mary, but didn't talk to her.

The natural way of proceeding is to ask the following question first:

(9.5) Did John meet Mary?

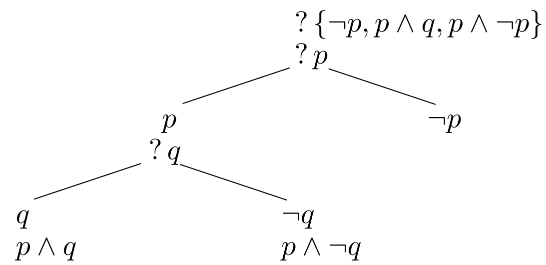
If the negative answer to (9.5) holds, the answer (9.2) to (9.1) is found. If, however, the affirmative answer to (9.5) holds, the following question should be asked:

(9.6) Did John talk to Mary?

The affirmative answer to (9.6) together with the (already established) affirmative answer to (9.5) give the answer (9.3) to the principal question (9.1), whereas the negative answer to (9.6) together with the affirmative answer to (9.5) yield the answer (9.4) to the principal question.

The following diagram presents the e-scenario involved:

Figure 1.




*Example 8:*

Let us now imagine that we are looking for an answer to the following principal question:

(9.7) Where did John leave for: Paris, London, Kiev, or Moscow?

on the basis of the following initial premises:

(9.8) If John departed in the morning, then he left for Paris or London.
(9.9) If John did not depart in the morning, then he left for Moscow.

(9.10) John left for London if he took his famous umbrella; otherwise he did not leave for London.

We can start with asking the following question:

(9.11) Did John depart in the morning?

If the negative answer to (9.11) is true, then by (9.9) John left for Moscow. If, however, the affirmative answer to (9.11) holds, then the following question arises:

(9.12) Where did John leave for: London or Paris?

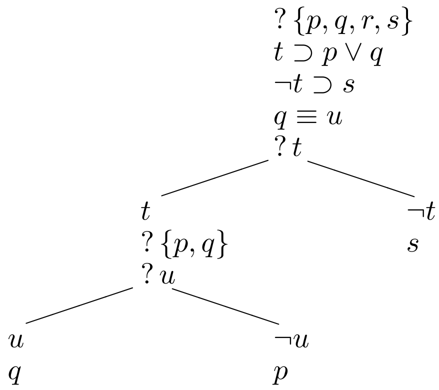
Yet, question (9.12) need not be asked. The following question:

(9.13) Did John take his famous umbrella?

is now raised by question (9.12) on the basis of the affirmative answer to (9.11) and the premises (9.8) and (9.10). The affirmative answer to (9.13) together with premise (9.10) yield that John left for London, whereas the negative answer to (9.13) together with the affirmative answer to (9.11) and premise (9.8) yield that John left for Paris.

The following diagram presents the appropriate e-scenario:

Figure 2.




*Example 9:*

Our third example is more abstract. Assume that $P_1$, $P_2$, $P_3$ are one-place predicates of $L_2$ and that $a$ is an individual constant of $L_2$. Let us imagine that we are looking for an answer to a question of the form:

(9.14) $?\, P_1 a$

("Does $a$ have the property $P_1$?") on the basis of the following initial premise:

(9.15) $\forall x(P_1 x \equiv P_2 x \wedge P_3 x)$

Figure 3 shows a possible e-scenario:

Figure 3.

$$\begin{array}{c}
?\, P_1 a \\
\forall x(P_1 x \equiv P_2 x \wedge P_3 x) \\
?\pm \mid P_2 a, P_3 a \mid \\
?\, P_2 a
\end{array}$$

$$\begin{array}{ll}
P_2 a & \neg P_2 a \\
?\, P_3 a & \neg P_1 a
\end{array}$$

$$\begin{array}{ll}
P_3 a & \neg P_3 a \\
P_1 a & \neg P_1 a
\end{array}$$

The principal question (9.14) on the basis of (9.15) implies:

(9.16) $?\pm \mid P_2 a, P_3 a \mid$

(cf. (8.22)). Question (9.16), in turn, implies:

(9.17) $?\, P_2 a$

(cf. (8.8)). Question (9.17) is now asked and answered. Since it has two direct answers, the scenario branches. The right branch contains the negative answer '$\neg P_2 a$' to (9.17) and, since '$\neg P_2 a$' and (9.15) entail '$\neg P_1 a$', ends with the negative answer to the principal question (9.14). The left branch contains the affirmative answer '$P_2 a$' to (9.16) followed by the question:

(9.18) $?\, P_3 a$

which is implied by question (9.17) (cf. (8.9)). Question (9.18) is asked and answered, and the scenario branches again. The leftmost branch contains the affirmative answer to (9.18) and ends with the affirmative answer to the principal question (9.14), which is now entailed by the results established so far and the premise (9.15). The right sub-branch contains the negative answer to (9.18) and ends with the negative answer to the principal question (9.14);

this answer is entailed by the answer just received (i.e. '$\neg P_3 a$') and premise (9.15).

Although e-scenarios can be represented by downward trees, they are defined as families of mutually interconnected *erotetic derivations* (e-derivations for short). An e-derivation corresponds to a branch of a tree: it starts with the principal question and ends with a direct answer to it. An auxiliary question which occurs in an e-derivation is either asked (is a query) or serves as a premise; each auxiliary question must be implied by a certain question which occurs earlier, possibly on the basis of some earlier d-wffs. A d-wff can enter an e-derivation for three possible reasons: (a) as an initial premise, or (b) as a direct answer to an auxiliary question, and (c) as a consequence of some d-wff(s) which satisfy the conditions (a) or (b). It is assumed that if an e-scenario involves an auxiliary question which is then answered in some way in an e-derivation, then the e-scenario contains also e-derivations in which the auxiliary question is answered in all the other possible ways; these e-derivations are identical up to the point at which the auxiliary question is asked, but start to differ at the level of answers to the auxiliary question (thus an e-scenario is "fair" with respect to queries).

To be more precise, e-scenarios can be defined as follows.

Let $L$ be a formalized language which fulfills the conditions specified in sections 3 and 4. We shall use the lower-case Greek letters $\varphi$, $\psi$, $\gamma$, with subscripts if needed, as metalinguistic variables for wffs (i.e. d-wffs and questions) of $L$. We have:

*Definition 12*: *A finite sequence* $\mathsf{e} = \varphi_1, ..., \varphi_n$ *of wffs is an* erotetic derivation of a direct answer $A$ to the question $Q$ on the basis of a set of d-wffs $X$ *iff* $\varphi_1 = Q$, $\varphi_n = A$ *and the following conditions hold:*

(1) *for each question* $\varphi_k$ *of* $\mathsf{e}$ *such that* $k > 1$*:*
  (a) $\mathsf{d}\varphi_k \neq \mathsf{d}Q$*, and*
  (b) $\varphi_{k+1}$ *is either a question or a direct answer to* $\varphi_k$*;*

(2) *for each d-wff* $\varphi_j$ *of* $\mathsf{e}$*:*
  (a) $\varphi_j \in X$*, or*
  (b) $\varphi_j$ *is a direct answer to* $\varphi_{j-1}$*, where* $\varphi_{j-1} \neq Q$*, or*
  (c) $\varphi_j$ *is entailed by a certain set of d-wffs such that each element of this set precedes* $\varphi_j$ *in* $\mathsf{e}$*;*

(3) *for each question* $\varphi_k$ *of* $\mathsf{e}$ *such that* $\varphi_k \neq Q$*:* $\varphi_k$ *is implied by a certain question* $\varphi_i$ *which precedes* $\varphi_k$ *in* $\mathsf{e}$ *on the basis of the empty set, or on the basis of a set of d-wffs such that each element of this set precedes* $\varphi_k$ *in* $\mathsf{e}$*.*

Note that by "precedes" we do not mean "immediately precedes".

An element $\varphi_k$ (where $1 < k < n$) of an e-derivation $\mathsf{e} = \varphi_1, ..., \varphi_n$ is a *query* of $\mathsf{e}$ if $\varphi_k$ is a question and $\varphi_{k+1}$ is a direct answer to $\varphi_k$. Note that e-derivations can involve auxiliary questions which are not queries and serve as erotetic premises only (recall that erotetic implication is not "transitive").

*Definition 13*: *A finite family* $\Phi$ *of sequences of wffs is an* erotetic search scenario for a question $Q$ relative to a set of d-wffs $X$ *iff each element of* $\Phi$ *is an e-derivation of a direct answer to* $Q$ *on the basis of* $X$ *and the following conditions hold:*

(1) $\mathsf{d}Q \cap X = \emptyset$*;*
(2) $\Phi$ *contains at least two elements;*
(3) *for each element* $\mathsf{e} = \varphi_1, \varphi_2, ..., \varphi_n$ *of* $\Phi$*, for each index* $k$ *such that* $1 \leq k < n$*:*
  (a) *if* $\varphi_k$ *is a question and* $\varphi_{k+1}$ *is a direct answer to* $\varphi_k$*, then for each direct answer* $B$ *to* $\varphi_k$*, the family* $\Phi$ *contains a certain e-derivation* $\mathsf{e}' = \psi_1, \psi_2, ..., \psi_m$ *such that* $\psi_j = \varphi_j$ *for* $j = 1, ..., k$*, and* $\psi_{k+1} = B$*;*
  (b) *if* $\varphi_k$ *is a d-wff, or* $\varphi_k$ *is a question and* $\varphi_{k+1}$ *is not a direct answer to* $\varphi_k$*, then for each e-derivation* $\mathsf{e}' = \psi_1, \psi_2, ..., \psi_m$ *in* $\Phi$ *such that* $\psi_j = \varphi_j$ *for* $j = 1, ..., k$ *we have* $\psi_{k+1} = \varphi_{k+1}$.

The e-derivations which are elements of an erotetic search scenario $\Phi$ for $Q$ relative to $X$ are called *paths* of $\Phi$, the question $Q$ is the *principal question* of $\Phi$, and the elements of the set $X$ are called *initial premises*. If a path $\mathsf{e}$ of $\Phi$ has a direct answer $A$ to $Q$ as its last element, we say that $\mathsf{e}$ *leads to* $A$. If $\Phi$ is an e-scenario for $Q$ relative to the empty set, we simply say that $\Phi$ is an e-scenario for $Q$.

An e-scenario for $Q$ relative to $X$ is said to be *complete* iff each direct answer to $Q$ is the endpoint of some path of the scenario, and incomplete otherwise. Figures 1 and 3 present examples of complete e-scenarios, whereas Figure 2 shows an incomplete e-scenario.

A *query* of an e-scenario is a query of a path of the e-scenario.

Roughly, a branching point of an e-scenario is a wff which occurs in the e-scenario in such a way that the tree-like diagram which presents this scenario branches immediately below the occurrence of the wff. To be more precise, we say that a wff $\gamma$ is a *branching point* of an e-scenario $\Phi$ iff there are: an index $k > 1$ and a subset $\Psi$ of $\Phi$ such that $\Psi$ contains at least two elements and for any $\mathsf{e} = \varphi_1, \varphi_2, ..., \varphi_n$, $\mathsf{e}' = \psi_1, \psi_2, ..., \psi_m$ (where $\mathsf{e} \neq \mathsf{e}'$) in $\Psi$ we have: (i) $\gamma = \varphi_k = \psi_k$, (ii) $\varphi_j = \psi_j$ for $j = 1, ..., k$, and (iii) $\varphi_{k+1} \neq \psi_{k+1}$.

The properties of e-scenarios were analyzed elsewhere (cf. Wiśniewski (2003)). Let us only mention the following:

*Fact 4*: Only questions can play the role of queries of e-scenarios.

*Fact 5*: Each query of an e-scenario is a branching point of the scenario; no d-wff can be a branching point.

*Fact 6*: Each e-scenario involves at least one query; each e-scenario has a major branching point, which is the first query of every path of the e-scenario.

*Fact 7*: A direct answer to the principal question cannot enter an e-scenario as the second element of any of its paths.

If $\mathsf{P} = <\mathsf{T}, \mathsf{U}>$ is an admissible partition of $L$ and a d-wff $A$ belongs to $\mathsf{T}$, we say that $A$ is true in $\mathsf{P}$. One can prove:

*Golden Path Theorem*: Let $\Phi$ be an e-scenario for a question $Q$ relative to a set of d-wffs $X$. Let $\mathsf{P}$ be an admissible partition of $L$ such that each d-wff in $X$ is true in $\mathsf{P}$ and $Q$ is sound in $\mathsf{P}$. Then the scenario $\Phi$ contains at least one path $\mathsf{e}$ such that:
(a) each d-wff of $\mathsf{e}$ is true in $\mathsf{P}$; and
(b) each question of $\mathsf{e}$ is sound in $\mathsf{P}$; and
(c) $\mathsf{e}$ leads to a direct answer to $Q$ which is true in $\mathsf{P}$.

The above theorem was proved in Wiśniewski (2003) for the propositional case. The proof for the generalized case goes along similar lines. Note that the Golden Path Theorem holds both for complete and incomplete e-scenarios.

For further examples of e-scenarios see Wiśniewski (2003).

### 9.3. *Search rules*

In order to introduce the concept of a search rule we need some auxiliary concepts.

We say that an e-scenario $\Phi$ for $Q$ relative to $X$ is in the *canonical form* iff there is no element of $X$ which occurs in $\Phi$ after the first branching point of $\Phi$.

One can prove that for each e-scenario $\Phi$ there exists an e-scenario $\Phi*$ in the canonical form which results from $\Phi$ by moving all the initial premises (i.e elements of $X$) occurring on paths of $\Phi$ after the first branching point to places before the first branching point. The e-scenarios presented above are already in the canonical form.

An e-scenario $\Phi$ for $Q$ relative to $X$ is *concise* iff $\Phi$ is in the canonical form and any d-wff which occurs on a path of $\Phi$ is: (i) an element of $X$, or

(ii) an answer to a query immediately succeeding the query, or (iii) a direct answer to $Q$ which occurs at the end of the path.

Each e-scenario in the canonical form can be transformed into a corresponding concise e-scenario by deleting all the d-wffs which do not fulfil the clauses (i), (ii) and (iii) specified above (one may prove that erotetic implication is retained). The e-scenarios presented in figures 1, 2 and 3 are already concise.

An e-scenario can involve auxiliary questions which are not queries. These questions are not requests for information and serve as erotetic premises only. Let $\Phi$ be a concise e-scenario for $Q$ relative to $X$. The *imperative counterpart* of $\Phi$ results from $\Phi$ by deleting all the auxiliary questions which are not queries of $\Phi$. Note that imperative counterparts are defined for concise e-scenarios only, and these, in turn, are in the canonical form.

Imperative counterparts of concise e-scenarios need not be e-scenarios (their constituents need not be e-derivations, since clause (3) of Definition 12 can be violated). Yet, the analogue of Golden Path Theorem holds for such counterparts.

The following shows the imperative counterpart of the e-scenario presented in Figure 3:

Figure 4.

$$? P_1 a$$
$$\forall x(P_1 x \equiv P_2 x \wedge P_3 x)$$
$$? P_2 a$$

$P_2 a$ | $\neg P_2 a$
$? P_3 a$ | $\neg P_1 a$

$P_3 a$ | $\neg P_3 a$
$P_1 a$ | $\neg P_1 a$

Assume that we have a tree-like diagram *D* of the imperative counterpart of a concise e-scenario. Let us now replace the non-logical constants/propositional variables in *D* with appropriate metalinguistic variables; of course, different non-logical constants/propositional variables should be replaced by distinct metalinguistic variables. The result can be viewed as a *search rule*. Here are examples of search rules based on the e-scenarios presented above:

Figure 5.

$? \{\neg A, A \wedge B, A \wedge \neg B\}$
$? A$

- $A$
  $? B$
  - $B$
    $A \wedge B$
  - $\neg B$
    $A \wedge \neg B$
- $\neg A$

Figure 6.

$? \{A, B, C, D\}$
$E \supset A \vee B$
$\neg E \supset D$
$B \equiv F$
$? E$

- $E$
  $? F$
  - $F$
    $B$
  - $\neg F$
    $A$
- $\neg E$
  $D$

Figure 7.

$? Pc$
$\forall x(Px \equiv Rx \wedge Tx)$
$? Rc$

- $Rc$
  $? Tc$
  - $Tc$
    $Pc$
  - $\neg Tc$
    $\neg Pc$
- $\neg Rc$
  $\neg Pc$

But what justifies us in saying that diagrams of the kind presented above can be called search rules? Let us take a look at the diagram presented in Figure 7. It conveys the following information: given that the initial premise holds, in order to answer a principal question of the form $? Pc$ an answer to question '$? Rc$' is needed first. If the established answer is a negative one, the negative answer to the principal question is obtained. If, however, the established answer is the affirmative one, an answer to question '$? Tc$' is also needed. If the established answer to '$? Tc$' is affirmative, then the affirmative answer to the principal question is found. If one establishes the negative answer to '$?\,Tc$', then the negative answer to the principal question holds. In other words, the above diagram provides us with a set of conditional instructions which tell us both what auxiliary questions should be asked and when they should be asked in order to answer the principal question; moreover, the diagram shows what should be done if such-and-such a direct answer to an auxiliary question appears to be acceptable and does so with respect to any direct answer to each auxiliary question. The same can be said about other search rules of the kind considered.

Erotetic search scenarios differ with respect to their complexity and as do the corresponding search rules. As long as we are concerned with practical applications, relatively simple search rules seem more useful. Note that there are search rules which do not involve any initial declarative premises; Figure 5 gives us an example. Here are more examples[15] (in order to save space, we present the diagrams in a "horizontal" form):

Figure 8.

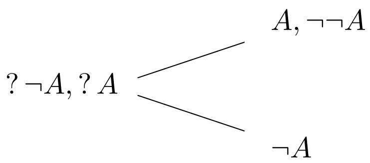
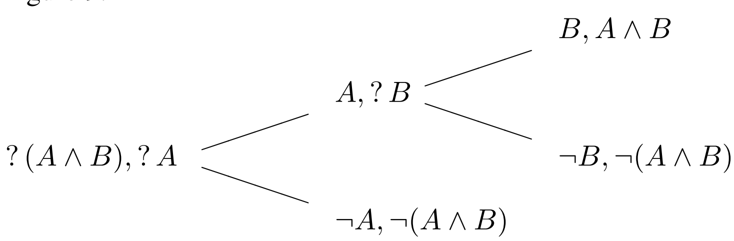



Figure 9.




[15] The following does not hold ($\otimes$ stands here for any of the connectives $\wedge$, $\vee$, $\supset$, $\equiv$):

(i) $\mathsf{Im}(?\,(A \otimes B), ?\,A)$.

But the following hold:

(8.6) $?\,(A \otimes B) \vdash_{\mathsf{Im}} ?\pm \mid A, B \mid$,
(8.8) $?\pm \mid A, B \mid \vdash_{\mathsf{Im}} ?\,A$.

The search rules presented in Figures 9–12 result from e-scenarios which have instances of (8.6) and (8.8) "between" principal questions and first queries; of course, these formulas do not occur in their imperative counterparts.

Figure 10.

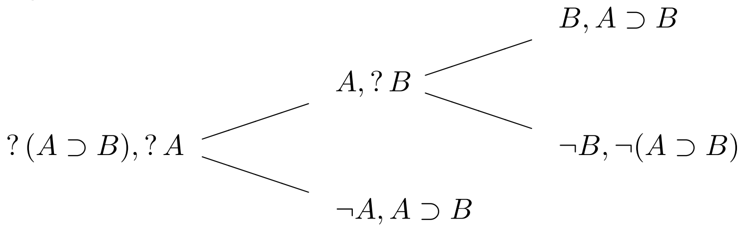



Figure 11.

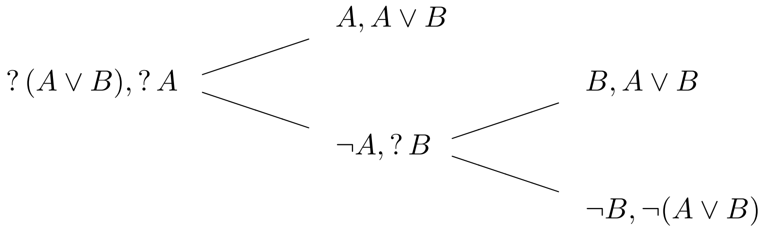
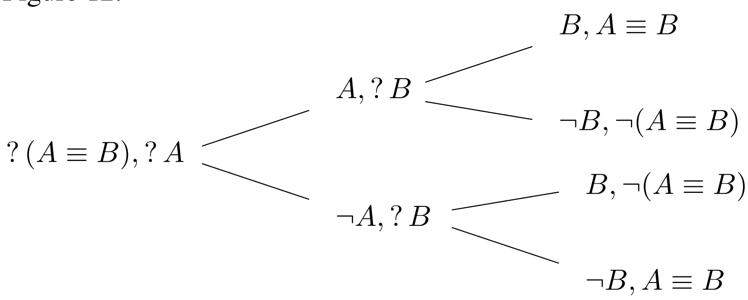



Figure 12.




## 10. *Related problems and applications*

The analysis of search scenarios has given rise to a certain proof method, called Synthetic Tableaux Method (STM). STM was developed by Urbański (cf. Urbański 2001, 2002, and (forthcoming)) for CPC and some non-classical propositional calculi; completeness results were proved. Looking from the erotetic point of view, a STM-style proof proceeds by answering the operative questions which are implied by the principal question (which has the form of a simple yes-no question) on the basis of the results obtained at previous steps. Yet, the reference to erotetic implication is only implicit: an STM-style proof is a well-defined proof-theoretical entity.

The problem of reducibility of questions to sets of ("operative") questions is important both to IEL and to STM. A semantical concept of reducibility is defined in Wiśniewski (1993); numerous results can be found in Wiśniewski (1994b), (1995a) and (forthcoming). In particular, the problem of reducibility of evoked questions to sets of questions which are implied by them on the basis of evoking sets was addressed and some general results were obtained (cf. Wiśniewski 1995a, Chapter 7.6). Leśniewski considered a more general notion of reducibility, namely the reducibility of a single question to a set of questions on the basis of a non-empty set of declarative premises; he received elegant and important results (cf. Leśniewski 1997, 2000).

Some concepts borrowed from IEL have been applied in the philosophy of science. Kuipers & Wiśniewski (1994) present an analysis of the so-called explanation by specification in terms of erotetic inferences, question generation and erotetic implication. Wiśniewski (1999a) introduces a relativized (with respect to background knowledge and an abnormic hypothesis in Bromberger's sense[16]) concept of a possible correct answer to a why-question; some IEL-based procedures for looking for answers of this kind are analyzed. Tworak (1993) addresses the problem of the cognitive importance of research questions by using, *int. al.*, evocation and erotetic implication. Sady (1990) applies the concept of question generation in rational reconstructions of some scientific discoveries.

As we have seen, IEL does not require Classical Logic (CL) as a basis. Yet, most of the exemplifications of the central concepts of IEL are based on CL and thus inherit all the merits and shortcomings of CL. For example, as long as the informativeness of a question is defined in terms of lack of CL-entailment (of any direct answer from the relevant body of knowledge), the logical omniscience paradox comes into play and no question is informative in an inconsistent environment. In practice, however, a scientist is usually unaware of many CL-consequences of his/her insights, and is able to operate in the presence of an inconsistency. Meheus (1999) makes an important step towards adapting evocation and question generation to the inconsistent case.

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[16] Cf. Sylvain Bromberger *On What We Know We Don't Know: Explanation, Theory, Linguistics, and How Questions Shape Them*, The University of Chicago Press, Chicago and London, and Center for Study of Language and Information, Stanford 1992.

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