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FINITE ACHIEVEMENTS

B.H. SLATER

The title of this paper relates, by opposition, to the 'infinite tasks' sometimes thought to be involved in connection with a number of paradoxes starting with Zeno's paradoxes of motion. The opposition between the finite and the infinite is very familiar, although, after all that has been said on the subject, what follows might still seem remarkably novel. The opposition between tasks and achievements comes from Ryle: Ryle separated tasks from achievements in connection with such pairs of activities as searching and finding, travelling and arriving, looking and seeing etc. in *The Concept of Mind*. In this paper I shall show that, contrary to some recent philosophical opinions (e.g. Earman and Norton, 1996), a finite achievement such as arriving at a place does not involve an infinite task, while travelling there.

The central point to be made concerns the possibility of covering, say, the open interval $]0, 1]$, with an infinity of closed intervals of the form $[1/2^n, 1/2^{n-1}]$. There is a little-noted fact about infinity in this case, since while, indeed, in the limit, no point in the open interval is omitted — so the closed intervals (timelessly) cover the open one — still it is an *open* interval, with no left-hand end, which therefore means that the task of *successively covering it* with the sub-intervals is itself endless.

The point might be put in terms of an ambiguity in 'complete'. The series of closed intervals 'completes' the open interval in the sense that it is co-extensive with it, but what it thus 'completes' is incomplete in a much more significant sense, since, like the series itself, it is endless. Were one, *per impossible*, to run over, successively, each of the sub-intervals one would have finished covering the whole open interval, but the openness of the whole interval itself prevents finalising its temporally successive covering by means of the infinitude of closed sub-intervals. One might, instead, cover $]0, 1]$ starting with some closed intervals of the same form as above (up to the one illustrated), and ending with the open interval $]0, 1/2^n]$. That would produce a finite series of sub-intervals, but the process of covering the last, smaller open interval with closed intervals would still be an unfinishable task, just because it is endless on the left.

By contrast one can complete the covering of something which has an end — like the closed interval $[0, 1]$ — with a finite series of closed sub-intervals, but that only reveals how crucial it is, to the avoidance of a number of paradoxes, that an end point is missed out with such infinitudes as those above. Benacerraf first showed this. Thus, in connection with a Thomson Lamp, which is switched on and then off as one passes through the infinite series of interval junctions above, Benacerraf would have said (c.f. Benacerraf 1962, p. 107):

> it follows only that there is no time between 1 and 0 at which the lamp was on/off and which was not followed by a time, also before 0, at which it was off/on. Nothing whatever has been said about the lamp at 0.

So it is the impossibility of reaching *the closure point* 0 the above way which is centrally what is to be noted once we move over to the certainty of covering $[0, 1]$ by means of a finite number of closed intervals — say all, except one, of the same form as that above plus the closed interval $[0, 1/2^n]$. There is a mention of such complementary intervals in Chihara 1965, but he does not develop the point appropriately. For the crucial question is: if one follows the previous infinitude of intervals, that lands one where? Certainly not at 0, but neither at any distance from it. If one is to get to the end point one must not follow the previous infinitude, in fact, since that lands one nowhere; one must, at a finite distance from 0, simply traverse the (closed) interval from there to the end.

There is more than a bare analogy, in other words, with the other philosophical context where Achilles and the Tortoise figure large: in the discussion following Lewis Carroll's article, about deduction (Carroll 1895). That there is a close analogy is immediately clear, for if, in place of moving from a finite number of premises A_1,...A_n to the conclusion A_0 one tries to insert another statement A_{n+1} which makes a further premise out of that rule of deduction, then one can be led to do so *ad infinitum*. But the analogy is truly even more exact, since if one is led into that infinite train of premises the result is that one does not *ever* draw the conclusion.

Consider a finite series of closed sub-intervals covering $[0, 1]$, of the kind mentioned before. Why is it only by means of such a finite number that one can complete the journey, and arrive at the closure point 0 — so that, by following the former infinity of closed intervals, one merely 'travels hopefully'? The infinite path does not arrive anywhere since, first, it clearly does not reach zero, but secondly, and more importantly, as above, to get, *per impossible* to the end of that path one would have to have got right next to zero, and there is no such place. The infinite activity is indeed a task, as is commonly said, but it is akin to the task of Sisyphus which has no termination: it is the infinite i.e. *unending* task associated, by some, confusedly with the job of covering the full, closed interval. But the finite activity is not (just) a task, since it also includes the achievement of the objective — and by entirely finitistic means, which are now revealed as the only means which could be employed.

The closed interval $[0,1]$ certainly includes the open interval $]0,1]$, but the latter is incomplete, and only becomes complete if the point 0 is added on the left. So the temporal process of successively covering the open interval with closed intervals of the first form is still an unfinishable task, just because it is endless. Reducing the time proportionately in which these successive intervals are traversed maybe means that it is all over *by* some finite time; but it still does not finish *at* any specifiable time. Where and when traversing the open interval finishes is the crucial point, and that is nowhere locatable in space or time, since it ends nowhere. Open intervals (like the single points which complete them), one has to realise, are *mathematical fictions*, and so are not anywhere in space or time (c.f. Benardete 1964, p. 272f).

The main thing which confuses seems to be the contrary thought that, while one traverses the finite number of intervals, still, surely, *the infinitude is there*, so one must, at least, have traversed that, amongst other things, making the infinite task completable, after all. But this line of argument involves is a red-herring, since, as Benacerraf's point shows, the fact that the associated infinitude of intervals does not reach 0 is crucial, and specifically it means that those intervals are not the intervals to attend to, if one wants to reach zero. When looking, instead, at traversing a series of intervals which arrive at the objective, the infinitude comes in another way than the traverse being a passage through all of it. The infinitude comes in merely in the indefinite number of closed intervals one can use to cover the closed interval $[0,1]$ (c.f. Goodstein 1951, p. 14). The interval $[0,1]$ may be covered, as before, by the series of closed intervals of the form $[1/2^n, 1/2^{n-1}]$, up to that particular one, together with the further interval $[0, 1/2^n]$, *for any* n, but each of those coverings only has a finite number of members. The point thus still is that, if one wants to reach the end, one must follow a series of closed intervals up to $[1/2^n, 1/2^{n-1}]$, for some specific n, with an interval of the form $[0, 1/2^n]$, not with the remaining intervals of the first form.

There have been several ramifications of thinking the reverse. Earman and Norton's 'simple infinity machines' are one extreme example of such developments, so correcting that notion will be sufficient to indicate further the necessity of finitism in this area. Earman and Norton set out their basic idea as follows (Earman and Norton 1996, p. 250):

> A *simple infinity machine* is just a Turing machine that is allowed to complete a countable infinity of steps and comprises the Slave part of the bifurcated supertask; the outcome of the calculation is read by the Master through signals from the Slave. The extra power of the machine derives solely from the fact that failure of the Slave Turing machine to halt is no longer uninformative. It no longer means that

> the machine is either about to halt or will never halt. In a simple infinity machine, it means the latter.

It is claimed 'A simple infinity machine can decide the truth of any proposition of number theory that is purely existentially or purely universally quantified in prenex normal form, where the relation quantified over is recursive', although 'Turing uncomputable tasks remain uncomputable for simple infinity machines' (Earman and Norton 1996, pp. 251–2).

But the question Earman and Norton fail to ask is *when* the Slave completes its computational part of the bifurcated supertask. This cannot be a point in time, as we have seen, because of the endlessness of the task. So how can it have knowledge of that time in order to formulate its signal? Does it rely on table tapping from the spirit world? There are sure to be some who will believe so, since, of course, there are plenty of Tortoises at deduction.

Dept. of Philosophy
The University of Western Australia
Nedlands
Australia
Western Australia 6009
E-mail: `slaterbh@cyllene.uwa.edu.au`

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