Logique & Analyse **167–168** (1999), 447–482

QUANTITATIVE CONFIRMATION, AND ITS QUALITATIVE CONSEQUENCES

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Introduction

In a previous paper (Kuipers, 1998)[1] we have developed, guided by the success perspective, a qualitative (classificatory and comparative) theory of deductive confirmation. In this paper we will present, in Section 1, the corresponding quantitative theory of confirmation, more specifically, the corresponding probabilistic theory of confirmation of a Bayesian nature, with a decomposition in deductive and non-deductive confirmation. It is again pure in the sense that all equally successful hypotheses profit from their success to the same degree. It is inclusive in the sense that it leaves room for confirmation of hypotheses with zero probability (p-zero hypotheses). In Section 2 the resulting qualitative theory of (general) confirmation, encompassing the qualitative theory of deductive confirmation, will be indicated. In the Appendix 1, it will be argued that Popper's quantitative theory of corroboration amounts to an inclusive and impure Bayesian theory of confirmation.

The quantitative approach to confirmation has a somewhat dubious character, since the assigned probabilities are, as a rule, largely artificial. Their main purpose is to lead to adequate qualitative (classificatory and comparative) judgments of confirmation. As far as *deductive* confirmation is concerned, we have seen in (Kuipers, 1998) that we do not need a quantitative approach for that purpose. However, since to date no independent or direct qualitative theory of *general* confirmation, or of *non-deductive* confirmation, has been developed, a quantitative approach is required for that purpose. Such a dependent or indirect qualitative theory of general and non-deductive confirmation will be presented in the second section.

Accordingly, we do not claim that the quantitative theory reflects quantitative cognitive structures concerning confirmation. Instead, they should primarily be conceived as quantitative explications of qualitative cognitive structures, to be used only for their qualitative consequences. As will be argued, the justification of these qualitative consequences is at least as good as

[1] Both papers will also appear, in a marginally revised form, in (Kuipers, 2000).

the justification of the quantitative explications 'under ideal circumstances', that is, when the probabilities make objective sense. Moreover, as in the qualitative case, it will also become clear that there is not one 'language of quantitative confirmation', but several, e.g. pure and impure ones, inclusive and non-inclusive ones. As long as one uses the same updating calculus for probabilities, it does not matter which confirmation language one chooses, the only important point is to always make clear which one one has chosen. Although speaking of confirmation languages hence is more appropriate, we will accept the current practice of speaking of confirmation theories.

1. *Quantitative confirmation*

In this section, a non-standard version will be presented of the so-called Bayesian theory of confirmation, guided by the success perspective. Quantitative confirmation will be decomposed into confirmation by a deductive or a non-deductive success, or simply deductive and non-deductive confirmation. Both will be localized in the so-called Confirmation Square. The degree of confirmation of a hypothesis by a piece of evidence will be equated with the plausible degree of success, which happens to be equivalent to the ratio of the posterior and prior probability when the latter is non-zero. The version of Bayesianism is non-standard in two senses[2]. First, and foremost, it is inclusive in the sense that it leaves room for a substantial degree of confirmation for 'p-zero' hypotheses when they are confirmed. Second, it is pure in the sense that equally successful hypotheses get the same degree of confirmation, irrespective of their prior probability.

1.1. *Non-deductive confirmation and the Confirmation Square*

The four possible (unconditional) deductive relations between hypothesis and evidence specified in the Confirmation Matrix in Section 1.1 of (Kuipers, 1998) have somewhat weaker probabilistic versions, for which we propose to use the same 'deductive' names.

$H \models E$	$\Rightarrow p(E/H) = 1$	Deductive Confirmation:	DC(H, E)
$H \models \neg E$	$\Rightarrow p(E/H) = 0$	Falsification:	F(H, E)

[2] Standard versions of Bayesian philosophy of science, leaving no room for confirmation of p-zero hypotheses, can be found in Horwich (1982), Earman (1992), Howson and Urbach (1989), Schaffner (1993, Ch. 5). These non-inclusive versions are pure or impure depending on whether they support the difference degree or the ratio degree of confirmation (see below), respectively.

$\neg H \models E \quad \Rightarrow p(E/\neg H) = 1$ Deductive Disconfirmation: DD(H, E)
$\neg H \models \neg E \Rightarrow p(E/\neg H) = 0$ Verification: V(H, E)

Here we assume that there is some defensible probability function p, i.e., p may well have subjective features, though then as much as possible in agreement with objective information. In line with Bayesian philosophers of science (Howson and Urbach 1989; Earman 1992), we will call $p(E/H)$ and $p(E/\neg H)$ likelihoods.[3]

A probabilistic theory of confirmation will be called *Bayesian* as soon as it assumes, explicitly or implicitly, some prior distribution, that is, probability values $p(H)$ and $p(\neg H) = 1 - p(H)$. As a rule, this is already the case when one of the probabilities $p(H)$, $p(\neg H)$, $p(E)$ or $p(H/E)$ is used, or both likelihoods $p(E/H)$ and $p(E/\neg H)$.[4]

According to the definition of conditional probability, $p(E/H) = p(E\& H)/p(H)$ is undefined when $p(H) = 0$. However, this does not exclude that $p(E/H)$ can be interpreted in this case. For example, in case H entails E, $p(E/H)$ is 1. Or consider the case that the hypotheses Hv for all possible values v in [0,1] for the probability of heads of a biased coin. Then $p(Hv) = 0$, but $p(En/Hv)$ makes perfectly sense for a sequence En of outcomes of n throws, viz. the corresponding binomial distribution. In this case, it is at most controversial for non-Bayesians whether and how $p(En/\neg Hv)$ can be meaningfully interpreted. For, in general, if $p(H) = 0$ then $p(E/\neg H) = p(E)$, and in any Bayesian approach it is assumed that $p(E)$ can be assigned a value, whether this is done in terms of the decomposition $p(H)p(E/H) + p(\neg H)p(E/\neg H)$ induced by H, hence $p(E/\neg H)$, or in terms of some other decomposition. From now on we will assume that both $p(E/H)$ and $p(E)$, and hence $p(E/\neg H)$, are interpreted, even if $p(H) = 0$. Similarly, there are cases where $p(H/E)$ can be interpreted when $p(E) = 0$. For instance, if E reports the specific value 0.2 of a quantity X taking values in the [0,1]-interval and H claims that the value of a similar quantity Y will be below 0.5, it may well be reasonable to assign, on the basis of the background beliefs, $p(H) = p(Y < 0.5) = 0.5$,

[3] That is, without assuming, as statisticians do, that H and $\neg H$ are simple hypotheses in the sense of generating a certain probability distribution. Hence, H and $\neg H$ may well be disjunctions of such simple hypotheses, in which case p is based on a prior distribution over the latter hypotheses and their corresponding conditional probability distributions. To be sure, H itself is primarily thought of as a non-statistical hypothesis. For the extrapolation of the Bayesian approach to statistical hypotheses, see e.g., (Howson and Urbach 1989) and (Schaffner 1993).

[4] Recall that $p(H/E)$ is equal to $p(H)p(E/H)/p(E)$, where $p(E)$ is equal to $p(H)p(E/H) + p(\neg H)p(E/\neg H)$. Note also that $p(E/\neg H)$ is equal to $p(E)p(\neg H/E)/p(\neg H)$.

$p(H/E) = p(Y < 0.5/X = 0.2) = 0.7$ and $p(E) = p(X = 0.2) = 0$.

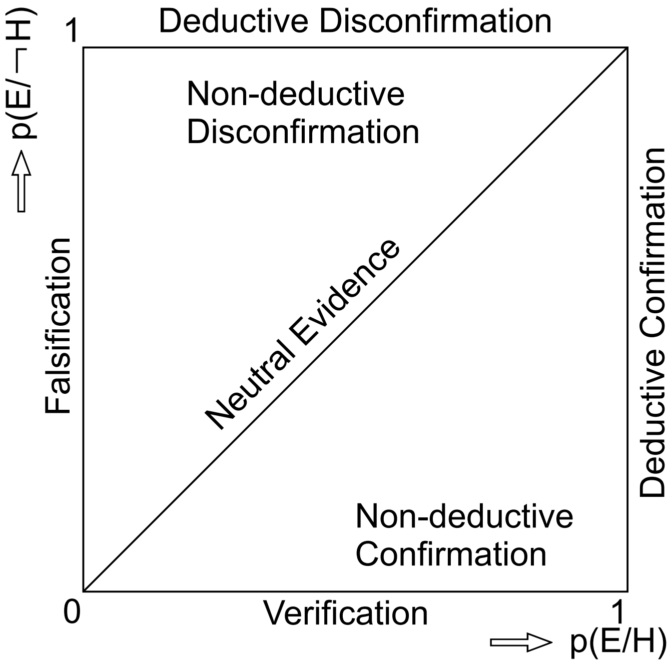

Figure 1: The Confirmation Square (CS)

Well then, by using the weaker ‘likelihood versions’, the four deductive relations between H and E can be depicted as the four sides of the unit square of likelihood pairs $\langle p(E/H), p(E/\neg H)\rangle$, henceforth called the *Confirmation Square* (CS), depicted in Figure 1.

The core of the ‘quantitative success theory of confirmation’ of Bayesian nature is completed by taking the interior of CS also into account. From the success perspective, the criteria

$p(E)$	$< p(E/H)$	Confirmation	C(H, E)
$p(E/H)$	$< p(E)$	Disconfirmation	D(H, E)

are the plausible criteria of confirmation and disconfirmation in general. The first condition, $p(E) < p(E/H)$, will be called the *S(uccess)-criterion* of confirmation. Note that the S-criterion coincides with the success definition of confirmation in general (SDC) in (Kuipers, 1998), viz., H makes E more plausible, as soon as we equate plausibility with probability. Note also that the depicted (/-)diagonal typically represents 'no confirmation' or neutral evidence:

$p(E) = p(E/H)$	Neutral Evidence:	NE(H, E)

To get a better view on extreme cases, represented by the sides of CS, and of the non-extreme cases, represented by the interior, we formulate first equivalent criteria of confirmation, disconfirmation and neutral evidence.

$p(E/\neg H) < p(E/H)$	Confirmation:	C(H, E)
$p(E/H) < p(E/\neg H)$	Disconfirmation:	D(H, E)
$p(E/\neg H) = p(E/H)$	Neutral Evidence:	NE(H, E)

In this way, the S-criterion for confirmation leaves clearly room for the extreme cases of verification, $p(E/\neg H) = 0$, and deductive confirmation, $p(E/H) = 1$. Similarly, the criterion for disconfirmation leaves room for the extreme cases of falsification, $p(E/H) = 0$, and deductive disconfirmation, $p(E/\neg H) = 1$.

As a consequence, the region of the interior of CS right/below (left/above) the diagonal typically represents non-extreme probabilistic successes of H $(\neg H)$. These non-extreme cases represent the remaining intuitive cases of confirmation and disconfirmation, respectively. They will be called non-deductive:

$0 < p(E/\neg H) < p(E/H) < 1$	Non-deductive Confirmation:	NC(H, E)
$0 < p(E/H) < p(E/\neg H) < 1$	Non-deductive Disconfirmation:	ND(H, E)

Note that, as in the deductive case, non-deductive disconfirmation of H amounts to non-deductive confirmation of $\neg H$.

If one wants to set apart verification and falsification as extreme cases of confirmation and disconfirmation, respectively, it is plausible to introduce the notions of non-extreme or proper confirmation and disconfirmation:

$0 < p(E/\neg H) < p(E/H)$	Proper Confirmation:	PC(H, E)
$0 < p(E/H) < p(E/\neg H)$	Proper Disconfirmation:	PD(H, E)

with some conceptually plausible consequences, in abbreviated form, indicating subsets of CS by the relevant condition:

$$\begin{aligned}
\mathrm{C}(H,E) &= \mathrm{V}(H,E) \cup \mathrm{PC}(H,E) \\
&\qquad \text{and } \mathrm{PC}(H,E) = \mathrm{DC}(H,E) \cup \mathrm{NC}(H,E) \\
\mathrm{D}(H,E) &= \mathrm{F}(H,E) \cup \mathrm{PD}(H,E) \\
&\qquad \text{and } \mathrm{PD}(H,E) = \mathrm{DD}(H,E) \cup \mathrm{ND}(H,E)
\end{aligned}$$

It is also fruitful to define conditional versions of non-deductive confirmation, proper confirmation and confirmation in general:

$0 < p(E/\neg H\&C) < p(E/H\&C) < 1$	cond. Non-ded. Confirmation:	$\mathrm{NC}(H,E;C)$
$0 < p(E/\neg H\&C) < p(E/H\&C)$	cond. Proper Confirmation:	$\mathrm{PC}(H,E;C)$
$p(E/\neg H\&C) < p(E/H\&C)$	cond. Confirmation:	$\mathrm{C}(H,E;C)$

When supplemented with plausible definitions of conditional (deductive and non-deductive) disconfirmation, each specific condition gives rise to its own confirmation square, the *conditional* CS.

In sum, CS not only depicts falsification, verification and neutral evidence but also suggests how to split proper confirmation and disconfirmation into both a (basically qualitative) deductive subtype and a (fundamentally quantitative, at least so it seems) non-deductive subtype. This interpretation of the unit square of likelihood pairs provides, as we will further illustrate, a quantitative explication of the general idea of (qualitative) confirmation, that is, the basic 'cognitive structure' regarding confirmation that is implicitly used by empirical scientists. However, since the required specific probabilities usually do not correspond to anything in reality, neither in the object of study, nor in the head of the scientist, consciously or unconsciously, they do not seem to directly reflect a quantitative cognitive structure. However, one may argue that there is something between a purely qualitative and a purely quantitative cognitive structure, viz., by certain elicitation procedures one obtains interval assignments of probabilities which may be interpreted as reflecting unconscious attitudes. These interval assignments might obey a cognitive structure in terms of intervals, but we will not pursue this possibility further.

Several aspects of CS will be treated in some detail. The following terminology will be very useful:

H is a *p-zero* hypothesis	$p(H) = 0$
H is a *p-one* hypothesis	$p(H) = 1$
H is a *p-normal* hypothesis	$0 < p(H) < 1$
H is a *p-uncertain* hypothesis	$p(H) < 1$

The analysis provides in fact a decomposition of the standard Bayesian theory of confirmation for p-normal hypotheses. Its criteria of confirmation and neutrality read, respectively:

$$p(H) < p(H/E) \qquad\qquad p(H) = p(H/E)$$

(see e.g., Carnap 1963[2], the new foreword, Horwich 1982, Howson and Urbach 1989). The confirmation criterion, stating that the posterior probability is larger than the prior probability, called the PP-criterion, is in perfect agreement with the common sense idea, expressed in the updating principle of plausibility (UPP) of (Kuipers, 1998), that confirmation, normally, increases, or leads to the increase of, the probability of the hypothesis. Assuming that H is p-normal, the PP-criterion is equivalent to the S-criterion, C(H, E), as is easy to check. In view of the "$p(E/\neg H) < p(E/H)$"-version of the S-criterion, its decomposition of Bayesian confirmation amounts to the following claim: assuming p-normality of H, the PP-criterion expressing Bayesian confirmation can be naturally decomposed into three mutually exclusive and together exhaustive possibilities in which the (equivalent) S-criterion can be satisfied: two extreme possibilities, viz., verification $(0 = p(E/\neg H) < p(E/H))$ and deductive confirmation $(p(E/\neg H) < p(E/H) = 1)$, and the non-extreme possibility, viz. non-deductive confirmation $(0 < p(E/\neg H) < p(E/H) < 1)$.

The important difference is that the S-criterion is also non-trivially applicable to p-zero hypotheses. Whereas the PP-criterion makes all evidence neutral with respect to p-zero hypotheses (for $p(H) = 0$ implies $p(H/E) = 0$), the S-criterion leaves perfectly room for confirmation of such hypotheses. However, since $p(H/E)$ remains 0, the confirmation is, as it were, not rewarded in this case. Note that the situation is different for p-one hypotheses. If $p(H) = 1$ then, assuming that E and H are compatible, $p(H/E) = p(H)$ and $p(E/H) = p(E)$. Hence, according to both criteria, p-one hypotheses cannot be confirmed. Note in this connection also that, in contrast to the fact that the confirmation of a p-normal hypothesis amounts to the disconfirmation of its negation, the confirmation of a p-zero hypothesis, according to the S-criterion, of course, does not amount to the disconfirmation of its negation according to any of the two criteria, which is easy to check. In view of the deviating behavior of the S-criterion regarding p-zero hypotheses, the S-criterion will be called *inclusive* and the PP-criterion *non-inclusive*. Hence, although the inclusive and the non-inclusive criteria are equivalent for the non-zero cases, they are incompatible for the zero cases. As we will see in Appendix 1, Popper's approach (Popper 1959, 1963, 1983) also presupposes the S-criterion, and hence is inclusive. Inclusive behavior is very important in our opinion. Although there may be good reasons (contra Popper, see Appendix 1) to assign sometimes non-zero probabilities to genuine hypotheses, it also occurs that scientists would sometimes assign in advance zero probability to them and would nevertheless concede that certain new evidence is in favor of them.

Whereas deductive confirmation has only one 'cause', the evidence is entailed by the hypothesis, non-deductive confirmation may have different causes. In the following we will restrict the attention to p-normal hypotheses and evidence. As Salmon (1969) already pointed out in the context of the possibilities of an inductive logic, a probability function may be such that E confirms H when H *partially* entails E. Here 'partial entailment' essentially amounts to the claim that the relative number of models in which E is true on the condition that H is true is larger than the relative number of models in which E is true without any condition.[5] For instance, in a 'color language' with at least four colors, p will be such that the evidence that a raven is black or white confirms the hypothesis that it is black or red. In general, one may require that a probability function satisfies the principle of partial entailment: if H partially entails $E(\neg E)$ then E confirms (disconfirms) H. Fortunately, it seems that a probability function usually satisfies this principle. However, and this was Salmon's main message, it is not at all guaranteed that such a function is such that E confirms H when H essentially is an *(inductive) extrapolation* of E, notably from past to future instances of a certain kind. For instance, one might like to have that the evidence that the first raven is black confirms the hypothesis that the second raven is black as well. In general, one may require that a probability function satisfies the principle of extrapolation (or induction): if H extrapolates upon $E(\neg E)$ then E confirms (disconfirms) H.[6] In (Kuipers, 1997, 2000) we study probability functions which satisfy both principles, e.g. Carnap's continuum of inductive methods. Of course, such functions are such that a hypothesis H which partially entails E *and* extrapolates upon E is confirmed by E. In sum, we may distinguish at least three causes or types of non-deductive confirmation: due to partial entailment, which might be called 'partial (deductive) confirmation', due to extrapolation, to be called 'inductive confirmation', and due to both factors.

[5] This formulation applies, strictly speaking, only to a language with a finite domain. However, in many cases it can be extended to infinite domains, provided E deals with a finite number of individuals.

[6] A problem with this principle is that the notion of extrapolation or 'going beyond the evidence' is not easy to define in a general way such that it is satisfactory from an inductive point of view, as Popper and Miller (1983) have pointed out. See Mura (1990) and Kuipers (2000, Chapter 4) for different proposals.

1.2. *The ratio-degree of confirmation*

Although the quantitative theory of confirmation presented thus far already allows qualitative judgments of deductive and non-deductive confirmation, for comparative purposes we also need a degree of confirmation. In (Kuipers, 1998) we have explicated 'confirmation' qualitatively as increase of plausibility of, in the first place, the evidence (SDC), and, in the second place, of the hypothesis (UPP). In the present probabilistic context, it is plausible to identify plausibility with probability, and hence, confirmation with increase of probability of the evidence, as we have noted, with the consequence, as far as p-normal hypotheses are concerned, that confirmation is rewarded by an increase of the probability of the hypothesis.

There are many possibilities for defining a degree of confirmation, several having some prima facie plausibility.[7] In the introduction we have already remarked that, as long as one uses the same updating calculus for probabilities, it does not matter very much which confirmation theory one chooses, and hence which degree of confirmation, the only important point is to always make clear which one one has chosen. In this section, we will restrict our attention to mainly one degree of confirmation, viz. the ratio degree of confirmation, with some reference to the standard and non-standard difference degree of confirmation.[8] Let us begin by the latter, $d(H,E) =_{def} p(H/E) - p(H)$, that is, the difference between the posterior and the prior probability of the the hypothesis. From the success perspective, $d'(H,E) =_{def} p(E/H) - p(E)$ is an at least as plausible difference measure for it expresses in a way to what extent E is a success of H. Since they usually give different values one has to choose between them.

The ratio degree of confirmation is usually presented as the ratio of the posterior and the prior probability, $p(H/E)/p(H)$. However, from the success perspective, the ratio $p(E/H)/p(E)$ is at least as plausible as indicator of the extent to which E is a success of H. The latter ratio may well be called the amount or degree of success of H on the basis of E. Fortunately, now we do not have to choose, for the two ratio measures are trivially equivalent, when they are defined, hence we define:

$$\begin{aligned} r(H,E) =_{def} p(H/E)/p(H)) &= p(E/H)/p(E) \\ &= p(H\&E)/(p(H)p(E)) \end{aligned}$$

[7] See Festa (1999a) for a lucid survey.

[8] Popper's arguments (Popper 1959) against $p(H/E)$ as degree of confirmation convinced even Carnap (1963^2, the new foreword) that the 'genuine' degree of confirmation should be identified with, or at least be proportional to $p(H/E) - p(H)$ or $p(H/E)/p(H)$.

to be called the r-degree or r-measure of success and confirmation. Note that the first and the third ratio are not defined, when $p(H) = 0$, and that the same holds for the second and the third ratio when $p(E) = 0$. Since p is, as a rule, not just an objective probability, both possibilities should not be excluded beforehand. Recall that we have assumed that $p(E/H)$ can be interpreted when $p(H) = 0$, and that $p(H/E)$ can be interpreted when $p(E) = 0$. Hence, $r(H, E)$ is almost always defined, that is, it is always defined, except when both $p(E)$ and $p(H)$ are zero, or when one of them is 0 such that the corresponding conditional probability cannot be interpreted, possibilities that will further be disregarded.

In the following, we will evaluate the r-degree of confirmation in some detail, partly in comparison with the d-degree and the d'-degree. To begin with, being almost always defined need not be a positive feature, that depends on the values that are assigned. For a first major advantage of r over d and d' we study their extreme behavior. Note first that r has the neutral value 1 and that d and d' both have the neutral value 0. Higher values indicate, of course, confirmation and lower values disconfirmation. Let us see what happens under the extreme conditions that $p(H)$ or $p(E)$ is zero. When $p(H) = 0$ d gets the neutral value. Hence d reflects the PP-criterion of confirmation, according to which a p-zero hypothesis is always neutrally confirmed. That is, an hypothesis that is excluded by p cannot be confirmed or disconfirmed by evidence; all evidence is, by definition, neutral for such hypotheses, a very strange situation indeed. Similarly, d' gets the neutral value whenever $p(E) = 0$. So, according to d' evidence that is impossible according to p cannot confirm nor disconfirm an hypothesis, but is always neutral. Note that in both cases, it would be less objectionable when the degree of confirmation would not be defined. It is the assignment of the neutral value which is conceptually unattractive.

It is easy to check that r may well assign a non-neutral value when either $p(H)$ or $p(E)$ is zero (assuming that $p(E/H)$, respectively $p(H/E)$, can be interpreted), and, as already remarked, it is undefined when $p(H) = p(E) = 0$. When $p(H)$ and $p(E)$ are both non-zero, $r(H, E)$ reflects both the S- and the PP-criterion of confirmation, it reflects the S-criterion when $p(H) = 0$ and the PP-criterion when $p(E) = 0$. Hence, we may say that the ratio-degree r shows *refined extreme behavior*, whereas d and d' show conceptually implausible extreme behavior.

To be sure, when $p(H) = 0$ and $r(H, E) > 1$, $r(H, E)$ expresses confirmation which is not rewarded, since $p(H/E)$ remains 0. Note that $r(H, E)$ equals $p(E/H)/p(E/\neg H)$[9] when $p(H) = 0$, since $P(E)$ then equals $p(E/$

[9] This ratio of likelihoods of H and $\neg H$ might be called the 'likelihood ratio', but we will not do so because this expression has a different meaning in statistics. There it means the ratio of the likelihoods of two alternative (but usually non-exhaustive) hypotheses assuming

$\neg H)$. Similarly, when $p(E) = 0$ and $r(H, E) > 1$, E is not recognized as confirming evidence, since $p(E/H)$ remains 0. In the case that $r(H, E) > 1$ and $p(H)$ and $p(E)$ are both positive, E is recognized as confirming evidence of H, in the sense that $p(E/H)$ has increased with respect to $p(E)$ by the factor $r(H, E)$, whereas H is rewarded for that success, in the sense that $p(H/E)$ has increased with respect to $p(H)$ by the same factor.

A second feature of the r-measure is its being a *P-incremental* measure[10] in the sense that it is (or can be written as) a function of the probabilities $p(H/E)$ and $p(H)$ which increases with increasing $p(H/E)$ and decreases with increasing $p(H)$[11]. It may also be called an *L-incremental* measure in the sense that it is (or can be written as) a function of the likelihoods $p(E/H)$ and $p(E)$ which increases with increasing $p(E/H)$ and decreases with increasing $p(E)$. Note that d is also P-incremental, but not L-incremental, whereas d' is L-incremental, but not P-incremental.

Next, the ratio of the r-degrees of confirmation of two hypotheses on the basis of the same evidence, $r(H1, E)/r(H2/E)$, just equals the ratio of the likelihoods, $p(E/H1)/p(E/H2)$.[12] This nicely fits the so-called likelihood ratio approach in statistics to comparing two statistical hypotheses with each other, assuming an underlying statistical model (see Note 9). Although d' is L-incremental, it is not easily connectable to this statistical practice.

An important further difference between r and both d and d' is that r is symmetric, that is, $r(H, E) = r(E, H)$, whereas d and d' are asymmetric: $d(H, E)$ is unequal to $d(E, H)$, in fact it is equal to $d'(E, H)$, and similarly for d'. Symmetry is particularly appealing in cases where the hypothesis is of the same nature as the evidence. Consider, for example, the hypothesis (H) that the outcome of a fair die will be even in relation to the evidence (E) that the outcome is larger than 1 and the reverse situation that the evidence reports an even die $(E' = H)$, and the hypothesis $(H' = E)$ states that the outcome will be larger than 1. An asymmetric degree of confirmation may

one underlying statistical model. However, the ratio $p(E/H)/p(E/\neg H)$ is also (unconditionally) equivalent to the ratio of the posterior odds, $p(H/E)/p(\neg H/E)$, and the prior odds, $p(H)/p(\neg H)$. For this reason, this ratio could also be conceived as an inclusive (and impure) degree of confirmation.

[10] The term is due to Festa (1999a).

[11] Since $p(H/E)$ itself is a function of $p(H)$, viz. $p(H) \cdot p(E/H)/p(E)$, this does not exclude that some P-incremental degrees of confirmation, e.g. the d-measure, increase under certain conditions with increasing $p(H)$. E.g. for deductive confirmation of H and H^* by E, $d(H, E) = p(H)(1/p(E) - 1) > d(H^*, E)$ iff $p(H) > p(H^*)$.

[12] Note that this ratio may be defined for two p-zero hypotheses and that values for $p(E/\neg H1)$ and $p(E/\neg H2)$ are not needed.

imply that E confirms H more (or less) than $E'(= H)$ confirms $H'(= E)$, and d and d' do so. The symmetry of r is, of course, directly related to the fact that $r(H, E)$ can be seen as a degree of mutual dependence between H and E, since independence is usually defined by the criterion $p(H\&E) = p(H)p(E)$[13].

Some special values of $r(H, E)$ are relatively simple. For instance, $r(H, E)$ increases from 0, for falsification, via $p(E/H)/[1 - p(H)p(\neg E/H)]$ for deductive disconfirmation, to 1, for neutral (including tautological) evidence, from which it increases further, via $1/p(E)$ for deductive confirmation, to $1/p(H)$, for verification. The last value is, moreover, the maximum degree of confirmation a hypothesis can get, viz. $1/p(H)$ for verification, e.g. when $E = H$. Note that this maximum is *hypothesis specific*, and that we have the plausible extreme consequence that verification of a p-zero hypothesis amounts to obtaining an infinite degree of confirmation. Similarly, $1/p(E)$ is the maximum degree of confirmation certain E can provide for an hypothesis, viz. by deductive confirmation, with the plausible extreme consequence that the degree of confirmation in the case of deductive confirmation by p-zero evidence is infinite.

1.3. *Comparing and composing degrees of confirmation*

Let us now turn to the comparative and composite behavior of $r(H, E)$ by presenting some trivial but crucial theorems, always assuming that H is p-uncertain ($p(H) < 1$).

We start by considering two pieces of evidence with respect to which a fixed hypothesis is equally successful in the sense that they provide the hypothesis with the same likelihood (e.g., 1 in the case of deductive confirmation):

Th.1: if $p(E/H) = p(E^*/H) > 0$ then
$r(H, E) > r(H, E^*)$ iff $p(E^*) > p(E)$
(iff $p(H/E) > p(H/E^*)$, if $p(H) > 0$)

Th.1 states that, when H obtains the same likelihood from two pieces of evidence, the degree of confirmation increases with decreasing prior probability of the evidence or, if $p(H) > 0$, equivalently, with increasing posterior probability of the hypothesis. Hence, under the mentioned condition, according to $r(H, E)$, H gets 'richer' from less probable (more surprising) evidence,

[13] This definition has some complications. Strictly speaking, it provides only a necessary condition for independence. It is nevertheless plausible to call, in general, the probabilistic expression $p(A\&B)/(p(A) \cdot p(B))$ the degree of mutual or inter-dependence of A and B. Carnap (1950/63, par. 66) has called it the (mutual) relevance quotient.

which agrees with scientific common sense; we will call this the *surprise bonus*. Note that, when $p(H) = 0$, this surprise bonus is not payed out in an increase of the posterior probability, for that remains zero.

Let us now turn to fixed evidence and two hypotheses, which are equally successful in the sense that they obtain the same likelihood from that evidence (again, e.g., 1 in the case of deductive confirmation):

Th.2: if $p(E/H) = p(E/H^*) > 0$ then
$r(H, E) = r(H^*, E)$
and
$p(H/E) > p(H^*/E)$ iff $p(H) > p(H^*)$

Th.2 shows in the first place that $r(H, E)$ is a (hypothesis-) neutral[14] or *pure* degree of confirmation, in the sense that two hypotheses which are equally successful in the sense that they make the evidence equally plausible, obtain a degree of confirmation which is independent of their prior probability. Th.2 states, moreover, that, assuming equal successfulness, the posterior probability increases with increasing prior probability. Note that the first feature is in sharp contrast to the 'impure' behavior of $d(H, E)$. Since $d(H, E)$ is equal to $p(H)(r(H, E) - 1)$, it favors plausible hypotheses, that is, it increases in the case of equal successfulness with the prior probability. On the other hand, d' is easily seen to be pure.

Restricting attention to deductive confirmation and identifying plausibility with probability, it follows directly from Th.1 and Th.2 that quantitative deductive confirmation, as measured by $r(H, E)$, satisfies the qualitative principles of deductive confirmation P.1 and P.2, respectively:

P.1 if E and E^* d-confirm H then E d-confirms H more than E^* iff E^* is more plausible than E in the light of the background beliefs

P.2 if E d-confirms H and H^* then E d-confirms H^* as much as H

Let us also look at some specific cases that have been put forward in favor of $r(H, E)$ or $d(H, E)$. Roberto Festa (1999a, p. 66) has suggested a version of the following counter-intuitive case against $d(H, E)$, and in favor of $r(H, E)$, when $p(H) > 0$. Compare $p(H/E) = 0.1$ and $p(H) = 0.0001$ with $p(H^*/E) = 0.9$ and $p(H^*) = 0.8$. Although the respective differences

[14] The term 'neutral' is already used within the presented theory of confirmation, viz., in the phrase 'neutral evidence', which makes that term less attractive for our present purposes.

are almost the same (≈0.1 and 0.1, respectively) the first case of confirmation is intuitively much more impressive than the second. It is easy to check that $r(H, E)$ is in agreement with this intuition (1000 and 9/8, respectively), which makes $r(H, E)$ superior to $d(H, E)$. For a real-life (aircraft) example of a formally similar nature, see (Schlesinger 1995, Section 4).

However, such specific intuitions may easily be countered by similar ones, pointing in the opposite direction. Consider the following case against $r(H, E)$ and in favor of $d(H, E)$, stemming from Eells and reported by Sober (1994). In a slightly modified form, consider $p(H/E) = 0.9$ and $p(H) = 0.1$ versus $p(H^*/E) = 0.001$ and $p(H^*) = 0.00001$. Though H may seem intuitively and according to $d(H, E)$ much more confirmed by E than H^* ($d(H, E) = 0.8$ versus $d(H^*, E)$ ≈0.001), the $r(H, E)$-definition leads to the reverse conclusion (9 versus 100).

Accordingly, such examples make clear that our intuitions are confused and that we can decide to reconsider our intuitions in the light of the fact that there is something to choose, viz., principles we may or may not want to subscribe to.

So let us return to general properties of the r-measure. First we will consider disjunctions of evidence and hypotheses. For the disjunction of two incompatible pieces of evidence, the r-degree of confirmation is the weighted sum of the separate degrees of confirmation:

Th.3.1: if $p(E\&E') = 0$ then
$$r(H, E \vee E') = \frac{p(E)}{p(E)+p(E')} r(H, E) + \frac{p(E')}{p(E)+p(E')} r(H, E')$$

Similarly, due to the symmetry of the r-degree with respect to E and H, the r-degree of confirmation of a disjunction of two incompatible hypotheses is the weighted sum of the degrees of the disjuncts:

Th.3.2: if $p(H\&H') = 0$ then
$$r(H \vee H', E) = \frac{p(H)}{p(H)+p(H')} r(H, E) + \frac{p(H')}{p(H)+p(H')} r(H', E)$$

Let us now turn to conjunctions. Let E and E' be mutually independent pieces of evidence in general and with respect to H. Then the degree of confirmation provided by the conjunction is the product of the separate degrees:

Th.4.1: if $p(E\&E') = p(E) \cdot p(E')$
and $p(E\&E'/H) = p(E/H) \cdot p(E'/H)$
then $r(H, E\&E') = r(H, E) \cdot r(H, E')$

Similarly, again due to the symmetry of the r-degree with respect to E and H, for prior and posterior mutually independent hypotheses:

Th.4.2: if $p(H\&H') = p(H) \cdot p(H')$
and $p(H\&H'/E) = p(H/E) \cdot p(H'/E)$
then $r(H\&H', E) = r(H, E) \cdot r(H', E)$

Finally, let us consider the 'addition' of an irrelevant piece of evidence E', defined by $p(H/E\&E') = p(H/E)$, or an irrelevant hypothesis, defined by $p(E/H\&H') = p(E/H)$.

Th.5.1: if $p(H/E\&E') = p(H/E)$ then $r(H, E\&E') = r(H, E)$
Th.5.2: if $p(E/H\&H') = p(E/H)$ then $r(H\&H', E) = r(H, E)$

In our opinion, the composite behavior of the r-measure, as expressed by Theorems 3–5, is very plausible.

In sum, we conclude that $r(H, E)$ is an attractive degree of confirmation. It shows refined extreme behavior, it is incremental with respect to the probability of the hypothesis as well as the evidence, it has hypothesis and evidence specific maxima, it realizes the surprise bonus, it is pure in the sense of being neutral with respect to equally successful hypotheses, independently from their prior probabilities, and it has plausible composite behavior.[15] Since it implies the qualitative principles of deductive confirmation, we may conclude from (Kuipers, 1998), that it can deal with the standard objections to deductive confirmation, with the raven paradoxes, and the grue problem. In the next section, we will further evaluate $r(H, E)$ with respect to qualitative consequences, again partly in comparison with other candidates.

We conclude this section with three technical points. First, accepting r as degree of confirmation, implies, of course, as explication of "E confirms H more than E^* confirms H^*": $r(H, E) > r(H^*, E^*)$. Second, as Milne (1995, 1996) has rightly argued, a near relative to r, viz., log $r(H, E)$, has some advantages over $r(H, E)$. E.g. its neutral value is 0. Third, for completeness and later use, we write down the *conditional* degree of confirmation corresponding to the unconditional one:

[15] See Jeffrey (1975) for a comparison of a couple of measures, including r, d, and d'. His emphasis is on r and d, and on second thoughts, that is, in his "Replies" he favors d over r, mainly because of its 'impure' character. In our opinion (see also the next section), the impact of different prior probabilities is perfectly accounted for in the resulting different posterior probabilities.

$$r(H,E;C) =_{def} p(H/E\&C)/p(H/C) = p(E/H\&C)/p(E/C) = p(H\&E/C)/(p(H/C)p(E/C))$$

It is easy to check that this conditional degree has similar properties to the unconditional one.

The quantitative theory of confirmation based on r will be called the r-theory of confirmation. Similarly for the d- and the d'-theory.

2. *Qualitative consequences*

In this section it will first be argued in some more detail than in Sections 1.3 and 1.4 that the 'r-theory' restricted to deductive confirmation implies the whole qualitative theory of deductive confirmation presented in (Kuipers, 1998). In this connection it will be particularly illuminating to write out the quantitative variant of the qualitative solution of the raven paradoxes in (Kuipers, 1998). This example illustrates, among other things, that the r-degree of confirmation can also be interpreted as a degree of severity of tests, in particular of HD-tests, with attractive qualitative consequences. Finally, we will investigate to what extent a corresponding qualitative theory of general and non-deductive confirmation can be derived and defended.

2.1. *Derivation of the qualitative theory of deductive confirmation*

The claim that the qualitative theory of deductive confirmation can be derived from the quantitative theory amounts, of course, to the claim that deductive confirmation is a subkind of quantitative confirmation that satisfies the comparative principles of deductive confirmation when plausibility is identified with probability. We have already seen that deductive confirmation amounts to an extreme kind of quantitative confirmation, due to the fact that $H \models E$ implies that $p(E/H) = 1$. The corresponding r-degree of confirmation is $1/p(E)$, which exceeds 1, hence indicates confirmation, as soon as E is probabilistically uncertain. We have also concluded already, on the basis of Th.1 and Th.2, that quantitative confirmation respects the comparative principles P.1 and P.2, when we identify plausibility with probability. Hence, quantitative confirmation entails all principles of the qualitative theory of deductive confirmation. In Section 2.4 we will review the extent to which it implies the general principles of qualitative confirmation presented in (Kuipers, 1998), viz., SDC, UPP, PS, and PCS.

In (Kuipers, 1998) we have also alluded to the reverse perspective on P.1 and P.2, that is, that they are made plausible by Bayesian considerations. For this purpose, it is important to note first that $r(H,E)$ and $d(H,E)$ are both popular among Bayesians. Hence, since both measures support P.1, this comparative postulate seems unproblematic for Bayesians. Moreover, since $r(H, E)$ is frequently suggested and used as an alternative to $d(H, E)$, and since $r(H, E)$ supports P.2, the latter comparative postulate is frequently implicitly assumed by Bayesians. However, it should be conceded that $d(H, E)$ is used at least as frequently as $r(H, E)$. Hence, for supporters of d, P.2 will only become acceptable as far as our general arguments in favor of r (above and below), and those of others, such as Festa (1999a), Schlesinger (1995), and Milne (1995, 1996), are convincing for them.

In this respect, it is interesting to study the way the r- and the d-measure deal with an irrelevant additional hypothesis H' in the case of deductive confirmation of H by E, that is, when $p(E/H) = 1$. Whereas $d(H\&H', E)$ becomes smaller than $d(H, E)$, by the factor $p(H\&H')/p(H)$, $r(H\&H', E)$ remains equal to $r(H, E)$. In general, if $p(E/H\&H') = p(E/H)$, the plausible condition for a, relative to E in the face of H, irrelevant additional hypothesis H', $d(H\&H', E) = (p(H\&H')/p(H))\cdot d(H, E)$, whereas $r(H\&H', E) = r(H, E)$ (Th.5.2). Hence, whereas r accounts for the irrelevance of H' in a straightforward way, d does so in a more complicated way, which may or may not be conceived as more sophisticated.

In order to question the latter suggestion we will conceive an 'objective case', assuming that the degree of confirmation should give satisfactory answers in cases where only objective probabilities are in the game, since our intuitions may then be assumed to be as sharp as possible. Consider the following urn-model of a two-step random experiment. A B-urn is an urn with precisely one black ball, a BB-urn an urn with precisely two black balls and a BW-urn an urn with one black and one white ball. First we randomly select an urn out of a collection of 1 B-urn, 4 BB-urns and 5 BW-urns, hence with objective probability 1/10, 2/5, 1/2, respectively. Next, in the selected urn, we randomly select balls with replacement. Suppose that the first n selections of the second type lead to a black ball. It is easy to check that this type of evidence deductively follows from, hence d-confirms, the hypothesis that the first selected urn is a B-urn, H-B, as well as the hypothesis that it is a BB-urn, H-BB. Now the question is whether this evidence (d-)confirms H-B more than H-BB. Since the evidence differentiates in no way between the two hypotheses, the 'r-answer' ('as much as') seems the most plausible one, and not the 'd-answer' ('less than'). Similarly, consider the disjunctive hypothesis 'H-B or H-BB', being weaker than its disjuncts, but nevertheless d-confirmed by the evidence. Again, the r-claim that it is as much confirmed as its disjuncts, seems more plausible than the d-claim that it is more confirmed. To be sure, the prior and hence the posterior probability of H-B is smaller than that of H-BB, and the latter, and hence the former, is smaller than that of the disjunctive hypothesis 'H-B or H-BB'.

Let us now consider the way the r- and the d-measure deal with irrelevant disjunctive evidence E' in the case of deductive confirmation of H by E. To avoid inessential complications, let us restrict attention to the case that E' is incompatible with E. Then $r(H, E \vee E')$ becomes smaller than $r(H, E)$ by the factor $p(E)/p(E \vee E') = p(E)/(p(E) + p(E'))$, hence decreases with increasing $p(E')$. Since $d(H, E) = p(H)(r(H, E) - 1)$, $d(H, E \vee E')$ decreases in a related way. In general, if $p(E'/H) = p(E')$, the plausible condition for irrelevant, for neutral, disjunctive evidence E', $r(H, E \vee E')$ and $d(H, E \vee E')$ both decrease with increasing $p(E')$; in view of Th. 3.1, the former does so in a more transparent way than the latter. In sum, as was to be expected, r and d behave rather similar with respect to irrelevant disjunctive evidence.

Combining the results for an irrelevant conjunctive hypothesis and an irrelevant disjunctive piece of evidence, we may conclude that the r-measure deals with both in a plausible way.

Let us now turn to the special qualitative applications or principles of (Kuipers, 1998), Section 2. It will be useful to list first the relevant corollaries of Th.1 and Th.2 with respect to conditional deductive confirmation:

Th.1c: if $p(E/H\&C) = p(E^*/H\&C^*) > 0$ then
$r(H, E; C) > r(H, E^*; C^*)$ iff $p(E^*/C^*) > p(E/C)$
(iff $p(H/E\&C) > p(H/E^*\&C^*)$, if $p(H) > 0$)

Th.2c: if $p(E/H\&C) = p(E/H^*\&C) > 0$ then
$r(H, E; C) = r(H^*, E; C) = p(E/H\&C)/p(E/C) = p(E/H^*\&C)/p(E/C)$
and
$p(H/E\&C) > p(H^*/E\&C)$ iff $p(H/C) > p(H^*/C)$
(iff $p(H) > p(H^*)$ if $p(C/H) = p(C/H^*) > 0$)

The condition "$p(C/H) = p(C/H^*) > 0$" amounts, of course, to the claim that the probability that C is, or will be, realized is independent of the hypothesis under consideration. Note that the unconditional versions of Th.1c and Th.2c arise by skipping C and C^* in the formulas.

In the next subsection we will show that the special principle $S^{\#}$.1c, dealing with a fixed hypothesis, e.g. the raven hypothesis, is realized by $r(H, E)$ as a special case of Th.1c if we are willing to express some relevant background beliefs by the probabilistic assumption:

A_p-ravens p is based on random sampling in the relevant universe

Finally, it is easy to check that S^Q.2c (an EMG EM-confirms "all E are Q" as much as "all E are G"), dealing with fixed evidence, e.g. in the emerald case, is trivially realized as a special case of (the first part of) Th.2c:

$$r(\text{“all E are Q”}, \mathrm{G}; \mathrm{EM}) = r(\text{“all E are G”}, \mathrm{G}; \mathrm{EM}) = 1/p(\mathrm{G}/\mathrm{EM})$$

If we are, moreover, willing to express the green/grue-case of the weak irrelevance assumption (WIA-emeralds) by the probabilistic assumption:

WIA$_p$(-emeralds) $\quad p(\text{“all E are G”}) > p(\text{“all E are Q”})$

it is easy to derive the probabilistic version of the refined intuition (4&5). Of course, if $p(\text{“all E are G”}) = 0$, the refined intuition cannot be realized, but the degree of confirmation of both hypotheses will remain $1/p(\mathrm{G}/\mathrm{EM})$.

Since, assuming A$_p$-ravens, S$^{\#}$.1c-ravens is realized by $r(H, E)$, the raven paradoxes are qualitatively solved in the same way as before, because all desired results already followed qualitatively, assuming S$^{\#}$.1c-ravens and A-ravens (see below). Similarly, in the light of the fact that S^{Q}.2c-emeralds is realized by $r(H, E)$, and assuming WIA$_p$-emeralds, the grue problem is qualitatively solved in the same way as before, since all desired results already followed qualitatively, assuming S^{Q}.2c-emeralds and WIA-emeralds.[16] As a matter of fact, Sober (1994) inspired us to our proposal for the refinement of Goodman's basic intuition, viz., (4&5) of (Kuipers, 1998), Section 2.2. In fact, he derived from WIA$_p$-emeralds the quantitative counterpart of that refinement.

In sum, the ‘r-theory’ of confirmation can generate the qualitative theory of deductive confirmation in the most encompassing way.

2.2. *The raven paradoxes reconsidered*

Whereas it is not interesting to write down the quantitative analysis of the grue problem, it is instructive, also for later purposes, to spell out the quantitative solution of the raven paradoxes. Recall RH, the hypothesis that all ravens are black. Table 1 introduces a matrix of numbers for the sizes of the four cells constituting the relevant conceptual possibilities.

	#R	#$\overline{\mathrm{R}}$	total
#B	a	b	$a+b$
#$\overline{\mathrm{B}}$	c	d	$c+d$
total	$a+c$	$b+d$	$a+b+c+d$

Table 1: Numbers of ‘raven possibilities’

[16] Note in particular that assigning the probability value 0 to “for all E: M iff G” amounts to adding the strong irrelevance assumption (SIA) as described in (Kuipers, 1998). In that case, the posterior probability and the posterior odds of the grue hypothesis are and remain 0.

Of course, these numbers are assumed to be finite but further unknown. There are only some comparative background beliefs. In particular, the assumption A-ravens stating that the number of ravens is much smaller than that of non-black objects, which amounts to $a + c << c + d$, and this is equivalent to $a << d$ (and hence to $a + b << b + d$). We assume, moreover, that a and b are positive and, of course, that c is 0 if RH is true and positive if RH is false. All results to be presented basically presuppose and use A_p-ravens, according to which testing is random sampling in the relevant universe. In the following, 'sampling' is to be read as 'random sampling' and the explicit occurrence of 'c' means that it 'resulted from' the condition that RH is false. Hence, from now on $c > 0$.

Recall that $r(H, E) = p(E/H)/p(E)(= p(H/E)/p(H))$.[17] Writing '$Q$' for $p(\mathrm{RH})$, which may or may not be assumed to be positive, and using the appropriate conditional versions whenever relevant (in which case C is, of course, supposed to be neutral 'evidence' for RH, that is, $p(\mathrm{RH/C}) = p(\mathrm{RH}) = Q$), the crucial expression becomes:

$$r(\mathrm{RH}, \mathrm{E}; \mathrm{C}) = \frac{p(\mathrm{E/RH\&C})}{p(\mathrm{RH/C})p(\mathrm{E/RH\&C}) + p(\neg\mathrm{RH/C})p(\mathrm{E}/\neg\mathrm{RH\&C})} = \frac{1}{Q + (1-Q)p(\mathrm{E}/\neg\mathrm{RH\&C})/p(\mathrm{E/RH\&C})}$$

The results are as follows[18]:

(1p) a black raven, a non-black non-raven and a black non-raven, resulting from sampling in the universe of objects (hence C tautologous), non-deductively confirm RH, all with the same r-value: $(a + b + c + d)/(a + b + Qc + d)$ (e.g., for a black raven, via $p(\mathrm{BR/RH}) = a/(a+b+d)$ and $p(\mathrm{BR}/\neg\mathrm{RH}) = a/(a+b+c+d)$)

(2p) a black raven resulting from sampling ravens cd-confirms RH with r-value $r(\mathrm{RH}, \mathrm{BR}; \mathrm{R}) = (a + c)/(a + Qc)$, via $p(\mathrm{BR/RH\&R}) = 1$ and $p(\mathrm{BR}/\neg\mathrm{RH\&R}) = a/(a + c)$, and a non-black non-raven resulting from sampling non-black objects cd-confirms RH with r-value $r(\mathrm{RH}, \overline{\mathrm{B}}\overline{\mathrm{R}}; \overline{\mathrm{B}}) = (c+d)/(Qc+d)$ (similar calculation)

[17] Note that, since $d(H, E) = p(H)(r(H, E) - 1)$, $r(H, E)$ is also the crucial expression in calculating the relevant d-values.

[18] The assumption that c is some positive number when RH is false is of course a simplification. However, it is easy to check that the proofs can be refined by conditionalization on the hypotheses that $c = 1, 2, 3, \ldots$, with the result that the claims remain valid, independent of the prior distribution for the hypotheses.

(3p) sampling black objects or non-ravens always leads to neutral evidence, i.e., r-value 1, for black objects, via $p(\text{BR}/\text{RH}\&\text{B}) = p(\text{BR}/\neg\text{RH}\&\text{B}) = a/(a+b)$

(4p) ad (2p): a black raven resulting from sampling ravens cd-confirms RH much more than a non-black non-raven resulting from sampling non-black objects, for $(a+c)/(a+Qc) >> (c+d)/(Qc+d)$ iff $a << d$, where the latter condition follows from A-ravens.

Note that all r-values, except those in (3p), exceed 1, and that this remains the case when $p(\text{RH}) = Q = 0$. It is easy to check that (4p) essentially amounts to a special case of Th.1c, realizing $\text{S}^{\#}$.1c-ravens: an RB R-confirms "all R are B" more than an $\overline{\text{RB}}$ $\overline{\text{B}}$-confirms it iff the background beliefs imply that #R < #$\overline{\text{B}}$.

It should be noted that this analysis deviates somewhat from the more or less standard Bayesian solutions of the paradoxes. In the light of the many references to Horwich (1982), he may be conceived to have given the best version. In Appendix 2 we argue that our solution, though highly similar, has some advantages compared to that of Horwich.

2.3. *The severity of tests*

The raven example provides a nice illustration of the fact that, at least in the case of a HD-test, the degree of confirmation can also be conceived as the 'degree of severity' of the test. When H entails E, $r(H, E)$ expresses the degree of success or the degree of confirmation H has obtained or *can obtain* from an experiment that results in E or non-E. In the latter reading, it expresses a *potential* degree of success. The smaller $p(E)$, the more success H can obtain, but the less probable it will obtain this success. Both aspects are crucial for the intuition of the severity of tests. The more severe a test is for a hypothesis, the less probable that the hypothesis will pass the test, but the more success is obtained when it passes the test. More specifically, the *degree of severity* of a HD-test is according to Popper (1959, Appendix *ix, 1963, Addendum 2, 1983, Section 32) (a measure increasing with) the probability that the test leads to falsification or a counter-example, or, to quote Popper (1983, p. 247), "the improbability of the prediction measures the severity of the test". This specification amounts to taking $p(\neg E)$ as the degree of severity, or some function increasing with $p(\neg E)$, as do Popper's proposals in Addendum 2 of (Popper, 1963). One of Popper's proposals for the degree of severity of an HD-test is the r-value $1/p(E)$, where it is again important that $p(E)$ is calculated before the experiment is performed, or at least before its result is known. Like Popper, we see no reason not to generalize this definition to $p(E/H)/p(E)$[19] for non-(conditionally) deductive tests. However, it is primarily HD-tests where we seem to have specific qualitative severity intuitions, some of which will be studied now.

First, rephrasing the results (1p)–(4p) concerning the raven paradoxes in the previous section in severity terms, the analysis explains and justifies, essentially in a deductive way, why scientists prefer, if possible, random testing of ravens, that is, randomly looking among ravens to see whether they are black, and, in general, choose that way of conditional random testing among the ones that are possible, which is the most severe.

Second, a standard objection to Bayesian theories of confirmation in general is the so-called 'problem of old evidence'. If we know already that E is true, and then find out that H entails E, the question arises whether E still confirms H. The problem is, of course, that our up to date probability function will be such that $p(E) = 1$, and hence $p(E/H) = p(E) = 1$ and $p(H/E) = p(H)$. Hence, the r-degree then leads to the neutral value 1. This reflects the intuition that there is no severe test involved any longer. Despite this severity diagnosis, E does nevertheless represent a success or confirming evidence of the degree $1/p'(E)$, where p' refers to the probability function before E became known or, similarly, the probability function based on the background knowledge minus E. The latter, counterfactual, defence is more or less standard among Bayesians (Howson & Urbach, 1989; Earman, 1992), but the additional severity diagnosis is not.

Third, there are two other intuitions associated with severity. The first one is the famous 'diminishing returns' intuition of Popper: "There is something like a law of diminishing returns from repeated tests" (Popper 1963, p. 240). It expresses the idea that the returns and hence the severity of repeated tests decreases in one way or another. Let us look at the raven example. Let Rn represent n random drawings (with replacement) of ravens and let Bn indicate that these n ravens are black. Of course we have that $p(\mathrm{B}n/\mathrm{RH}\&\mathrm{R}n) = 1$. Moreover, replacing $p(\mathrm{RH})$ again by Q, $p(\mathrm{B}n/\mathrm{R}n) = p(\mathrm{RH}) \cdot p(\mathrm{B}n/\mathrm{RH}\&\mathrm{R}n) + p(\neg\mathrm{RH}) \cdot p(\mathrm{B}n/\neg\mathrm{RH}\&\mathrm{R}n) = Q + (1-Q)p(\mathrm{B}n/\neg\mathrm{RH}\&\mathrm{R}n)$. Suppose first that $Q > 0$. Assuming that $\neg$RH implies that there is a positive probability $(1-q)$ of drawing a non-black raven, $p(\mathrm{B}n/\neg\mathrm{RH}\&\mathrm{R}n) = q^n$ goes to 0, for increasing n, and hence $r(\mathrm{RH}, \mathrm{B}n; \mathrm{R}n)$ will increase to $1/Q$. Hence, $r(\mathrm{RH}, \mathrm{B}n+1; \mathrm{R}n+1) - r(\mathrm{RH}, \mathrm{B}n; \mathrm{R}n)$ has to go to 0, that is, the additional returns or degree of confirmation obtained by an extra (successful) test goes to 0. In terms of severity, the additional degree of severity by an extra test goes to 0. A similar diminishing effect arises, of course,

[19] Popper calls the r-value in general the 'explanatory power' of H with respect to E. Although Popper does not do so, it would have been plausible for him to call it the 'explanatory success' as soon as E has turned out to be the result of the test. We simply call it the degree of success.

when we consider the ratio $r(\mathrm{RH}, \mathrm{B}n+1; \mathrm{R}n+1)/r(\mathrm{RH}, \mathrm{B}n; \mathrm{R}n)$, which will go to 1. However, if $Q = 0$ the situation is different: $r(\mathrm{RH}, \mathrm{B}n; \mathrm{R}n) = 1/p(\mathrm{B}n/\neg\mathrm{RH}\&\mathrm{R}n) = 1/q^n$. Hence, r increases without limit, so does the extra returns/ confirmation/ severity $1/q^{n+1} - 1/q^n = (1/q^{n+1})(1-q)$, whereas the ratio $r(\mathrm{RH}, \mathrm{B}n+1; \mathrm{R}n+1)/r(\mathrm{RH}, \mathrm{B}n; \mathrm{R}n)$ remains constant $(1/q)$. In sum, the r-measure perfectly reflects the intuition of diminishing returns, assuming that $Q = p(\mathrm{RH})$ is positive.

The last severity intuition to be considered, may be called the 'superiority of new tests', that is, the idea that a new test is more severe than a mere repetition. It is a specific instance of the more general 'variety of evidence' intuition. However, it appears to be not easy to give a rigorous explication and proof of the general intuition (Earman 1992, p. 77–79). But for the special case, it is plausible to build an objective probabilistic model which realizes the intuition under fairly general conditions. The set-up is a direct adaptation of an old proposal for the severity of test (Kuipers, 1983). Let us start by making the intuition as precise as possible. Suppose that we can distinguish types of (HD-)test-conditions and their tokens by means other than severity considerations. E.g. ravens from different regions, and individual drawings from a region. Any sequence of tokens can then be represented as a sequence of N and M, where N indicates a token of a new type, i.e. *a new test-condition*, and M a token of the foregoing type, i.e. *a mere repetition*. Each test can result in a success B or failure non-B. Any test sequence starts, of course, with N. Suppose further that any such sequence Xn, of length n, is probabilistic with respect to the outcome sequence it generates. Note that RH is still supposed to imply that all n-sequences result in Bn, and hence that one non-B in the outcome sequence pertains to a falsification of RH. A plausible interpretation of the intuition now is that the severity of a XnN-sequence is higher than that of a XnM-sequence, that is:

$$\frac{1}{Q+(1-Q)p(\mathrm{B}n+1/\neg\mathrm{RH}\&\mathrm{X}n\mathrm{N})} > \frac{1}{Q+(1-Q)p(\mathrm{B}n+1/\neg\mathrm{RH}\&\mathrm{X}n\mathrm{M})}$$

which is equivalent to:

$$p(\mathrm{B}n+1/\neg\mathrm{RH}\&\mathrm{X}n\mathrm{N}) < p(\mathrm{B}n+1/\neg\mathrm{RH}\&\mathrm{X}n\mathrm{M})$$

to be called the superiority condition.

The remaining question is whether this condition holds under general assumptions. For this purpose, we will construct an urn-model which reflects the ideas of new tests and mere repetitions. Suppose there is a reservoir with an unknown finite number of urns, each containing an unknown finite number of balls, which number may or may not differ from urn to urn. Our hypothesis to be tested states that all balls in the reservoir, and hence in each urn, are black. Restricting our attention to random selections of urns and balls *with* replacement, the possibilities for probabilistic test sequences are as follows: start with a random selection of an urn, draw randomly and successively a number of balls out of that urn with replacement, replace the urn and start over again, with the same or a different number of ball selections out of the next urn. It turns out to be non-trivial (see Kuipers 1983, 219–220) to prove the superiority condition assuming one very plausible condition, viz. if RH is false, the ratio of black balls may not be the same in all urns.

In sum, in the case of deductive confirmation, the ratio degree of confirmation may well be conceived as the degree of severity of the HD-test giving rise to the confirmation, for it satisfies the main qualitative features associated to current severity intuitions.[20]

2.4. *Qualitative non-deductive confirmation*

The plausible question now arises whether it is possible to give a qualitative explication of non-deductive confirmation, that is, an explication of non-deductive confirmation in terms of ‘plausibility’. It will be easier to concentrate first on confirmation in general, or *general* confirmation, after which *non-deductive* confirmation can be identified with general confirmation of non-deductive nature.

But first we will check whether and to what extent the r-theory realizes the success definition of general confirmation and the further general principles that were presented Section 1.2 of (Kuipers, 1998). For this purpose, we have to replace ‘E confirms H' by $r(H, E) > 1$ and ‘plausibility’ by ‘probability’. According to the *success definition of confirmation* (SDC), we should have that $r(H, E) > 1$ iff $p(E/H) > p(E)$. This holds, by definition, whenever $p(E) > 0$. According to the *updating principle of plausibility* (UPP) we should have that $p(H/E) > p(H)$ iff $r(H, E) > 1$, which holds whenever $p(H) > 0$.[21] The joint consequence, that is, the *principle of symmetry* (PS), $p(H/E) > p(H)$ iff $p(E/H) > p(E)$, holds whenever both $p(H)$ and $p(E)$ are positive. Finally, $r(H, E)$ realizes the *principle of*

[20] In view of the nature of our analysis and the relation $d(H, E) = p(H)(r(H, E) - 1)$, it is clear that $d(H, E)$ also realizes the severity intuitions dealt with, though in a somewhat less transparant way.

[21] Note that the principle of partial entailment, suggested at the end of Section 1.1, may be seen as a special case of SDC or UPP as soon as we assume that “H partially entails E” implies that H makes E more plausible or that E makes H more plausible, respectively. Similarly, the principle of inductive extrapolation, also suggested there, is realized by UPP as soon as we assume that “H inductively extrapolates upon E” implies that E makes H more plausible. In this case, the implication that H makes E more plausible does not seem as natural as the reverse implication, hence, the principle is not as easy to see as a special case of SDC.

comparative symmetry (PCS), now pertaining to some trivial equivalences of the conditions $r(H^*, E) > (or >>, or =) r(H, E)$, straightforwardly by its two-sided definition, where the inequalities only hold as far as the relevant prior probabilities are non-zero and the relevant conditional probabilities can be interpreted under 'p-zero conditions'.

For the remaining comparative principles we first state trivial generalizations of Th.1 and Th.2:

Th.1G If $0 < p(H) < 1$ and $p(E^*/H) =\geq> p(E/H) > 0$ then
$\frac{r(H,E^*)}{r(H,E)} = \frac{p(E^*/H)}{p(E/H)} \frac{p(E)}{p(E^*)} = \frac{p(H/E^*)}{p(H/E)} =\geq> \frac{p(E)}{p(E^*)}$

Th.1G suggests

P.1G a) If H makes E^* *as plausible as* E then E^* confirms H *as much as* E if (and only if) E^* is *as plausible as* E
b) If H makes E^* *at least as plausible as* E then E^* confirms H *at least as much as* E if E^* is *at most as plausible as* E
c) If H makes E^* *more plausible than* E then E^* confirms H *more than* E if E^* is *less plausible than* E

Th.2 can be generalized to:

Th.2G If $0 < p(H) < 1$ and $p(E/H^*) =\geq> p(E/H) > 0$ then
$\frac{r(H^*,E)}{r(H,E)} = \frac{p(E/H^*)}{p(E/H)} =\geq> 1$ and
$\frac{p(H^*/E)}{p(H/E)} = \frac{p(E/H^*)}{p(E/H)} \frac{p(H^*)}{p(H)} =\geq> \frac{p(H^*)}{p(H)}$

Th.2G suggests

P.2G a) If H^* makes E *as plausible as* H then E confirms H^* *as much as* H (with the consequence that the relative plausibility of H^* with respect to H *remains the same*)
b) If H^* makes E *at least as plausible as* H then E confirms H^* *at least as much as* H (with the consequence that the relative plausibility of H^* with respect to H *remains at least the same*)
c) If H^* makes E *more plausible than* H then E confirms H^* *more than* H (with the consequence that the relative plausibility of H^* with respect to H *increases*)

It is not difficult to check that P.1 and P.2 are subcases of P.1G and P.2G, respectively.[22]

In sum, the qualitative explication of general confirmation can be given by SDC, UPP, PCS, P.1G and P.2G. As announced, it is now plausible to present general confirmation of a non-deductive nature as the resulting qualitative explication of non-deductive confirmation.

Although it is apparently possible to give qualitative explications of general and non-deductive confirmation, we do not claim that these explications are independent of the corresponding quantitative explications. In particular, we would certainly not have arrived at P.1G and P.2G without the quantitative detour. To be sure, this is a claim about the discovery of these principles; we do not want to exclude that they can be justified by purely non-quantitative considerations.

Similarly, although it is possible to suggest that the two (qualitative) proper connotations formulated for deductive confirmation can be extrapolated to general and non-deductive confirmation, we would only subscribe to them, at least for the time being, to the extent that their quantitative analogues hold. However, in this respect the quantitative situation turns out to be rather complicated, hence, its re-translation in qualitative terms becomes even more so. Fortunately, the proper connotations looked for do not belong to the core of a qualitative theory of general confirmation.

Accordingly, although there is no intuitively appealing qualitative explication of general and non-deductive confirmation, there is a plausible qualitative explication of their core features in the sense that it can be derived via the quantitative explication. In other words, we have an indirectly, more specifically, quantitatively justified qualitative explication of general and non-deductive confirmation.

It is important to argue that its justification is at least as strong as the justification of the quantitative explication 'under ideal circumstances', that is, when the probabilities make objective sense. At first sight, it may seem that we have to take into account our relativization of the quantitative explication by emphasizing and criticizing the artificial character of most of the probabilities. However, this is not the correct evaluation. As soon as we agree that the quantitative explication is the right one in cases where these probabilities

[22] And hence their respective deductive applications, i.e., the special principles S.1 (if $H \models E \models E^*$ then E d-confirms H more than E^*) and S.2 (if $H^* \models H \models E$ then E d-confirms H^* as much as H). Since the special conditional principles dealing with ravens and emeralds concerned specific types of (conditional) deductive confirmation, we do not need to generalize them.

make (objective) sense[23], the qualitative consequences are justified in general, since they are not laden with artificial probabilities. In other words, the justification of the qualitative explication is at least as strong as the justification of the quantitative explication in cases were the relevant probabilities make sense.

An interesting question is to what extent the d-theory, counting $d(H, E) > 0$ as confirmation, leads to another explication of general and non-deductive confirmation. It is almost evident that d realizes SDC, UPP, PS, PCS, and P.1G for p-normal hypotheses, that it does not leave room for confirmation of p-zero hypotheses, that is, it is non-inclusive, and finally that it is impure in the sense that it is in conflict with P.2G. More specifically, the d-theory favors more probable hypotheses among equally successful ones. In other words, the d-theory gives rise to an alternative explication of deductive, general and non-deductive confirmation. We leave the question of whether the two resulting sets of principles should be considered as expressing the 'robust qualitative features' of two different concepts of confirmation or simply as two different ways in which the intuitive concept of confirmation can be modelled as an open problem to the reader.

3. *Acceptance criteria*

Finally, we will briefly discuss the acceptance of hypotheses in the light of quantitative confirmation. The subject of probabilistic rules of acceptance has received much attention in the last decades. For a lucid survey of 'cognitive decision theory', see Festa (1999b). As Festa documents, there is a strong tendency to take 'cognitive utilities', such as information content and distance from the truth into consideration. However, in our set-up, we only need rules of acceptance in the traditional sense of rules for 'inductive jumps', that is, rules that use conditions for acceptance that may be assumed to give good reasons for believing that the hypothesis is true simpliciter. The story of theory evaluation and truth approximation, presented in (Kuipers, 2000), only presupposes such traditional rules of acceptance, in particular, for general observational hypotheses (first order jumps) and for comparative hypotheses, comparing success (comparative second order jumps) and

[23] Think of a context in which not only the evidence results from some kind of random sampling, but also the population resulted from some earlier random sampling in a larger universe, and hence the division of true and false hypotheses. The urn-model argument in favor of the P.2-feature of the r-degree of confirmation in Section 1.3 and the urn-model illustration of the superiority of new tests in Section 2.3 were of this kind.

truth approximation claims of theories (comparative referential and theoretical jumps). Hence, let us finally consider the role the degree of confirmation may play in the acceptance of hypotheses as presumably true.

Let us first note that the acceptance of H (as true) on the basis of E is a ‘non-Bayesian move’, that is, assuming that E does not verify H, i.e., $p(H/E) < 1$, acceptance of H amounts to the replacement of $p(H/E)$ by the new prior probability $p'(H) = 1$, at least for the time being. It is also plausible to think that, if $p(H) > 0$, and if there is something like a threshold for acceptance, this threshold is independent of $p(H)$. That is, there is assumed some number ϵ (<1/2), such that H is accepted when $p(H/E) > 1 - \epsilon$. As is well-known, the suggested rule immediately leads to the lottery-paradox: for a sufficiently large lottery, one may know that there is just one winning ticket, and at the same time, have to accept for each of the tickets that it is not the winning one, hence that there is no winning ticket. However, the paradox is based on a priori reasoning, hence evidence and posterior probabilities do not play a role in it. So let us assume that the Bayesian approach to confirmation can formally be combined with a sophisticated kind of non-Bayesian high probability rule of acceptance independent of the prior probabilities (see e.g. (Pollock 1990) for some interesting attempts).

The question now is, what role does the degree of confirmation play in such rules? The answer is, of course, *none*, for the degree of confirmation amounts to an expression of the increase of the probability of the hypothesis, in a pure or an impure form, and not to the resulting posterior probability, which is simply calculated by Bayes’ rule. Hence, whether we construe the degree of confirmation in one way or another, it does not matter for the acceptance of a hypothesis as ‘true simpliciter’.

To be sure, it may well be that our confirmation intuitions are laden with a mixture of the suggested two aspects, that is, the increase and the resulting probability. From the ‘pure’ point of view, we will say that a more probable hypothesis is not more confirmed by the same deductive success than a less probable one, it just gets a higher posterior probability, and hence will earlier pass the threshold for acceptance. Specifically, the conjunction intuition and the refined grue intuition are done justice by the fact that the corresponding strange hypotheses will obtain at most a small non-zero posterior probability, never high enough to pass the threshold, as long as we do not find rock-blocks of cheese on the moon or color changing emeralds.

Accordingly, the Bayesian approach to posterior probabilities guarantees that different definitions of the degree of confirmation will not lead to differences in acceptance behavior, as long as the resulting posterior probabilities are crucial for the rules of acceptance.

However, p-zero hypotheses will not get accepted in this way, since their posterior probability remains zero. So, let us see what role the r-degree of confirmation might play in acceptance rules for p-zero hypotheses. We have already remarked that, although it may make perfect sense to assign non-zero probabilities to genuine hypotheses, it nevertheless occurs that scientists would initially have assigned zero probability to certain hypotheses, of which they are nevertheless willing to say that they later have come across confirming evidence for them, and even that they have later decided to accept them. Now one may argue that this can be reconstrued in an 'as if' way in standard terms: if the scientist would have assigned at least such and such a (positive) prior value, the posterior value would have passed the threshold. To calculate this minimal prior value, both $d(H,E)$ and $r(H,E)$ would be suitable. However, only $r(H,E)$ is a degree for which this 'as if' degree would be the same as the 'original' degree, for $r(H,E)$ does not explicitly depend on $p(H)$[24]. In contrast to this feature, the original d-degree of confirmation assumes its neutral value 0.

If one follows this path, it is also plausible to look for a general acceptance criterion that does justice to the, in most cases, relative arbitrariness of the prior distribution. Let us, for that purpose, first assume that for cases of objective probability one decided to take as the acceptance threshold $1-\epsilon$, for $0<\epsilon<1/2$. One plausible criterion now seems to be the r-degree of confirmation that is required for the transition from $p(H)=\epsilon$ to $p(H/E)\geq 1-\epsilon$, that is, $r(H,E)=(1-\epsilon)/\epsilon$. The suggested criterion can be used for p-normal as well as p-zero hypotheses. However, as Jeffrey (1975) rightly remarks, most genuine scientific hypotheses not only start with very low initial probability, but will remain to have a low posterior probability. Hence, if ϵ is very small, they may not pass the threshold. However, passing the threshold is essentially independently defined from $p(H)$. For deductive confirmation, it is easily checked to amount to the condition $p(E)<\epsilon/(1-\epsilon)$. Hence, for somebody for whom $p(H)=\epsilon$, deductive success E should be almost as surprising as H itself. Whether the criterion is useful in other cases has still to be studied.

[24] Of course, $r(H,E)$ may be conceived as depending on $p(H)$, by using $p(E) = p(H)p(E/H)+p(\neg H)p(E/\neg H)$ to calculate $p(E)$. However, nothing forces us to use this particular 'decomposition' of $p(E)$. The relative independence of $r(H,E)$ from $p(H)$ may be conceived as an additional, pragmatic advantage of the r-degree: people may agree about it, without having to agree about $p(H)$.

Concluding remarks

The possibility of a quantitative, i.c. probabilistic, theory of confirmation is one thing; its status and relevance is another. Although probabilistic reasoning is certainly practiced by scientists, it is also clear that specific probabilities usually do not play a role in that reasoning. Hence, in the best instrumentalist traditions, as remarked before, the required probabilities in a quantitative account correspond, as a rule, to nothing in reality, i.e., neither in the world that is studied, nor in the head of the scientist. They simply provide a possibility of deriving the qualitative features of scientific reasoning.

If our reservations amounted to the claim that the quantitative accounts are not yet perfect and have still to be improved, it would be plausible to call them tentative explanations and even justifications of the corresponding kinds of qualitative reasoning. However, nothing of that kind seems to be the case. Hence, it remains questionable to what extent these formal accounts can be said to reveal quantitative cognitive structures that underlie scientific reasoning. The situation would change in an interesting way if the r-theory itself, or some alternative quantitative theory, could be given a justification. In particular, we do not exclude that such a justification could be given in terms of functionality for truth approximation. However, although (Kuipers, 2000) deals with truth approximation, it does not touch the problem of such a justification.

In the meantime, we may only conclude that the r-theory should primarily be conceived as a quantitative explication of a qualitative cognitive structure, to be used only for its qualitative consequences. As has been argued, the justification of these qualitative consequences is at least as good as the justification of the quantitative explication 'under ideal circumstances', that is, when the probabilities make objective sense.

Appendix 1: Corroboration as inclusive and impure confirmation

As is well-known, Popper preferred to talk about '(degree of) corroboration', instead of '(degree of) confirmation', but the question is whether his views essentially deviate from the Bayesian approach. Jeffrey (1975) argued already that this is not the case. In this appendix, we will more specifically argue that Popper's quantitative theory of corroboration amounts to an inclusive and impure Bayesian theory.

Popper's main expositions about corroboration can be found in Popper (1959, 1963, 1983), where Section 32 of Popper (1983) summarizes his main ideas. He formulates six conditions of adequacy for a quantitative degree of corroboration, here denoted by $c(H, E)$, which he conceives as the best proposal. We will first list (the core of) these conditions, in which it is important to realize that $p(\neg H)$ and $p(\neg E)$ may be conceived as measures for the (amount of) empirical content of H and E, respectively.

(i) $-1 \leq c(H,E) \leq p(\neg H) \leq 1$
(ii) $-1 = c(H\&\neg H, E) = c(H, \neg H) \leq c(H,E) \leq c(H,H) \leq p(\neg H) \leq 1$
(iii) $c(H \vee \neg H, E) = 0$
(iv) if H entails E and E^* and if $p(E) < p(E^*)$ then $c(H,E^*) < c(H,E)$
(v) if (H^* entails H such that) $0 < p(H^*) < p(H) < 1$ then $c(H,H) < c(H^*,H^*)$[25]
(vi) if (H^* entails H such that) $0 \leq p(H^*) < p(H) < 1$ and $p(E/H^*) \leq p(E/H)$ then $c(H^*,E) < c(H,E)$

The following definition is the simplest one fulfilling these six conditions Popper has found.

$$c(H,E) = \frac{p(E/H)-p(E)}{p(\neg H)p(E/H)+p(E)}$$

Note that $c(H,H) = p(\neg H)$ and that $c(H,E) = p(\neg E)/(p(\neg H) + p(E))$ in the case of 'deductive corroboration', that is, when H entails E.

Note first that $c(H,E)$ is inclusive, in the sense that it can assign substantial values when $p(H) = 0$ and $p(E/H)$ can nevertheless be interpreted. In that case, $c(H,E)$ amounts to $(p(E/H)-p(E/\neg H))/(p(E/H)+p(E/\neg H))$, which reduces to $p(\neg E/\neg H)/[1+p(E/\neg H)]$ in the deductive case. The inclusiveness of $c(H,E)$ is important in view of a specific dispute of Popper with the Bayesian approach as far as it assigns non-zero probabilities to genuine universal hypotheses. However, several authors have argued that Popper's arguments against p-normal hypotheses (Popper 1959, Appendix *vii and *viii) fail, e.g., Earman (1992, section 4.3), Howson and Urbach (1989, section 11.c) and Kuipers (1978, Ch. 6).[26]

It is easy to check that $c(H,E)$ satisfies the qualitative principle P.1 of deductive confirmation rephrased in terms of deductive (d-)corroboration:

P.1cor if E and E^* d-corroborate H then E d-corroborates H more than E^* iff E^* is more plausible than E in the light of the background beliefs

[26] Moreover, there is the interesting suggestion of Jeffrey (1975, p. 150) to assign infinitesimal numbers, developed in non-standard analysis, to p-zero hypotheses in the standard sense.

Surprisingly enough[27], it satisfies the (rephrased) impure alternative to P.2 favoring more probable hypotheses when equally successful:

P.2Icor if E d-corroborates H and H^* then E d-corroborates H more than H^* iff H is more plausible than H^* in the light of the background beliefs

To study the qualitative features of 'general' corroboration, we will look in some detail at the conditions (i)–(vi). The first three conditions deal with quantitative special values, and are mainly conventional, except that (i) and (ii) together require that, for each H, $c(H,H)$ is the maximum value, by Popper called the degree of corroborability, which should not exceed $p(\neg H)$. Although Popper restricts (iv) to d-corroboration, giving rise to P.1cor, his definition of $c(H,E)$ satisfies the generalization of P.1cor to

P.1Gcor if H makes E at least as plausible as E^* and if E is less plausible than E^* in the light of the background beliefs, then E corroborates H more than E^*

which corresponds to P.1G. Condition (v) amounts, in combination with (i) and (ii), to the idea that a less probable hypothesis should be able to get a higher maximum degree of corroboration than a more plausible one. The Bayesian measures $d(H,E)$ and $r(H,E)$ also satisfy this idea, where $d(H,E)$ has the same maximum value, viz. $d(H,H) = p(\neg H)$, while that of $r(H,E)$, $r(H,H)$, equals $1/p(H)$. Finally, condition (vi) amounts to the qualitative idea

P.2IGcor if H makes E at least as plausible as H^* and if H is more plausible than H^* in the light of the background beliefs, then E corroborates H more than H^*

Recall that the d-degree of confirmation was impure as well, more specifically, also favoring plausible hypotheses. Hence, it is no surprise that it also satisfies the analogues of (vi), P.2Icor and P.2IGcor.

The foregoing comparison suffices to support the claim that the resulting qualitative theory of Popper roughly corresponds to the impure qualitative

[27] Note that from the informal expositions of Popper one might sometimes get the idea that he is pleading for the *opposite of P.2Icor*, favoring less plausible hypotheses when equally successful, that is, the stronger hypothesis should be praised more by the corroborating evidence than the weaker one. However, he is well aware of this consequence of (vi), for he speaks (Popper 1983, p. 251) of an aspect in which degree of corroboration resembles probability. Hence, it may be assumed that Popper, at least on second thoughts, subscribed to P.2Icor.

Bayesian theory based on $d(H, E)$ for p-normal hypotheses. However, in contrast to $d(H, E)$, $c(H, E)$ is inclusive.

Appendix 2: Comparison with standard analysis of the raven paradox

As suggested in Section 2.3, there is a more or less standard Bayesian solution of the paradoxes of which Horwich (1982) may be assumed to have given the best version[28]. Hence, we will compare our solution with his. After criticizing Mackie's account (Mackie 1963) and pointing out that an unconditional approach will miss conditional connotations of the (first) raven paradox, he introduces the idea of conditional sampling, and obtains roughly the same confirmation claims as reported in (2p) and (4p). However, as we will explain, his precise quantitative versions of (2p) and (4p) are wrong, and he misses the core of (1p) and (3p). The latter shortcoming is mainly due to overlooking a plausible relation between the two relevant matrices. The former shortcoming is due to a systematic mistake in the intended calculation of r-values.

There is one difference in Horwich's approach which is not really important. He does not interpret the matrix as a survey of the sizes of the cells, but as reporting subjective probabilities of randomly selecting a member of the cells. It is easy to transform our matrix in this sense by dividing all numbers by their sum $(a + b + c + d)$, without changing any of the results.

The first serious difference is the following. We use the matrix for two purposes, with $c > 0$ for calculating $p(\mathrm{E}/\neg\mathrm{RH}\&\mathrm{C})$-values, to be called the $\neg$RH-matrix, and with $c = 0$ for calculating $p(\mathrm{E}/\mathrm{RH}\&\mathrm{C})$-values, to be called the RH-matrix. The latter provides the numerator of $r(\mathrm{RH}, \mathrm{E}; \mathrm{C}) = p(\mathrm{E}/\mathrm{RH}\&\mathrm{C})/p(\mathrm{E}/\mathrm{C})$, whereas its denominator $p(\mathrm{E}/\mathrm{C})$ is calculated by

$$\begin{aligned}(*)\ p(\mathrm{E}/\mathrm{C}) &= p(\mathrm{RH}/\mathrm{C})p(\mathrm{E}/\mathrm{RH}\&\mathrm{C}) + p(\neg\mathrm{RH}/\mathrm{C})p(\mathrm{E}/\neg\mathrm{RH}\&\mathrm{C}) \\ &= Qp(\mathrm{E}/\mathrm{RH}\&\mathrm{C}) + (1 - Q)p(\mathrm{E}/\neg\mathrm{RH}\&\mathrm{C})\end{aligned}$$

Recall that the simplification derives from the plausible assumption that $p(\mathrm{RH}/\mathrm{C}) = p(\mathrm{RH}) = Q$, that is, C is neutral evidence for H.

Horwich, instead, interprets the $a/b/c/d/$-matrix as directly prepared for $p(\mathrm{E}/\mathrm{C})$-values, to be called the HOR-matrix. Hence, he leaves open whether c is zero or positive. This is problematic, however, for some of his general conclusions (see below) only hold if one assumes that c in the HOR-matrix is positive, hence that RH is false, hence that it is in fact the $\neg$RH-matrix. As a consequence, though he intends to directly base the $p(\mathrm{E}/\mathrm{C})$-value occurring

[28] He does not explicitly deal with the second paradox, but implicitly the situation is clear.

in $r(\mathrm{RH}, \mathrm{E}; \mathrm{C})$ on the HOR-matrix, he bases it in fact on the ¬RH-matrix. However, a genuine Bayesian approach requires to calculate $p(\mathrm{E})$-values on the basis of (*), and hence on both the RH- and the ¬HR-matrix, to be called *the proper calculation*.

The second main difference is that Horwich introduces an independent RH-matrix for calculating $p(\mathrm{E}/\mathrm{HR}\&\mathrm{C})$, to be indicated as the HOR-RH-matrix, with α, β, $\gamma = 0$, δ, representing the relevant probabilities under the assumption that RH is true. Horwich waves away the idea of a relation between his two matrices. However, in his probability interpretation of the $a/b/c/d$-matrix, it seems rather plausible to take α, β, and δ proportional to a, b, and d, respectively, such that they add to 1. This corresponds to identifying the non-zero values in our RH-matrix with a, b, and d in the size interpretation of our ¬RH-matrix, as we in fact did. Why should the relative probabilities for two cells differ depending on whether a third cell is empty or non-empty? When two matrices are related in the suggested way they will be said to be *tuned*. Hence, our RH- and ¬RH-matrix are tuned and our results (1p)–(4p) directly follow from the proper calculation presupposing these tuned matrices.

Let us call (r-)values tuned when they are based on two tuned matrices, otherwise they are called untuned. Due to the improper calculation, Horwich calculates in fact untuned values for the ratio $p(\mathrm{E}/\mathrm{HR}\&\mathrm{C})/p(\mathrm{E}/\neg\mathrm{HR}\&\mathrm{C})$, to be called the r_0-value (with index 0, for it corresponds to the r-ratio when $p(\mathrm{RH}) = Q = 0$), instead of the intended untuned r_+-values, $p(\mathrm{E}/\mathrm{RH}\&\mathrm{C})/p(\mathrm{E}/\mathrm{C})$, based on some $Q > 0$. Of course, when the calculated values are tuned one gets tuned r_0-values instead of tuned r_+-values.

Now we can sum up Horwich's deviations from his intended proper Bayesian treatment of the raven paradoxes. By the improper calculation, Horwich got the wrong r-values of (2p) and (4p); he got in fact the r_0-values, $r_0(\mathrm{RH}, \mathrm{BR}; \mathrm{R}) = (a + c)/a$ and $r_0(\mathrm{RH}, \overline{\mathrm{B}}\overline{\mathrm{R}}; \overline{\mathrm{B}}) = (c + d)/d$, which he did not intend, for he apparently assumes throughout that Q is non-zero. Note that the intended qualitative results of (2p) and (4p), i.e., (2) and (4) of Section 2.1 of (Kuipers, 1998), only follow when c is positive, for if $c = 0$, both r_0-values are 1. Hence, contrary to his suggestion of using the HOR-matrix (in which c may or may not be 0), Horwich uses in fact the ¬RH-matrix (with $c > 0$) for the (improper) calculation of $p(\mathrm{E}/\mathrm{C})$.

Moreover, by not assuming tuned matrices, he missed the results (1p) and (3p). Regarding (1p), he does not even calculate the values for unconditional confirmation corresponding to the uniform one we obtained, viz., $(a + b + c + d)/(a + b + Qc + d)$. The improper calculation would have given, assuming the size-interpretation of all numbers, $\alpha\mathrm{F}/a$, $\beta\mathrm{F}/b$, and $\delta\mathrm{F}/d$, with $F = (a+b+c+d)/(\alpha+\beta+\delta)$, for a black raven, a non-black non-raven and a black non-raven, respectively. If these values are not tuned they differ from each other. If they are tuned, they assume a uniform value, but the wrong one, viz., the corresponding r_0-value $(a + b + c + d)/(a + b + d)$. Finally, regarding (3p), Horwich does calculate the (r_0-)values for sampling non-ravens and black objects corresponding to the uniform one we obtained, viz., 1. If these values are not tuned, they differ for all four possible results, a non-raven that is black or non-black, and a black object that is a raven or a non-raven. However, if they are tuned, this gives, more or less by accident, the right uniform value 1, reported in (3p), for the r_0- and the r_+-value for neutral confirmation are both equal to 1.

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