

PARACONSISTENT LOGIC IN A HISTORICAL PERSPECTIVE¹

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Let us learn from the lessons of history. Let us grant to those who work in any special field of investigation the freedom to use any form of expression which seems useful to them; the work in the field will sooner or later lead to the elimination of those forms which have no useful function. *Let us be cautious in making assertions and critical in examining them, but tolerant in permitting linguistic forms.*

R. Carnap [1950], p. 221.

Introduction

In classical logic, from a contradiction one can deduce anything; hence, the despair it engenders. Contradictions trivialise; one has, thus, to banish them. It is natural then to put the question whether it is possible to develop a logic in which contradictions can be mastered, in which there are inoffensive or, at least, not dangerous contradictions. The creation of paraconsistent logic by the first author of the present paper (da Costa), more than thirty years ago, brought an affirmative answer to this question. We shall retrace here the history of this invention that has contributed to the subversion of the usual conception of logic.

1. Gestation: 1910-1963

The period of gestation of paraconsistent logic began in 1910 A.D., although we do not exactly know the epoch of fecundating, that might perhaps be traced as far back as the fourth century B.C. - or so Łukasiewicz believed. The latter published in 1910 his celebrated work on the principle of contradiction in Aristotle (see Łukasiewicz [1910]). According to Łukasiewicz, Aristotle himself did not fundamentally believe in the absolute value of the principle of contradiction; moreover, Łukasiewicz's de-

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tailed analysis of the arguments formulated by Aristotle in order to 'justify' the principle of contradiction shows that they are unsatisfactory.

By the same time, Vasiliev had visualised the creation of an imaginary or non-Aristotelian logic, in analogy to Lobatchevski's imaginary or non-Euclidean geometry (see Vasiliev [1910]). Later, the Polish logician Jaśkowski developed his *discussive logic*, intending to take into account contradictory debates (see Jaśkowski [1948]).³

Boole intended to describe mathematically the *laws of thought*. His revolutionary approach would turn against himself: the mathematisation of such laws would put into doubt their very existence. Just as Euclidean geometry does not describe the laws of absolute space, Boolean logic does not spell out the laws of absolute thought.

The mathematisation of logic has led to its *relativisation*. In the first half of the present century, several systems were devised, the majority of which has a negation that does not satisfy the so-called laws of thought. This could be done by making explicit and decomposing this celebrated law, the principle of contradiction.

The analysis of this principle is articulated within several systems (deduction systems, theory of matrices, algebras) that seem to be more fundamental than the principle itself. On the other hand, such a law is no longer a basic reference, for one studies, in particular, systems without negation.

If, by that time, the relativity of the contradiction principle had been clearly acknowledged, a truly paraconsistent logic was not constructed.

³ As far as we understand, paraconsistency as such is not simply an incidental feature of a system, but a property resulting from the deliberation of dealing with inconsistencies avoiding trivialisation. This seems to us to be an important point that, if disregarded, may lead to grave historical distortions. For instance, it is for this reason, as opposed to certain writers, that we do not take Orlov even as a forerunner of paraconsistent logic, although he built a relevant system that turned out to be paraconsistent (for references and details, see da Costa, Béziau and Bueno [1995], p. 598, note 1).

Similarly, from our viewpoint, Vasiliev did not discover paraconsistent logic as such. He definitely was a *forerunner* of this kind of logic, given his explicit concern with dealing with certain kinds of inconsistencies in a certain way (see Arruda [1977], [1980], and [1989], pp. 102-103). But, of course, there still remains a great distance from this to the proper discovery of paraconsistent logic. (Vasiliev's work was not really understood until the seventies, when the first author read an abstract of a paper of his written in English, and perceived that he had the intuition of paraconsistent logic. Then he suggested that one of his students, A.I. Arruda, investigate Vasiliev's works.) On the other hand, Jaśkowski has not pconstructed any discussive logic at the quantificational level. This was done by L. Dubikajtis and the first author in the sixties.

Others forerunners of paraconsistent logic are David Nelson (see Nelson [1959]) and T.J. Smiley (see Smiley [1959]).

Several reasons could be presented for that; in particular, an ideological one.

Contradictions were celebrated by Marx, Hegel and Heraclitus. These authors not only intended that contradictions actually exist, but claimed that they are the essence of everything. However, their ideas were not presented in a formal way. One wonders whether this would be possible. Fifty years ago, on the one hand, Soviet thinkers used to consider the new logic as a grave mistake of the bourgeoisie, incapable of taking into account the fundamentally 'dialectical' nature of reality; on the other hand, someone like Popper did not hesitate in claiming, in the name of logic, that dialectic was nothing but a misconceived enterprise, given that, from his viewpoint, one could not represent dialectics, according to logic (see Popper [1940] and Popper [1943]).

The devising of paraconsistent logic would end the ideological debate, showing that, to some extent, neither of the sides was entirely correct, as usually happens in this kind of debate. This logic was thus not necessarily welcomed. As one of the present authors remarked a long time ago, paraconsistent logic has contributed to the demystification of contradictions (see da Costa [1980]), assigning to them a human face. They are no longer the god of the Bolshevik, nor the devil of the 'open society'.

2. *Birth and infancy*: 1963-1976

The birth of paraconsistent logic corresponds to its appearance, strictly speaking, as a theory, i.e., as a mathematical theory, studied in itself in a systematic way, and scientifically acknowledged. It is thus that the first author, after writing a thesis on paraconsistent logic (see da Costa [1963]), published, from 1963 onward, with the help of Marcel Guillaume and other open minded French mathematicians, a series of notes in *Comptes Rendus de l'Académie des Sciences de Paris*. (The first note was da Costa [1963b]; the reference to other notes can be found, for instance, in D'Ottaviano [1990].)

The first author has formulated several paraconsistent logics, but some of his works have focused on the propositional logic C_1 and its extensions to predicate logic, as well as, based on the latter, the formulation of a 'higher

logic', i.e., a paraconsistent set theory.⁴

The C_1 system was first presented by the first author as a Hilbert type system. From the outset, several questions can be formulated: Is it decidable? What kind of semantics can one add to it? Is it algebraisable? Could it be formulated as a natural deduction system or a sequent calculus?

It was only very slowly that such questions have been answered. During the first decade, few results were obtained; the field was yet in a period of tentative works. The first author proposed an algebraic counterpart of his systems and developed, thus, what he called 'Curry's algebras' (see da Costa [1966]). However, despite his interest in these systems, they do not supply a clear answer to the issue of the algebraisation of paraconsistent logic. In fact, one can doubt whether algebras having non-monotonic operators truly deserve such a name (algebra).

Another considerable difficulty concerns the development of a paraconsistent set theory. At first glance, the interest of such a theory consists in putting forward an abstraction principle that does not require any amputation, a principle that states that, for every predicate, there exists a set which is its extension. One knows that such a principle leads to Russell paradox, which means that it is inconsistent. One might then perhaps wonder whether a paraconsistent logic could be employed in order to save the furniture. Unfortunately, Curry-Moh Shaw-Kwey's paradox shows that this principle is trivial in a logic with a fairly weak implication and without negation (see Arruda and da Costa [1966]).

The hopes of employing a paraconsistent logic along these lines were not, however, completely useless. On the one hand, one can develop a logic with an implication that does not lead to Curry's paradox; Arruda in particular has worked in such a direction (cf., for example, Arruda and da Costa [1974]). On the other hand, one can work with a theory such as Zermelo-Fraenkel or Quine's with a Russell set (see da Costa and Béziau [1995], da Costa and Bueno [1995], da Costa, Béziau and Bueno [1997]).

Throughout this first period, paraconsistent logic seemed a monstrous abnormality, and one wondered whether it would survive. Nonetheless, it was directly related to fundamental problems connected with the very nature of logic. And those that sweep them away, employing Quine's (and others') arguments to the effect that one was not here concerned with logic,

⁴ From our point of view, a system of logic, to really constitute a logic, has to be developed at least to encompass quantification and equality, given the role of logic in the articulation of conceptual systems. And, to this extent, the first author seems to have been the first logician to have done so with paraconsistent logic. His research in this field, begun in 1954, was developed in a completely independent way from the works of Vasiliev, Łukasiewicz and Jaśkowski. At that time, in Brazil, the works of these logicians were inaccessible to him.

refused to reflect on the very challenge brought by the development of non-classical logics and, in particular, of paraconsistent logic: 'What is logic?'.

2.1. *Interlude: on da Costa's paraconsistent systems*

According to some writers, the first author has not spelled out the 'philosophical' rationale for including in his **C**-systems (presented, for instance, in da Costa [1974]), the constraint that this logic should not contain $\neg(\alpha \wedge \neg\alpha)$ as a logical truth. On this regard, we wish to point out that, from our viewpoint, when presenting a formal system, one does not need to be concerned with the formulation of philosophical rationales for the mathematical constraints introduced. One should not confuse the mathematical development of logical systems with their philosophical interpretation: these are, in fact, quite distinct issues. After all, we do not live any longer in Frege's period, in which metaphysical and logical issues were always intertwined. If there are those who do wish to intertwine them, of course, they are free to do so. However, a criticism based on such a decision, addressed to someone who does not intend to conflate these kinds of problems, misses the mark of course.

Nonetheless, even if this were not the case, the first author clearly pointed out, from a *mathematical* perspective, such a rationale: there are mathematical reasons, related to the construction of the systems, to demarcate between well-behaved and not well-behaved propositions, and that is why the constraint on rejecting $\neg(\alpha \wedge \neg\alpha)$ as a logical truth was advanced (see da Costa [1974]). Moreover, as opposed to any particular philosophical concerns, the main consideration underlying such a proposal consisted in presenting a logical framework in which the presence of contradictions does not lead to trivialisation, meeting thus, initially at least, a mathematical (not a philosophical) demand.

These, however, are by no means the only kind of systems formulated by the first author. He has developed several paraconsistent systems other than those referred to above; for instance, the first two absolutely relevant logics, founded on the propositional calculi **P** and **P***, or on a similar system, and not on **R**. (Arguing as Priest [1987] does, we could try to show that **P** or **P*** is the 'true' relevant logic, and not **R**!) Moreover, in Arruda and da Costa [1970] some paraconsistent logical systems completely different from the **C_n** logics are presented.

It has also been showed that there are infinitely many paraconsistent systems of set theory, most of them stronger than the classical ones; for instance, stronger than **ZF**. In this sense, paraconsistent logic is not weaker than classical logic, but stronger. Those systems, in addition, were introduced by means of rigorous axiomatisations (and formalisations). It was

even showed that these inconsistent systems are nontrivial if, and only if, the classical systems are not inconsistent (see da Costa [1986]).

One of the reasons behind the construction of paraconsistent set theories was the need for a semantics for paraconsistent systems; indeed, to some extent, only such set theories could naturally function as a basis for these semantical theories (as opposed to classical set theory, whose use in this context would clearly be philosophically untenable!). Though there is, in a certain sense, no semantics for a paraconsistent logic without a paraconsistent set theory, we do not know, leaving aside the group of our close collaborators, any logician that tried to proceed in this way.

In this regard, we wish to point out that if the first author's own works on paraconsistent logic were first presented in terms of axiom systems, this was so exactly because by that time (1954) there was no paraconsistent set theory in which to formulate appropriately a semantics for this logic. So, in our view, in this case there is no real privilege of proof theory over semantics; but only this contingency in the development of paraconsistent logic, and its associated philosophical requirement.⁵ Another reason for the construction of paraconsistent set theories was to settle the foundations for a paraconsistent mathematics - an important point if one intends to apply adequately and in detail the paraconsistent framework.

Thus, given such a pluralism, it should be clear that we could not accept the claim that the C-systems are the only game in town: the kind of constraints formulated, as opposed to Priest's suggestion (see Priest [1996], p. 5), are not *a priori*; indeed, such an issue is not even considered as long as matters of application are not addressed. So, to a certain extent, there seems to be here a confusion between pure logic and applied logic; in the latter, though not necessarily in the former, *a priori* matters might not be the most important aspects of the questions being examined (see da Costa and Bueno [1995], and da Costa and Bueno [1996a]). Moreover, such systems are not thought of as capturing the *true nature* of the world, nor of logic, of logicity or whatever. In the first instance, they were just devised with the aim of putting forward a particular logical system meeting certain theoretical constraints. Could such constraints be changed? Of course, as in fact they have already been with the formulation of alternative systems - several of them, for instance those concerned with issues from artificial intelligence (as we shall see), clearly received their motivation from the exigencies of application.

Given such a plurality of logics, each of them with their associated domain, from our viewpoint, it could hardly be argued that there exists a 'true' one - nor does such a move even seem necessary. We do not consider

⁵ For further comments on semantics, see da Costa, Bueno and Béziau [1995].

the systems based on the propositional calculi C_n as the 'true' paraconsistent logics. In fact, we do not think that there are 'true' logics (see, for instance, da Costa and Bueno [1996a]); in our opinion, a given system may be applicable under certain conditions (owing in part to pragmatic and heuristic factors), but not under others. For instance, classical logic, just as paraconsistent logic for that matter, has its domains of application, and just in these domains we can say whether it is adequate or not. Moreover, we can think of each logic as being, as it were, an element of a covering of a topological manifold 'representing' our knowledge. However, as far as we know, it is still an open question whether such a covering contains a finite subcovering, and in particular, a unique one (that is, to pursue the analogy still further, whether in particular the topological manifold is 'compact').

Similar remarks might be presented to cope with an objection that Priest [1996] presents to the first author's constraints on his C -systems. Indeed, according to da Costa, these logics should contain as much of classical, or intuitionistic, logic as does not interfere with its paraconsistent nature. However, from Priest's viewpoint, such a condition is too strong, for 'it assumes that a paraconsistent logician must have no objection to other aspects of classical or intuitionistic logic, and this is clearly not true. For example, a relevant logician might well object to paradoxes of implication, such as $\alpha \rightarrow (\beta \rightarrow \alpha)$ ' (p. 5). Now, it seems to us, this would be the case only if the C -systems were thought of as the only kind of paraconsistent logics at the disposal of the paraconsistent logician, as somehow capturing the true nature of paraconsistency, so that either the paraconsistent logician inevitably accepts the main features of these systems, or rejects altogether the paraconsistency project. Nevertheless, as we have just remarked, we are in total disagreement with the underlying essentialist approach to logic (even to paraconsistent ones!) assumed in such a move. As we noted, there are several alternative paraconsistent logics, quite distinct from the C -systems, some of them for instance of a relevant nature. To this extent the relevant logician might adopt those (paraconsistent) logics that are most adequate for his or her needs.

As a matter of fact, paraphrasing Cantor, from the viewpoint of the pure development of a formal system, the freedom is complete - if one is not satisfied with certain imposed constraints, it is simple: just change them! But, and we stress once again this point, a critique of the constraints (that were eventually adopted for the construction of certain paraconsistent logic) in terms of essentialist assumptions, is far from being adequate, given that it depends on a particular view of logic whose basic features are not accepted.

3. *Baptism and adolescence*: 1976-1991

Thirteen years after being born, paraconsistent logic still had not got an appropriate name. The expression 'inconsistent formal system', employed up to that time, was ambiguous and unsatisfactory, leaving the impression that, in such logics, one could derive a proposition and its negation (these are called today strongly paraconsistent).

Being aware of the difficulty, the first author requested the help of a friend, the Peruvian philosopher Francisco Miró Quesada Cantuarias. Later, he described the event in these terms:

Several years ago, I needed a convenient and meaningful denomination for a logic that did not eliminate contradictions from the outset as being false, i.e., as absolutely unacceptable. Miró Quesada helped me.

On the one hand, it should be recalled that, by that time, all logics unavoidably condemned contradictions. The new logic in which I worked faced too much resistance, it was badly divulged, and those that got to know it were in general sceptics.

By that time I wrote to Miró Quesada, who saw the new logic with great enthusiasm, requesting a name for it. I remember as it was today that he answered with three proposals: it could be called metaconsistent, ultraconsistent or paraconsistent. After commenting on these possible denominations, he stated that, from his viewpoint, he preferred the latter. The term paraconsistent sounded splendid and I begun to use it, suggesting that people interested on this logic did the same.

Two or three months later, the miracle took place; the term spread through the world, all the centres directly or indirectly related to logic, from northern to southern hemisphere, begun to employ it. I believe that few times in the history of science (definitely in the history of logic) something similar has happened, for not only the word run the whole world, but the very logic called by Miró Quesada 'paraconsistent' received a formidable push. It became one of the most discussed theories of logic of our time. (da Costa [1992])

Quesada officially proposed the term in the Third Latin American Congress of Mathematical Logic, held in Campinas (Brazil) in 1976.

The Greek prefix 'para' means 'by the side of'; the term 'paraconsistent logic' conveys thus the idea that this logic is complementary, rather than rival, to classical logic. For this reason, the defenders of contradiction took this term as being too weak (see, for instance, Asenjo [1991]) and advanced other names, *dialetheic* logic - not *dialectical* logic (Priest/Routley) -, that studies true contradictions, or transconsistent logic (Priest), that supplies a conceptual surplus similar to that of the transfinite (cf. Priest [1987]).

Although the term paraconsistent logic has imposed itself due to its conciliatory character, which definitely reflects its neutrality, and allows a liberal view of contradiction, paraconsistent logic can also be adopted by those that believe that the world is actually contradictory, or by those only concerned with yielding contradictory bits of information, independently of any ontological assumption. Thus, and perhaps due to its *passe partout* name, paraconsistent logic knew a new flight with its applications to computer science in the 80's. In fact, some paraconsistent logics were conceived especially to handle certain problems of artificial intelligence (see da Costa and Subrahmanian [1989], and da Costa, Henschen, Lu and Subrahmanian [1992]).

Some authors claim that 'one can [...] subscribe to the use of paraconsistent logics in some contexts without believing that inconsistent information or theories may be true' (Priest [1996], p. 7). Given our agnosticism with regard to the existence of true contradictions (see da Costa and Bueno [1996a]), we wonder in what contexts (if any) one would have to believe that inconsistent theories are true in order to use a paraconsistent logic. Indeed, these are quite dissimilar issues: the application of paraconsistent logic, on the one hand, and the philosophical interpretation of it, on the other. In order to apply such a logic, one is not required to take sides at the philosophical level, but roughly speaking, to examine the particular features both of the domain of application and of the logic to be adopted in order to determine whether (and to what extent) they 'match', as it were - but no further commitment to the existence of true contradictions seems to be in order here. Of course, if one wishes to carry additional metaphysical baggage on one's trip, there is no problem; but there are those who would rather travel lighter...

In fact, from our viewpoint, it might be interesting to advance a comprehensive agnosticism in connection with some issues raised by paraconsistency. In particular, as opposed to Priest's view (see Priest [1987]), it could be argued that no commitment to the existence of true contradictions is necessary in order to present a reasonable philosophical perspective of paraconsistent logic. Moreover, one should not forget that, at the moment, it is by no means established that true contradictions in fact exist, or even that, with further conceptual developments, classical logic could not handle them (indicating how they could be dissolved). Nonetheless, and we stress this point, even if one shows that classical logic could deal appropriately with such contradictions, this of course would not have established its truth. (How could such a truth be ascertained anyway?) By the same token, despite being successful in its treatment of certain inconsistencies, the truth of paraconsistency could not possibly be fixed in such a way. We might thus be eventually led to the view that the notion of truth (at least in its strongest

senses) may not be so crucial for our understanding of logic as has been usually supposed.

Nonetheless, at the 'mathematical' level, we can of course develop strong paraconsistent systems. At this level there are 'true' contradictions, as opposed to real or empirical ones (the actual world would be contradictory).

Anyway, after its baptism it truly began to be developed, that is, the first important results appeared. M. Fidel proved the decidability of C_1 by algebraic methods (see Fidel [1977]); there is also the construction of a semantics for C_1 , a valuation semantics, based on truth and falsity, but which is not truth-functional (cf. da Costa and Alves [1976]). In the beginning of the 80's, C. Mortensen proved that C_1 cannot be algebraisable (see Mortensen [1980]). The second author, from an aborted approach from A.R. Raggio (see Raggio [1968]), developed a system of sequents for C_1 and proved the cut elimination for these system (see Béziau [1990a]).

The increasing number of works in paraconsistent logics reached a remarkable point with the publication, in 1982, of a special issue of *Studia Logica* (no. 43), entirely dedicated to this logic, and finally, with the first monograph (actually, a collection of papers), edited by Priest, Routley and Norman in 1989: *Paraconsistent Logic: Essays on the Inconsistent*, published by Philosophia at an unbelievably expensive price.

4. *The age of reason*: 1991

In 1991, fifteen years after its baptism and twenty-eight after its birth, paraconsistent logic was accepted in the category of theories admitted by the mathematicians: a special section is created for it in *Mathematical Reviews*. This is something that cannot be disregarded; it means that paraconsistent logic is a research domain whose importance is acknowledged by the mathematicians, despite ideological divergences. It does not mean, nonetheless, that the matter is established, but on the contrary, that it is an open domain and can receive interesting elaborations.

It is worth remarking that although paraconsistent logic is mathematically acknowledged, it still persists as a 'dark shadow' for certain philosophers, who do not hesitate in employing the most old-fashioned arguments in order to deny its existence (as is the case of Slater [1995]; see the second author's review of this paper in *Mathematical Reviews*).

Paradoxically, one of the open problems is the definition of paraconsistent logic. The first author's original definition is indeed approximate or, at least, unilateral; it states that a one-place operator η is paraconsistent if there is a formula a such that the theory $\{a, \eta a\}$ is non-trivial or, in terms of valuations, such that a and ηa are simultaneously true. Of course, this condition is not enough to make such an operator a *negation*. This problem

is approached by putting forward the constraint that such an operator η should have all the properties of classical negation compatible with this condition.

The difficulties with such a move are double-headed: on the one hand, until now no one knows what are the properties compatible with such an exigency and, on the other hand, this exigency has to be strengthened, as Urbas has clearly pointed out, introducing the notion of strict paraconsistency: one cannot deduce from the theory $\{a, \neg a\}$ any formula of the form $\neg b$ (as in Johansson's minimal logic, that no one wishes to consider as a paraconsistent logic) or of any other non-tautological form (see Urbas [1990]).

For the future of paraconsistent logic, one can outline the following research lines:

- *To strengthen the logic C_1* . The logic C_1^+ , as put forward by the second author, supplies an answer: this is a stronger logic than C_1 , in which one can define a congruence relation (see Béziau [1990b]).
- *To develop a paraconsistent model theory*. One difficulty is that if the replacement theorem is not valid, two isomorphic structures are not necessarily equivalent; the very notion of structure has to be reconsidered (see da Costa, Béziau and Bueno [1997]).
- *To develop a paraconsistent set theory* (see da Costa, Béziau and Bueno [1997]).
- *To develop a paraconsistent mathematics* (see da Costa [1989], da Costa [1996b], Mortensen [1995], and, for a discussion of the latter, da Costa and Bueno [1996b]).
- *To develop further applications of paraconsistent logic* to computer science, artificial intelligence, law, everyday life and so on; see the applications of the Toulouse's group (in Carnielli, del Cerro and Marques [19??]), and the examples examined by Subrahmanian (for instance, in da Costa and Subrahmanian [1989]) and by the second author (in Béziau [1996]).

5. From paraconsistent logic to universal logic

If the question 'What is logic?' is formulated today, the answer is quite different from that which could be presented a century ago.

Logic was once considered as the study of the laws of thought. The mathematical study of logic has shown that there are not more laws of thought than laws of algebra. And just as the creation of ever more bizarre algebras, such as non-commutative ones, has led to the development of universal algebra, the creation of ever more weird logics, such as paracon-

sistent ones, has contributed to the development of a universal logic (cf. Béziau [1995]).

The first author himself has worked in that direction, developing a valuation theory, a general semantics that can be used to supply paraconsistent logic with a semantics (see da Costa and Béziau [1994], and da Costa and Béziau [1996]). Similarly, in the 20's, Polish logicians, following Łukasiewicz's interest on polyvalent logics, developed matrix theory (see Łoś [1949]).

One notes here an *ascent towards abstraction*, typical of mathematics and even of human intelligence. Slowly, one gets rid of the superfluous, focusing on the essential, isolating it from the concrete form in which it initially appears. Paraconsistent logic shows thus that one should demarcate between the trivial and the inconsistent, and that the notion of triviality is more basic. By the same token, one distinguishes between the notion of implication and of deduction, stressing the priority of the latter. In this way, the fundamental concepts of logic are set free in an ever clearer way, in order to eventually appear in the brightness of their simplicity.

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