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THE GENERAL FORM OF A PROPOSITION IN AN ESPECIALLY PERSPICUOUS LANGUAGE

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0. *Introduction*

The topic of this paper arises from Wittgenstein's *Tractatus* and his approach in it to the problem of how thought relates to the world. §1 contains a very brief sketch of Wittgenstein's perspicuous language project and four fundamental assumptions that underlie it. His ultimate aim is to describe the general form of a proposition in a perspicuous language. In §2, by generalizing one interpretation of the language Wittgenstein himself sketches out in T4.31, T4.4, T4.442, and T5.101[1], Wittgenstein's project is carried to completion for perspicuous languages which (a) make it immediately clear in every case which propositions follow from which, (b) have exactly one name per object, and (c) have exactly one propositional sign per proposition. The form the propositional signs of any such language take is described. Finally, in §3, a Tractarian semantics of states of affairs is set out for these especially perspicuous languages, even though the attempt to give a semantics that relates all propositions and the world is, as Wittgenstein observes, profoundly misdirected.

1. *Thought and the World*

Thoughts about the world and their linguistic counterparts, propositions, are like pictures in two ways:

(P1) *In having truth-values*. Pictures agree with the world or fail to do so, are correct or incorrect, true or false (T2.21).

(P2) *In representing*. Pictures of the world have ties to it that are independent of their truth or falsehood (T2.13ff, T2.17).

[1] Citations like these refer to passages in the *Tractatus*. The Pears-McGuinness translation [2] is used.

Wittgenstein's discussion of pictures is in great part an elaboration of these two ideas.

Concerning (P2), irrespective of whether a picture is correct or incorrect, true or false, parts of reality ('objects') serve as anchors to which elements of the picture are attached.[2] Cables, as it were, run from elements of the picture to anchors in reality, and the picture is moored to reality this way, indeed the *same* way, whether it agrees with reality or disagrees, is correct or incorrect, true or false.[3] Only when a picture is anchored to reality can it then be judged whether or not it correctly depicts reality. Indeed, without the cables, its connections to reality, what remains fails to *be* a picture.

Besides the 'objects' which anchor pictures, the world for Wittgenstein contains entities of one other sort. After all, since the objects pictures are anchored to are there whether the pictures portray truly or falsely, something else besides these objects must be responsible for truth and falsehood: facts. This exhausts the analogy between pictures and thoughts.[4]

Wittgenstein next draws a parallel between propositions and thoughts. The ideal is to have a perspicuous language which 'mirrors' reality, in which all true and false thoughts about the world can be expressed. In such a language there will be one and only one representative, one name, for each object to which a thought is anchored (T5.53). Furthermore, in a perspicuous language the difference between names and propositional signs will be clear (T3.143), there will be no ambiguity (T3.322-T3.325), and it will also be clear which thought is being asserted by which propositional sign (T4.002). There are further virtues a perspicuous language may have: (a) in it what is essential to a language representing thoughts will be preserved and what is merely accidental will to the greatest possible extent be pared away (T3.34, T3.341, T3.3411, T3.344, T3.3441, T4.0621, T5.254,

2 Reality must not be confused with space. No assumption is made that the 'objects' that anchor pictures are or are not spatial.

3 In *Problems of Philosophy* [1], Russell, too, held there to be 'object-like' entities basic to thoughts that subsist or exist independently of whether the thoughts are true or false. The present king of France is *not* such an entity. Sense-data, memory-data, data of reflection, facts, and universals are. The proper analysis of the thought, 'The present king of France exists (is bald),' decomposes it into sense-data, memory-data, and universals, all of which exist whether the thought is true or false. Where Russell differs from Wittgenstein is in holding that the things thoughts are about *are* the components of the thoughts while Wittgenstein holds components of thoughts *to represent* them.

4 That there are no *a priori* pictures is the last proposition under (T2), the last proposition devoted to pictures alone. (T3) comes next, and with it Wittgenstein begins his discussion of thoughts and of propositions. Tautologies and contradictions are not pictures of reality (T4.4611).

T5.44) and (b) it will be immediately clear from the propositional signs which propositions follow from which (T5.13, T5.131). In the following, T4.2, T4.442, and T5.101 are taken to furnish a basis for a perspicuous language that has all of these virtues —a language, indeed, in which there is exactly one way to express each thought about the world.[5]

Wittgenstein makes one further assumption:

> (P3) *Analyzability*. Each proposition in a perspicuous language must be analyzable into picture-like elementary propositions which (a) consist of representations of objects (names) and (b) are true or false independently of each other (T4.221, T4.211, T4.25, T4.26).

The demand in (P3) that there be elementary propositions which are true or false independently of each other, or to put it semantically, that there be independent states of affairs, is the most controversial of the three touched upon thus far. Wittgenstein reads as if he thinks it obvious that reduction to independent states of affairs, to elementary propositions, is possible (T2.061, T2.062, T4.221, T4.2211, T4.411, T5.5562). In any event, (P1), (P2), and (P3) serve as the basis of Wittgenstein's highly abstract analysis of the relation between thought and reality.

One final point should be mentioned:

> (P4) *Limits*. My thoughts about the world are *all* I have that is true or false; I cannot escape outside thought to think about the relation between it and reality (T5.556-T5.5562).

Humans may in fact have thoughts that are in some sense infinite —thoughts with visual parts, for example— but they do not have an infinite hierarchy of thoughts about the relation of thoughts and reality, no infinity of 'higher' thoughts about the relation of 'lower' thoughts to reality. So since getting outside our thoughts to think about our thoughts' relations to reality is impossible, semantics of the normal kind is impossible for a language meant to represent *all* possible thought about the world. All we can

[5] Whether the propositional signs in T4.442 and T5.101 were intended to be taken as parts of such a language is controversial and, in my opinion, beyond resolution. And despite what Wittgenstein says in T5.1311, 'When we infer q from $p \vee q$ and $\sim p$ the relation between the propositional forms of '$p \vee q$' and '$\sim p$' is masked, in this case, by our mode of signifying. But if instead of '$p \vee q$' we write '$p \mid q \,.\, \mid . \, p \mid q$', and instead of '$\sim p$', '$p \mid p$'($p \mid q$ = neither p nor q) then the inner connexion becomes obvious' the use of the Sheffer stroke does not make what follows from what obvious at all. It is certain *not* immediately clear that q follows from $p \mid q \,.\, \mid . \, p \mid q$ and $p \mid p$.

do is to pay heed to what our thoughts *show* us about reality as we approach their limits from the inside.

2. *Especially Perspicuous Languages*

Let us say that an *especially perspicuous* language is one which has, among other desiderata, exactly one name for each object in reality, exactly one propositional sign for each proposition about reality, and a structure that makes it immediately clear which propositions follow from which. The key to providing the general form a proposition for such a language lies in:

> T4.4 A proposition is an expression of agreement and disagreement with truth-possibilities of elementary propositions.

T4.31 provides an illustration of truth-possibilities by showing the truth-possibilities of two 'elementary propositions' :

p	q
T	T
F	T
T	F
F	F

In T4.442, then, a proposition, an expression of agreement and disagreement with truth-possibilities of two such elementary propositions, is represented by the propositional sign:

'p	q	'
T	T	A
F	T	A
T	F	
F	F	A

or better:

'p	q	'
T	T	A
F	T	A
T	F	D
F	F	A

In these, instead of Wittgenstein's '*T*'s and '*F*'s in the last column, I have used '*A*' to signify agreement and '*D*' disagreement, thus distinguishing agreement and disagreement from truth possibilities.

It is customary among those of us who have taken and taught beginning logic courses to think of truth-tables as pertaining to semantics. The sentences of the languages we are used to are the '*p*'s, '*q*'s, '$p \supset q$'s etc., that appear on the top lines of truth tables. Not so here. Wittgenstein calls the truth-table in T4.442 a 'propositional sign' and, indeed, puts the whole thing in quotation marks. The attachment of '*T*'s and '*F*'s to '*p*'s and '*q*'s and '*A*'s and '*D*'s to various lines representing truth-possibilities is part of the *syntax* of a perspicuous language of propositions as they are characterized in T4.4. Wittgenstein then writes this propositional sign in abbreviated form as '(*TTFT*)(*p*,*q*)'[6] and goes on to use this abbreviated notation in T5.101 to list all sixteen propositional signs of the form '$(X_1X_2X_3X_4)(p,q)$'. Following this, in T5.11-T5.131, he proceeds to explain which propositions follow from which and how the fact that one proposition follows from a number of propositions can be seen from the very structure of the propositions themselves —which in *this* notation is definitely true.

Notice that Wittgenstein's explanation of 'following from' is very clear when read in the context of the sixteen propositions of T5.101. But if the language is taken to contain not only these sixteen propositional signs but 'p' and

'p	'
T	A
F	D

as well, the explanation in T5.11-T5.131 ceases to be so simple and clear. What exactly are the sentences of the language Wittgenstein is describing

6 '(AADA)(*p*,*q*)'.

in T4.442 and T5.101? Does this language also contain as propositional signs

'p	'
T	*A*
F	*D*

and 'p'? Is the result of substituting the propositional sign

'p	'
T	*A*
F	*D*

for 'p' in

'p	*q*	'
T	*T*	*A*
F	*T*	*A*
T	*F*	*D*
F	*F*	*A*

a propositional sign as well?

I do not find definitive answers to these questions in the *Tractatus*. Wittgenstein does state that he uses the letters '*p*', '*q*', '*r*' to indicate elementary propositions (T4.24). So it would seem that we are entitled to conclude that the '*p*' and '*q*' in the propositional sign in T4.442 are themselves propositional signs. Yet '*p*'s and '*q*'s appear not only in each of the sixteen propositional signs in T5.101 but also in the explanations in English and in Russellian notation corresponding to them —save in the two cases: (*ADAD*)(*p*,*q*) and (*AADD*)(*p*,*q*), where '*p*' is the sole entry after the first and '*q*' after the second. Then, too, there are many, many occurrences of '*p*'s and '*q*'s prior to and after T4.24 which are definitely not meant to stand for elementary propositions. I just do not know what to make of all this.

I propose to take the language of T4.442 and T5.101 as containing the sixteen propositional signs listed in T5.101 *and no more*, so that there is exactly one propositional sign for each proposition. In this notation one propositional sign, for example the one in T4.442, takes the place of an infinite host of symbols that are equivalent to each other in standard logical notation: '$p \supset q$', '$\sim p \vee q$', '$\sim(\sim p \wedge \sim q)$', '$(p \vee \sim p) \wedge (p \supset q)$', '$(q \vee \sim q) \wedge (p \supset q)$', '$(p \wedge \sim p) \vee (p \supset q)$', '$\sim\sim(p \supset q)$', '$\sim\sim\sim\sim\sim(p \wedge \sim q)$', '$p|p.|q:|:p|p.|q$', '$p|p.|q:|:p|p.|q.:|:.p|p.|q:|:p|p.|q::|::p|p.|q:|:p|p.|q.:|:.p|p.|q:|:p|p.|q$', etc. And as T5.101-T5.121 attest, it *is* perfectly clear in the proposed notation which propositions follow from which. Moreover, in this notation, internal relations like *following from,* as well as the various internal relations 'represented' in other, more familiar notations by propositional connectives, are also clear. In joint denial, for instance, that a particular propositional sign has a '*D*' when and only when all the members of a set of propositional signs have '*A*' is plain from the signs themselves.

The propositional signs

'*p*	*q*	'
T	*T*	*A*
F	*T*	*D*
T	*F*	*A*
F	*F*	*D*

and

'*p*	*q*	'
T	*T*	*A*
F	*T*	*A*
T	*F*	*D*
F	*F*	*D*

represent the language's two elementary propositions. This Wittgenstein makes clear in T5.101. The '*p*' and '*q*' within them also, of course, represent elementary propositions but '*p*' and '*q*' are not themselves well-formed propositional signs *in the language I am now describing*. They are abbreviations for different well-formed arrays of names (T4.22, T4.221, T4.23) which are not propositional signs in this language. A '*T*' under '*p*' will say that the objects named in array '*p*' are linked, concatenated, an '*F*' under it that they are not. Ditto for '*q*'. The need for '*A*'s and '*D*'s becomes clear when one thinks of the many different things about linkages and non-linkages that may be asserted or denied. It is easy to see how the whole of logical space for a world having two elementary propositions is given in each one of the sixteen propositional signs (T3.42).

Since this analysis is, like Wittgenstein's, meant to be completely general, we need not address questions of detail, questions of application (T5.557) like that of whether there can be two or more well-formed arrays of the exact same names (whether there can be more than one possible linkage of the same set of objects).

The language we have been considering presupposes an ontology with just two possible linkages among the objects there are. To arrive at the general form of a proposition in this notation —however many objects there may be to be named and however many possible linkages of them there may be (T4.5, T5.47, T5.471, T5.472, T5.512)— all that is needed is a generalization of the example Wittgenstein sets out in T4.442.[7]

This generalization presupposes a rudimentary notion of propositional complexes and their composition. Propositions do have components, perhaps denumerably many, perhaps even non-denumerably many. No assumption is made in the following about how many. Let us use '$x(y)$' to say that y is a component of x. '$x+y$' will be used to denote a complex with exactly two components, x and y. For our purposes, there need be no distinction between $x+y$ and $y+x$. That transitivity holds for componenthood is doubtful. If $x(y)$ and $y(z)$, why should $x(z)$? Reflexivity $[x(x)]$ definitely does not hold (T3.332).

We are now ready to describe in a general way a whole family of especially perspicuous languages. Each such language is determined by a non-empty set, N, of well-formed arrays of names. Given N the set P^* of *truth-possibilities* is easily definable:

$$P^* = \{Y{:}(\forall X \in N)[(Y(T+X) \vee Y(F+X)) \wedge \sim(Y(T+X) \wedge Y(F+X))] \wedge (\forall Z)[Y(Z) \supset (\exists X \in N)(Z = T+X \vee Z = F+X)]\}.$$

In words, P^* is the set of truth possibilities. Each truth possibility Y contains as its only components assignments of exactly one of 'T' or 'F' to each well-formed array of names X in N i.e., T+X or F+X.

Given P^* the set P of *propositions* can then be defined:

$$P = \{Y{:}(\forall X \in P^*)[(Y(X+A) \vee Y(X+D)) \wedge \sim(Y(X+A) \wedge Y(X+D))] \wedge (\forall Z)[Y(Z) \supset (\exists X \in P^*)(Z = X+A \vee Z = X+D)]\}.$$

In words, all the propositions Y in P have as their only components assignments of exactly one of 'A' (agreement) or 'D' (disagreement) to each truth possibility X in P^* i.e. X+A or X+D. This definition characterizes all the propositions of the language determined by N for every set N of well-formed arrays of names.

[7] The characterization that follows is of the second type that Wittgenstein lists in T5.501 —"giving a function *fx* whose values for all values of *x* are the propositions to be described". In T6 Wittgenstein himself attempts to characterize the general form of a proposition in the third way by means of the 'N' operator by "giving a formal law that governs the construction of the propositions" ultimately from elementary propositions.

In the simplest case where $N=\{p\}$, $P^*=\{T+p,\ F+p\}$ and $P=\{((T+p)+A)+((F+p)+A),\ ((T+P)+D)+((F+p)+A),\ ((T+p)+A)+((F+p)+D),\ ((T+p)+D)+((F+p)+D)\}$.

For each language in this family, one proposition in it, call it T, agrees with all truth possibilities, i.e. $(\forall X\in P^*)T(X+A)$, and one proposition, call it F, disagrees with all truth possibilities, i.e., $(\forall X\in P^*)F(X+D)$. The former will always be true, the latter always false.

Unlike the familiar languages of logic texts, it is also easy to tell in this kind of language when one proposition follows from others. Where K is a set of propositions, let $\tau(K)=\{X+A\!:X\in P^*\wedge(\forall Y\in K)Y(X+A)\}$. $\tau(K)$ is what Wittgenstein would call the set of truth-grounds (agreement-grounds), truth possibilities that received 'A' as their assignment) common to all the members of K (T5.101). A proposition B, then *follows from* K $(K\vdash B)$ if $\tau(K)\subseteq\tau(\{B\})$, i.e., if its agreement-grounds contain those common to the members of K.

Also, in these languages there is always a class of propositions that can fairly be called 'world propositions', propositions each one of which completely describes a possible world. Although unremarked by Wittgenstein, this class is easily defined. B is a world proposition, $W(B)$, if $B\in\{X\in P\!:\ (\exists!Y\in P^*)X(Y+A)\}$, i.e., if it contains exactly one component with an 'A' in it. A world proposition agrees with one and only one truth possibility.

Finally, the characterization of such especially perspicuous languages can easily be generalized to remove the assumption that elementary propositions have to be independent. A language can be taken to be a pair <N, P^{**}>, where N is (as before) a non-empty set of arrays of names and $\wedge\subset P^{**}\subseteq P^*$. The cases in which $P^{**}\subset P^*$ and both the sets N and P^* are finite are those in which certain truth possibilities (those in P^*-P^{**}) are deleted —certain horizontal lines in the truth table notation of T4.442 are omitted. The set of propositions P is now defined using P^{**} instead of P^*:

$$P=\{Y\!:(\forall X\in P^{**})[(Y(X+A)\vee Y(X+D))\wedge\sim(Y(X+A)\wedge Y(X+D))]\\ \wedge(\forall Z)[Y(Z)\supset(\exists X\in P^{**})(Z=X+A\vee Z=X+D)]\}.$$

3. *States of Affairs Semantics*

A *logical space* for a perspicuous language determined by N is a pair <O, R>. O is a set of arrays of 'objects' isomorphic to N and R is a one to one function from N onto O that pairs members of N with their isomorphs in O. Given O, the set W of possible worlds can be defined:

$$W = \{Y: (\forall X \in O)((LX \in Y \vee UX \in Y) \wedge \sim (LX \in Y \wedge UX \in Y))\}.$$

'LX' is to be read 'the elements of X are linked' and 'UX' 'the elements of X are unlinked'. X is a state of affairs when LX, a negative fact when UX (T2.01, T2.03, T2.06). A possible world is determined by the set of possible arrays of the objects there are and by the determination concerning each such array that it is linked or unlinked.

$<O, R, Aw>$ is a model for N if $<O, R>$ is a logical space, and $Aw \in W$. Aw is called 'the actual world' of the model $<O, R, Aw>$. Aw determines which states of affairs exist and which do not exist (T2.04 T2.05). A model is a logical space together with a determination of which possible world is the actual one.

Let $T^* \in P^*$ be such that for all $p \in N$ and $o \in O$, if Rpo, then $T^*(T+p)$ iff $Lo \in Aw$ and $T^*(F+p)$ iff $Uo \in Aw$. T^* is the truth possibility in the language determined by N that represents Aw. In the finite case it is the line of the truth table that represents the actual world's existing and not existing states of affairs.

Finally, for all $p \in P$, p is *true* on $<O, R, Aw>$ if $p(T^*+A)$ and p is *false* on $<O, R, Aw>$ if $p(T^*+D)$. True propositions agree with the truth possibility that represents the actual world's existing and not existing states of affairs; false ones disagree.

For languages $<N, P^{**}>$ in which independence is not assumed, the relation R will be extended. For all $X \in P^*$ and $w \in W$, RXw iff for all $p \in N$ and $o \in O$ such that Rpo, (1) $X(T+p)$ iff $Lo \in w$ and (2) $X(F+p)$ iff $Uo \in w$. The set of worlds for the logical space $<O, R>$ for $<N, P^{**}>$ is then:

$$W^{**} = \{X\text{: there is some } Y \in P^{**} \text{ such that } RYX\}.$$

The actual world Aw of models will be taken from W^{**} instead of W.

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