

THE DEONTIC BRANCHING TIME: TWO RELATED CONCEPTIONS

Patrice BAILHACHE

1. Introduction.

Deontic logic is concerned with time. It has been often remarked that, for example, obligations have sense only when they look *future actions*¹. In [Bailhache 1991] was presented a system, called RS5-DS5, which mixed alethic, deontic and temporal modalities. In fact, this system stems from [Bailhache 1983]² and was conceived independently from other works on the same issue, using a branching time due to A.N. Prior³. But among the other attempts to capture the future character, [Åqvist & Hoepelman 1981] is of a particular interest. There are obvious similarities between Åqvist's conception and mine, there are also important differences which render the comparison between the two systems fruitful. More precisely, before making this comparison, I was inclined to think that the concept of *conditional obligation* was not primitive, being analysable in terms of pure obligation, necessity and time. In this article I would like to show how the problem seems to me rather questionable.

2. Main semantic ingredients of RS5-DS5⁴

Operators of the system are alethic modalities (necessity and possibility, \Box and \Diamond), temporal modalities under the form of the operator R_t = it is realized at time t that, and deontic modalities (obligation and permission, O and P).

¹ [Åqvist & Hoepelman 1981], p. 190. Also [Bailhache 1991], p. 8, etc.

² [Bailhache 1983] was published only as copies sold by the University of Nantes in 1986 and is still available under this form.

³ [Prior 1967].

⁴ I shall not present the axiomatic counterpart of the system, which is not indispensable here. Only some axioms or theorems will be mentioned if necessary.

The semantics handles paths which was noted ρ_i, ρ_j, \dots and which I prefer to write now simply as w for reasons of convenience. Here, such paths will be named also *worlds*. They should be conceived as sets of successive states of a world, so that

$$w = \{w(t): \text{for every real } t\}$$

Two triadic relations, R and S , over the set of real numbers (time) and $W \times W$ (W being the set of worlds) allow to express the alethic and deontic properties with respect to time⁵.

$Rtww'$ can be interpreted as *the world w' is accessible to the world w from the time t* , which means also that the paths w and w' are confused at least up to the instant t . In other words, $w(t') = w'(t')$ for every $t' \leq t$.

Similarly, $Stww'$ can be read as *the world w' is permissible for the world w with respect to the time t* , which means that the paths w and w' are confused at least up to the instant t , and $w'(t')$ is "good" for every $t' > t$. Of course the goodness under consideration is rather formal, being able to capture morality, law, technical norms and so on. This is a well-known feature in deontic logic.

Both relations should verify certain properties in order to express the structure of onticity and deonticity with respect to time.

First R is a ramified equivalence relation:

- reflexivity: $\forall w Rtww$ ⁶
- symmetry: $\forall w \forall w' (Rtww' \Rightarrow Rtw'w)$
- transitivity: $\forall w \forall w' \forall w'' [(Rtww' \& Rtw'w'') \Rightarrow Rtww'']$
- ramification: $\forall t \forall t' \forall w \forall w' [(t' < t \& Rtww') \Rightarrow Rt'ww']$

All these properties are obvious⁷.

Then S is serial, post-reflexive and post-ramified:

⁵ For convenience we make use of the same letter, R , to denote the triadic alethic relation and the realization operator.

⁶ Quantifiers in shadowed characters are metalinguistic.

⁷ See [Bailhache 1991], p. 72 for comments.

- seriality: $\forall t \forall w \exists w' Stww'$
- post - reflexivity: $\forall t \forall w \forall w' (Stww' \Rightarrow Stw'w')$
- post - ramification:

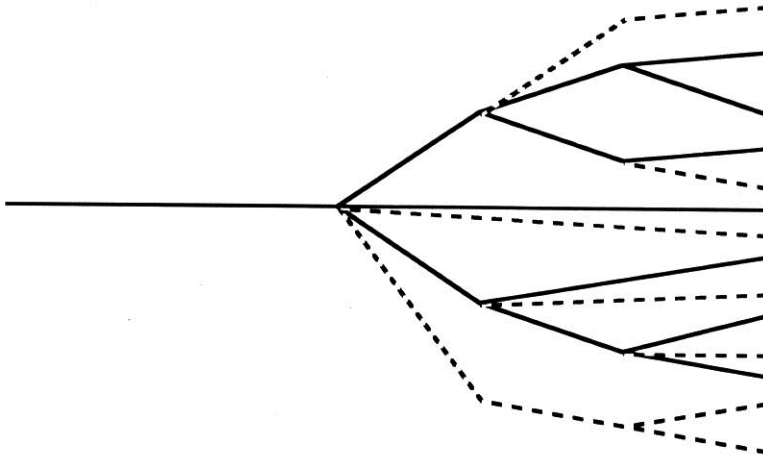
$$\forall t \forall t' \forall w \forall w' \forall w'' [(t' \leq t \& St'ww' \& Stw'w'') \Rightarrow St'w'w'']$$

In addition, there are three properties common to *R* and *S*:

- SR - implication: $\forall t \forall w \forall w' (Stww' \Rightarrow Rtww')$
- RS - transitivity: $\forall t \forall w \forall w' \forall w'' [(Rtww' \& Stw'w'') \Rightarrow Stww'']^8$
- RS - post - implication:

$$\forall t \forall t' \forall w \forall w' \forall w'' [(t' < t \& St'ww' \& Rtw'w'') \Rightarrow Stw'w'']$$

These semantic properties, apparently complicated, correspond to a very natural structure, easily expressed with a graph. Paths which are only alethic alternative of one another appear as branches of a tree; “good” alternative paths are branches of a special kind (in broken lines):



The RS-post-implication graphically implies that a “good” path cannot lead, towards the future, to a not “good” path. This feature is essential.

⁸ This property has been forgotten in [Bailhache 1991], p. 80. But it is really necessary, as explained in [Bailhache 1983], p. 384 and 759. RS-transitivity is bound to the axiom of RS5-DS5, $Op \rightarrow \Box Op$ ([Bailhache 1991], p. 81).

Similarly, one would think that the post-symmetry of *S* or a property like as RS-symmetry must be added to the semantics of RS5-DS5, but it can be proved that the post-symmetry of *S* is a consequence of the symmetry of *R* ([Bailhache 1983], p. 766).

Permissible paths are sets of permissible states of worlds all of which must be used as models for action in any future time. Therefore, the structure cannot admit permissible paths, good just after the point of divergence, which would become ordinary, i.e. not good, later. With close attention one realizes that if it was the case it would be justified not to consider the very beginning of such a path as good. "Goodness" is not evaluated in a unique instant nor in a limited duration. An isolated individual, of course, can have a deontically varying behaviour which, however, provides no absolute indication for his intrinsic value (if it is permitted to speak of such a notion at all). Neither a permissible path represents a model of action *for an isolated individual*. On the contrary, it represents a model for a world treated as a whole⁹.

Valuations in RS5-DS5 are rather obvious. A formula should be evaluated on a path at some time. $\models_{w,t} A$ means that A is true on w at t (in some model). If it is true in every model for every path and at every time, then it is valid. There are the rules of valuation for *realization at t*, *necessity* and *obligation*:

$$\begin{aligned}\models_{w,t} R_t A &\Leftrightarrow \models_{w,t'} A \\ \models_{w,t} \Box A &\Leftrightarrow \forall w' \in W (Rtw' \Rightarrow \models_{w',t} A) \\ \models_{w,t} OA &\Leftrightarrow \forall w' \in W (Stww' \Rightarrow \models_{w',t} A)\end{aligned}$$

With this system it becomes possible to cancel most of traditional deontic paradoxes. For example, the formula of the so-called paradox of *derived obligation*,

$$O \neg p \rightarrow O(p \rightarrow q),$$

is deducible and valid in every standard deontic system, although it expresses the irrational fact that if something is forbidden ($O \neg p$), then it is obligatory that if this thing is performed (p) then everything (q) can be performed. But taking into account the time in this puzzle, a correct analysis drops the paradox. Let us consider e.g. the temporally pure propositions:

p_i = Peter steals this wallet
 q_i = Peter murders William,

⁹ For more details see [Bailhache 1991], p. 78.

and take the three dates t'' , t' and t following one another in the given order, t' representing the date of Peter's stealing, and t that of his murder. One can easily prove in RS5-DS5 that

$$R_{t''}OR_{t'}\neg p_i \rightarrow R_{t''}O(R_{t'}p_i \rightarrow R_tq_i)$$

is valid. If at the date t'' the state of affairs $R_{t'}\neg p_i$ is obligatory, then *at the same date* t'' the state of affairs $R_{t'}p_i \rightarrow R_tq_i$, i.e. $R_{t'}\neg p_i \vee R_tq_i$ also belongs to the set of true propositions of the permissible paths branched on the real path at the date. But, in fact, this does not correspond to the paradoxical interpretation of the first formula, in which there is implicitly introduced an obligation which holds at the moment where the initial interdiction is transgressed. In other words, the implication:

$$R_{t''}OR_{t'}\neg p_i \rightarrow R_{t'}O(R_{t'}p_i \rightarrow R_tq_i)$$

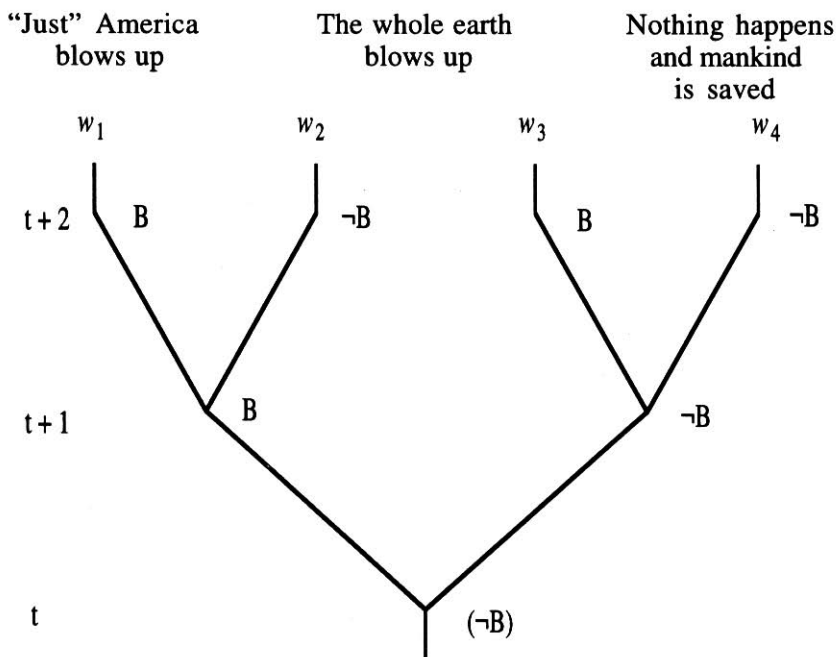
is expected to be valid, with $R_{t'}$ in front of its consequent instead of $R_{t''}$. But it is not at all the case, as easily proved by a model where two distinct permissible paths are branched on the real path at the instants, t'' and t' . Briefly, confusing the dates at which norms hold makes the paradox possible, and put again in proper temporal order the mistake vanishes¹⁰.

The more complex well-known "*contrary-to-duty imperatives*" paradox, due to Chisholm, can be solved in a similar way. This paradox seems to me important because it reflects some features of deontic or legal conducts. I shall now present it in a version given by [Åqvist & Hoepelman 1981], which will constitute an introduction to their system DARB.

¹⁰ As a matter of fact according to the intended interpretations the paradox can result from still other mistakes. But that connected with time is perhaps the most usual and the most important.

3. Chisholm's paradox in the version of [Åqvist & Hoepelman 1981]

The paradox is built on the following diagram which illustrates the possible evolution of world according as John presses a certain button of a "devilish" machine :



B = John presses the button of the devilish machine

p = B at $t+1$

q = B at $t+2$

Assuming that John presses the button at time $t+1$, the following four propositions are intuitively true *at time t*:

1. $O\neg p$
2. p
3. $p \rightarrow Oq$
4. $O(\neg p \rightarrow \neg q)$

1 is justified because at time t the best evolution is that of w_4 , which implies $\neg B$ at time $t+1$, i.e. $\neg p$. 2 is a fact. 3 describes what John has to

do at time $t+2$ if he has not chosen the best path at time $t+1$ (he has to press button at time $t+2$ so that "just" America blows up, not the whole earth). Similarly 4 describes what he has to do if he has chosen the good path at time $t+1$.

Note that 4 cannot be written as $\neg p \rightarrow O\neg q$, like 3, for this proposition is a trivial consequence of p already assumed in 2¹¹.

From 2 and 3 derives Oq by modus ponens; from 1 and 4 derives $O\neg q$ by axioms of standard deontic logic. The couple Oq , $O\neg q$ is in contradiction with the axiom $Op \rightarrow \neg O\neg p$ of standard deontic logic.

Introducing time into the formulation of the puzzle, one can immediately check that contradiction disappears. Consider the future operator:

\oplus = it will be the case at the next instant that

The four propositions above can now be written:

1. $O\oplus\neg B$
2. $\oplus B$
3. $\oplus B \rightarrow \oplus O\oplus B$
4. $O(\oplus\neg B \rightarrow \oplus\oplus\neg B)$

from which no contradiction are derivable. The best proof of this is that the model corresponding to the diagram validates each of the formulae. However, there is another important result: the dissymmetry between the two alternative formulations of *commitment*, $p \rightarrow Oq$ and $O(p \rightarrow q)$, can be cancelled in Åqvist's system DARB, to the benefit of a *conditional obligation*, $O(q/p)$, which is not reducible to usual obligation. After having presented DARB briefly (only in its semantic counterpart) and compared it with RS5-DS5, I shall return to this issue and give details on what I mean by *reduction* here.

4. Åqvist's system DARB: some semantic features

This system handles a discrete time illustrated by two primitive operators, \oplus and \ominus ¹² and uses a "prohairetic" (preference) selection operator, Δ , which is the true source of deonticity. ΔA should be understood as "what is good in A " or "the best of A ". There are also alethic modal operators of necessity and possibility, \Box and \Diamond . Åqvist reads

¹¹ For more details, see [Åqvist & Hoepelman 1981], p. 191-192.

¹² For \oplus see above. \ominus should be read "it was the case at the last instant that".

them "it is necessary (possible) on the basis of the past that". Other operators can be considered as defined from \oplus , \ominus , Δ and \Box . Particularly, *conditional obligation* and (*absolute*) *obligation*:

$$O(A/B) = \Box(\Delta B \rightarrow A)$$

$$OA = O(A/T), \text{ where } T \text{ is the true proposition}$$

Semantically, models of DARB (i.e. valuations in deontically enriched trees based on integral temporal frames) resemble to those of RS5-DS5 very much.

— The notion of path is entirely similar, except that Åqvist prefers to call a path a *world*.

— The alethic relation $Rtw w'$ is noted $w \approx_t w'$ in DARB¹³. Two worlds such that $w \approx_t w'$ are said to have the same history up to t ($w(t') = w'(t')$ for every $t' \leq t$, as in RS5-DS5).

— There is no deontic relation S but a *choice function*, $\text{opt}()$, from the power set of W (the set of worlds or paths) into itself, whose approximate reading is:

$\text{opt}(X)$ = the set of the "good" paths of X

In order to capture the features of *choice*, this function must obey the following conditions for any $X, Y \in W$:

- (a) $\text{opt}(X) \subseteq X$
- (b) $\text{opt}(X) = \emptyset \Rightarrow X = \emptyset$
- (c) $\text{opt}(X) \cap Y \subseteq \text{opt}(X \cap Y)$
- (d) $\text{opt}(X) \cap Y \neq \emptyset \Rightarrow \text{opt}(X \cap Y) \subseteq \text{opt}(X) \cap Y$

There are valuations of main operators:

$$\models_{w,t} \oplus A \Leftrightarrow \models_{w,t+1} A$$

$$\models_{w,t} \Box A \Leftrightarrow \forall w' \in W (w \approx_t w' \Rightarrow \models_{w',t} A)$$

$$\models_{w,t} \Delta A \Leftrightarrow w \in \text{opt}(\|A\|_{w,t})$$

$$\text{where } \|A\|_{w,t} = \left\{ w' \in W : w \approx_t w' \ \& \models_{w',t} A \right\}$$

= the set of paths of the same history as w at t and where A is true at t .

¹³ Semantic properties of \approx are differently presented in [Åqvist & Hoepelman 1981], but are equivalent to those of R .

$$\begin{aligned}
& \models_{w,t} O(A/B) \Leftrightarrow \text{opt}(\|B\|_{w,t}) \subseteq \|A\|_{w,t}^{14} \\
& \models_{w,t} OA \Leftrightarrow ([w]_{\approx} \subseteq \|A\|_{w,t} \\
& \quad \text{with } [w]_{\approx} = \{w' \in W : w \approx w'\} \\
& \quad = \text{the set of paths of the same history as } w \text{ at } t
\end{aligned}$$

Here I shall not dwell on the axiomatic counterpart of DARB. For certain reasons Åqvist is not sure that his axiomatics is complete¹⁵. I explain below why I think so, but for other reasons.

5. DARB examined from the point of view of RS5-DS5: similarities and differences.

To begin with, let us reformulate the rule of valuation for obligation in DARB. The inclusion $\text{opt}([w]_{\approx}) \subseteq \|A\|_{w,t}$ can be developed as¹⁶:

$$\begin{aligned}
& w' \in \text{opt}([w]_{\approx}) \Rightarrow w' \in \|A\|_{w,t} \\
& \text{which is} \\
& w' \in \text{opt}([w]_{\approx}) \Rightarrow w' \in \{w'' \in W : w \approx w'' \& \models_{w'',t} A\}
\end{aligned}$$

In this inference the part of the consequence $w' \in \{w'' \in W : w \approx w''\}$, i.e. $w \approx w'$, is trivial since $[w]_{\approx}$ contains exactly paths w' such that $w \approx w'$ and $\text{opt}([w]_{\approx})$ is a subset of $[w]_{\approx}$. Thus the "vital" part of the inference is

$$\begin{aligned}
& w' \in \text{opt}([w]_{\approx}) \Rightarrow w' \in \{w'' \in W : \models_{w'',t} A\} \\
& \text{that is, simplifying the consequent:} \\
& w' \in \text{opt}([w]_{\approx}) \Rightarrow \models_{w',t} A
\end{aligned}$$

So, the rule of valuation for obligation can be formulated:

¹⁴ Of course, this rule of valuation, as the next one, can be deduced from the rules of valuation for implication (not given here), necessity and prohaireticity (Δ), using the definition of conditional obligation in terms of these operators ($O(A/B) = \Box(\Delta B \rightarrow A)$)

¹⁵ [Åqvist & Hoepelman 1981], p. 200.

¹⁶ To simplify notations, we drop the metalinguistic quantifier $\forall w'$ in front of the inference, here and in the following lines.

$$\models_{w,t} OA \Leftrightarrow \left(w' \in \text{opt}([w] \approx) \Rightarrow \models_{w',t} A \right)$$

But if we compare this formulation with the corresponding rule in RS5-DS5,

$$\models_{w,t} OA \Leftrightarrow \forall w' \in W (Stww' \Rightarrow \models_{w',t} A),$$

we reach the conclusion that *Stww'* of RS5-DS5 must be interpreted as $w' \in \text{opt}([w] \approx)$ of DARB and vice-versa. This decision will ensure us that properties of obligation in the two systems are most likely similar since their rules of valuation are identified.

At present, a convenient way to compare DARB and RS5-DS5 consists in checking the semantic properties of *R*, *S* and *R-S* of RS5-DS5 in DARB, interpreting *Rtww'* as $w \approx w'$ and *Stww'* as $w' \in \text{opt}([w] \approx)$. We are going to prove that all the properties are verified except RS-post-implication.

Checking the properties of the relation *R* is a trivial matter which is left to the reader. Thus let us examine the remaining properties, of *S* and *R-S*.

Seriality $[\forall t \forall w \exists w' Stww']$

According to (b) of *opt* ():

$$\text{opt}(X) = \emptyset \Rightarrow X = \emptyset$$

$[w] \approx \neq \emptyset$, since $[w] \approx$ contains at least w . Therefore $\text{opt}([w] \approx) \neq \emptyset$
and

$$\forall w \exists w' w' \in \text{opt}([w] \approx)$$

Q.E.D.

Post-reflexivity $[\forall t \forall w \forall w' (Stww' \Rightarrow Stw' w')]$

If $w' \in \text{opt}([w] \approx)$, then $w \approx_t w'$, i.e. $[w] \approx = [w'] \approx$

and $w' \in \text{opt}([w'] \approx)$

Q.E.D.

Post-ramification

$$\{ \forall t \forall t' \forall w \forall w' \forall w'' [(t' \leq t \& St' ww' \& Stw' w'') \Rightarrow St' w' w''] \}$$

Omitting quantifiers, the formula to be proved can be written

$$(t' \leq t \& w' \in \text{opt}([w] \approx_t) \& w'' \in \text{opt}([w'] \approx_{t'})) \Rightarrow w'' \in \text{opt}([w'] \approx_t).$$

From

$$w' \approx_t w'' \Rightarrow w' \approx_{t'} w''^{17}$$

immediately derives

$$[w'] \approx_t \subseteq [w'] \approx_{t'}.$$

More difficult is to prove that $\text{opt}([w'] \approx_t) \subseteq \text{opt}([w'] \approx_{t'})$.

In general

$$X \subseteq Y \Rightarrow \text{opt}(X) \subseteq \text{opt}(Y)$$

is false. But in the present case $X = [w'] \approx_t$, $Y = [w'] \approx_{t'}$ and $w' \in \text{opt}([w] \approx_t)$ (second hypothesis of the formula to be proved).

if $X \subseteq Y$, then $Y \cap X = X$. Thus:

$$\text{opt}(X) = \text{opt}(Y \cap X) \subseteq \text{opt}(Y) \cap X \subseteq \text{opt}(Y)$$

The first inclusion is justified by the property (d) of $\text{opt}()$ provided that $\text{opt}(Y) \cap X \neq \emptyset$, i.e. $\text{opt}([w'] \approx_{t'}) \cap [w'] \approx_t \neq \emptyset$. I prove this:

$$\begin{aligned} w' \in \text{opt}([w] \approx_t) &\Rightarrow w' \approx_t w, \text{ i.e. } [w'] \approx_t = [w'] \approx_t, \text{ and} \\ w' \in \text{opt}([w] \approx_t) &\Leftrightarrow w' \in \text{opt}([w'] \approx_t). \text{ Thus } w' \in \text{opt}([w'] \approx_t). \\ \text{Now, } w' \in [w'] \approx_t &\text{ (trivial).} \\ \text{Finally, } w' \in \text{opt}([w'] \approx_t) \cap [w'] \approx_t &\neq \emptyset. \end{aligned}$$

Thus it has been proved that $\text{opt}([w'] \approx_t) \subseteq \text{opt}([w'] \approx_{t'})$, i.e.

$$w'' \in \text{opt}([w'] \approx_t) \Rightarrow w'' \in \text{opt}([w'] \approx_{t'}),$$

¹⁷ Cf. condition (c) of [Åqvist & Hoepelman 1981], p. 196.

provided that $t' \leq t$ and $w' \in \text{opt}([w] \approx)$.

Q.E.D.¹⁸

SR-implication $[\forall t \forall w \forall w' (Stww' \Rightarrow Rtww')]$

According to (a) of $\text{opt}()$:

$$\text{opt}(X) \subseteq X.$$

Therefore

$$w' \in \text{opt}([w] \approx) \Rightarrow w' \in [w] \approx, \text{ i.e. } w' \approx w.$$

Q.E.D.

RS-transitivity $\{\forall t \forall w \forall w' \forall w'' [(Rtww' \& Stw'w'') \Rightarrow Stww'']\}$

Omitting quantifiers, the formula amounts to

$$[w \approx w' \& w'' \in \text{opt}([w'] \approx)] \Rightarrow w'' \in \text{opt}([w] \approx).$$

$$w \approx w' \text{ is } [w] \approx = [w'] \approx. \text{ Therefore, if } w'' \in \text{opt}([w'] \approx)$$

$$\text{then } w'' \in \text{opt}([w] \approx).$$

Q.E.D.

RS-post-implication

$$\{\forall t \forall t' \forall w \forall w' \forall w'' [(t' < t \& St'ww' \& Rtw'w'') \Rightarrow Stw'w'']\}$$

Omitting quantifiers, the formula can be written

$$(t' < t \& w' \in \text{opt}([w] \approx) \& w'' \approx w') \Rightarrow w'' \in \text{opt}([w'] \approx).$$

It is not verified in the semantics of DARB. Consider the model already exhibited for the contrary-to-duty imperatives paradox (Section 3) and take t' (of our formula) = t (of the schedule) = 1 (it is the first moment when the four paths are confused), t (of our formula) = $t + 1$ (of the schedule) = 2; then:

$$(1 < 2 \& w_4 \in \text{opt}([w_1] \approx) \& w_3 \approx w_4) \Rightarrow w_3 \in \text{opt}([w_4] \approx).$$

The antecedent is true, the consequent false (w_4 being "better" than w_3), which proves that RS-post-implication does not obtain.

¹⁸ This can be illustrated by a model similar to the previous one for the contrary-to-duty imperatives paradox, but where w_3 is as good as w_4 at the first instant, i.e.

$$\{w_3, w_4\} = \text{opt}([w_i] \approx) \text{ Then:}$$

$$(w_3 \in \text{opt}([w_2] \approx) \& w_4 \in \text{opt}([w_3] \approx)) \Rightarrow w_4 \in \text{opt}([w_3] \approx).$$

Remark 1

Note that the fact is obvious from a graphical point of view. *RS*-post-implication expresses that every alternative future branch of a good branch is itself good. But this is not true in *DARB*: in the previous schedule, between the first and the second instants the right branch is "good", at least better than the left one. However, after the second instant this branch may lead to the worst state of affairs (the whole earth blows up). Only the right part of the branch leads to the best state of affairs (nothing happens). In *DARB* a relatively good branch may lead to a not good future branch. This is precluded in *RS5-DS5*, where a good state of a world should be "really" good; it should not contain the possibility of a "decadence", it must be used as models for action in any future time. This is the essential difference between the semantics of *DARB* and *RS5-DS5*.

Remark 2

In [Bailhache 1983] it has been proved that post-ramification corresponds to the axiom

$$t' < t \Rightarrow R_t O(R_t O R_{t'} p \rightarrow R_t O R_{t'} p)$$

In *DARB*, which has no continuous time as *RS5-DS5*, this axiom can be expressed by the formula

$$O(O \oplus A \rightarrow \oplus OA).$$

One can easily prove it by means of a model and *reductio ad absurdum*. Let 1 and 2 be two successive instants. The proof is straightforward:

$O(O \oplus A \rightarrow \oplus OA)$ false in $w(1)$ (assumption)

Therefore there is at least one w' such that $w' \in \text{opt}([w] \approx)$ and $O \oplus A \rightarrow \oplus OA$ false in $w'(1)$.

(i) Thus $\oplus OA$ is false in $w'(1)$, i.e. OA false in $w'(2)$.

Therefore there is at least one w'' such that $w'' \in \text{opt}([w'] \approx)$ and A false in $w''(2)$.

(ii) Also $O \oplus A$ is true in $w'(1)$.

However, by a proof similar to that used above to establish the post-ramification:

$$w'' \in \text{opt}([w'] \approx) \Rightarrow w'' \in \text{opt}([w'] \approx)$$

And therefore $\oplus A$ is true in $w''(1)$, i.e. A true in $w''(2)$.

Contradiction.

Q.E.D.

In the study of 1981, Åqvist & Hoepelman present their axiomatics only as "an attempted axiomatic formulation"¹⁹, confessing that the system is not proved to be semantically complete, nor entirely free from redundancies. I surmise that at least the valid formula $O(O\oplus A \rightarrow \oplus OA)$ is not provable in their system, which would establish the non-completeness²⁰.

6. Conclusion: DARB and RS5-DS5 examined from the point of view of their normative expressive power

At first sight, both conceptions of DARB and RS5-DS5 have good reasons to capture fundamental deontic intuitions. In particular, the presence of time in the systems makes possible to express conditional norms. For example, at a certain time 1 it is obligatory that if at another later time 2 the state of affairs B is realized, then the state of affairs A is realized at a third more later time 3. The formalization in DARB raises no question since, in a way, the system was build to this end:

$$O(\oplus\oplus A / \oplus B)$$

Thus, without entering into details here, we could make a systematic use of conditional obligation $O(/)$. For example, Chisholm's paradox could be formulated

- 1 $O\oplus\neg B$
- 2 $\oplus B$
- 3c $\oplus O(\oplus B / B)$
- 4c $O(\oplus\oplus\neg B / \oplus\neg B)$

¹⁹ [Åqvist & Hoepelman 1981], p. 200

²⁰ The formula can be written in primitive terms:

$$\Box[\Delta T \rightarrow (\Box(\Delta T \rightarrow \oplus A) \rightarrow \oplus\Box(\Delta T \rightarrow A))]$$

If it is a theorem, the formula without the main necessity operator is also a theorem:

$$\Delta T \rightarrow (\Box(\Delta T \rightarrow \oplus A) \rightarrow \oplus\Box(\Delta T \rightarrow A)).$$

To prove this implication one has to use the axiom A33 $\Box\oplus A \rightarrow \oplus\Box A$, which suggests that one starts by proving

$$\Delta T \rightarrow (\Box(\Delta T \rightarrow \oplus A) \rightarrow \Box\oplus(\Delta T \rightarrow A)).$$

But this expression is not valid.

Since the condition is not future²¹ in 3c, it is equivalent to

$$3 \quad \oplus B \rightarrow \oplus O \oplus B$$

which was adopted above. As for 4c, impossible to be reduced to a formula of the structure of 3 or 4, we ask that (i) it is not a trivial consequence of 2 and (ii) its conjunction with 1 implies the consequence $O \oplus \oplus \neg B$. Both these conditions can be proved to be met²².

²¹ See [Åqvist & Hoepelman 1981], p. 217, theorem T48, also [Åqvist 1990], p. 134-135.

²² Proof of condition (i), very easy, rests on a countermodel similar to that of Chisholm's paradox (Section 3), but wherein $\neg B$ (resp. B) is true in $w_3(t+2)$ (resp. $w_4(t+2)$) instead of B (resp. $\neg B$); details are left to the reader. The condition (ii) cannot be proved by invoking that 4c amounts to 4, this equivalence being false, because 4c contains a future condition ($\oplus \neg B$). A semantic proof that

$$(O \oplus B \wedge O(\oplus \oplus A / \oplus B)) \rightarrow O \oplus \oplus A$$

is valid in DARB can be given. However, Åqvist suggests us the following simpler proof. To begin with, he shows that

$$O(A/B) \rightarrow O(B \rightarrow A)$$

is a thesis of DARB, as follows (where invoked axioms and rules come from [Åqvist & Hoepelman 1981], p. 201):

- | | |
|--|---------------------------|
| 1. $O(A/B)$ | hyp. |
| 2. $\Box(\Delta B \rightarrow A)$ | 1, A39 |
| 3. $\Box(\Delta B \leftrightarrow \Delta(T \wedge B))$ | AO, R4, A34 |
| 4. $\Box(\Delta(T \wedge B) \rightarrow A)$ | 2, 3, S5 for \Box |
| 5. $\Box((\Delta T \wedge B) \rightarrow A)$ | 4, A37 |
| 6. $\Box(\Delta T \rightarrow (B \rightarrow A))$ | 5, Propositional Calculus |
| 7. $O(B \rightarrow A)$ | 6, A41 |

Q.E.D.

Then since

$$(OB \wedge O(B \rightarrow A)) \rightarrow OA$$

is a thesis of DARB as in the standard system, by Propositional Calculus:

$$(OB \wedge O(A/B)) \rightarrow OA$$

is proved in DARB. Thus the instance

$$(O \oplus B \wedge O(\oplus \oplus A / \oplus B)) \rightarrow O \oplus \oplus A$$

is a thesis, too. Finally, by soundness of DARB ([Åqvist & Hoepelman 1981], p. 219), the formula is also valid. Q.E.D.

Note in passing that $O(A/B) \rightarrow O(B \rightarrow A)$ is a rather attractive formula from the intuitive point of view. I report discussion on it, as on conditional norms in general, in future research.

A deontic language which is rich enough should be able to "plan" deontically the future with conditional norms like $O(\oplus \oplus A / \oplus B)$. A very exciting feature of DARB is just that it permits to express forward-looking norms of this sort. Thus at the first instant of Chisholm's paradox, we have

$$4c \quad O(\oplus \oplus \neg B / \oplus \neg B)$$

and also

$$3'c \quad O(\oplus \oplus B / \oplus B),$$

an obligation which is useless at the second instant, but tells us *at the first instant* what the good course of events could be if the norm 1 *were* violated at the second instant. Because of its relative uselessness it does not appear in the paradox.

To turn now to RS5-DS5, Chisholm's paradox becomes

- 1b $R_1 O R_2 \neg B$
- 2b $R_2 B$
- 3b $R_2 B \rightarrow R_2 O R_3 B$
- 4b $R_1 O(R_2 \neg B \rightarrow R_3 \neg B)$

From 1b and 4b we conclude $R_1 O R_3 \neg B$, from 2b and 3b $R_2 O R_3 B$. These conclusions are perfectly compatible. But what about conditional obligations with future conditions? Since a genuine conditional operator $O(/)$ is not available in RS5-DS5, the sentences 4c and 3'c can be expressed only by

- 4b $R_1 O(R_2 \neg B \rightarrow R_3 \neg B)$
- 3'b $R_1 O(R_2 B \rightarrow R_3 B)$

Unfortunately, $R_1 O R_2 \neg B$ being true (1b), the second conditional obligation which can be rewritten $R_1 O(R_2 \neg B \vee R_3 B)$ is a trivial consequence of 1b. The true reason of this comes from the very conception of deontic future in RS5-DS5. John pressing the button at 2 cannot be a condition for deontic actions viewed from the instant 1 since the good path precludes it. On the contrary, it is possible in the semantic of DARB which has no absolute good paths, but only some ones better or less bad than others. Thus conditional obligations with a future condition are expressible in DARB, apparently not in RS5-DS5. Nevertheless, the semantic concep-

tion of RS5-DS5 is not absurd. Are obligations with future conditions really inexpressible in this system?

Département de Philosophie, Nantes

REFERENCES

- Åqvist L. & Hoepelman J., 1981, "Some theorems about a 'tree' system of deontic tense logic", in R. Hilpinen, *New Studies in Deontic Logic*, Reidel, pp. 187-221.
- Åqvist L., 1990, "Deontic tense logic: restricted equivalence of certain forms of conditional obligation and a solution to Chisholm's paradox", in G. Schurz & G.J.W. Dorn (ed), *Advances in Scientific Philosophy, Essays in Honour of Paul Weingartner...*, Poznan Studies in the Philosophy of the Sciences and the Humanities, Rodopi, pp. 127-141.
- Bailhache P., 1983, *Normes et modalités, essai d'analyse formelle du devoir-être*, thèse d'Etat, defended May 83, 10 at the Université de Paris IV (Sorbonne), 785 p., reprinted in *Les normes dans le temps et sur l'action*, Université de Nantes, 1986.
- Bailhache P., 1990, "Perspectives ontiques et temporelles pour la logique déontique (axiomatique, sémantique et complétude)", *Acta Universitatis Lodzensis, Folia philosophica* 7, p. 5-18
- Bailhache P., 1991, *Essai de logique déontique*, Vrin, coll. "Mathesis", Paris, 232 p.
- Prior A.N., 1967, *Past, Present and Future*, Oxford, x-217 p.