

BIVALENCE, CONTRADICTION AND THE LOGIC OF CHANGE

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ABSTRACT. Is a non-bivalent logic consistent? And if not, why not? Lately there have been authors who are quick to reply in the negative regarding the first question. And who then, in replying to the second, actually propose that it is not the fault of *presently* available non-bivalent logics, that they fail to so qualify, but that somehow *all* non-bivalent logics must eventually turn up inconsistent, this inconsistency being an implicit or even a direct consequence of having rejected the Law of the Excluded Middle. Thus in these logics the Law of Non-Contradiction is violated *because* the Law of the Excluded Middle is violated. This idea is not exactly new and can be traced back to the dialectical philosophy of G.F. Hegel, who explicitly declares that in denying the Law of the Excluded Middle one will immediately find one's self saying something self-contradictory. This, if true, would be pretty ominous for all systems of non-bivalent logic, especially those which never *intended* to meddle with the Law of Non-Contradiction.

This essay consists of a re-examination of these claims and of their rejection. Two very recent versions of them are discussed, one by Mr. Karpenko, the other by Prof. G.H. von Wright, the former only indirectly, the latter however very directly so. Prof. von Wright's arguments, purporting to prove that during a certain physical process (a transition) the Law of Non-Contradiction fails *because* the Law of the Excluded Middle also fails, are at length discussed and criticized. My starting point is that, properly speaking, one cannot *ever* say something self-contradictory in denying the Law of the Excluded Middle, for the very simple reason that, if a sentence is *neither* true *nor* false, it cannot obviously be *both* true and false.

This self-evident truth should salvage some, if not all, of the non-bivalent logics constructed or yet to be constructed. But stated thus it still lacks a definite content. To give it such a content and even to illustrate it, I turn to discuss a different model for physical change, one which can be extracted from the field of quantum mechanics, in the specific way that its principal assumptions are formulated by Prof. C.A. Hooker. Prof.

Hooker's work in question is exclusively about quantum mechanics. However, as I maintain, it can be proved to have extremely important (and happy) consequences for the very foundations of Logic. I argue that this model is the utter negation of everything which Prof. von Wright claims for his own. Even of its validity. Among its various interesting consequences is the one which inescapably leads to the conclusion that, during a quantized transition, the Law of Non-Contradiction and the Law of the Excluded Middle will become *mutually exclusive*. (Though only in such a transition). And since mutual exclusion is a symmetric relation, not only should the Law of the Excluded Middle be false when the Law of Non-Contradiction is true, in these specific circumstances, but also the converse; namely, should the Law of the Excluded Middle be false, then the Law of Non-Contradiction should *as a consequence* be true. Which is precisely what I am after.

1. *Contradiction and the Law of the Excluded Middle*

In a relatively recent publication ⁽¹⁾ a view was put forward by Prof. G.H. von Wright which allows, if not indeed encourages, the reader to interpret it as a contention that in a logic where the Law of the Excluded Middle (LEM hereafter) is no longer assumed as valid, the Law of Non-Contradiction (LNC hereafter) cannot be upheld either. The gist of that argument was that LNC is violated *because* LEM is violated. This dual and mutually dependent violation Prof. von Wright goes on to relate with Hegel's dialectical logic. (See below).

In all fairness, I think I should add that the suggested connection between the failure of LEM and that of LNC is not quite as straightforward as a careless reading of the foregoing paragraph may lead one to believe (or as a careless reading of the author's arguments might). To be precise, Prof. von Wright mentions a thing or two which might even count as a kind of reservation towards the very view which I am attributing to him.* Yet having said this, I cannot but insist that there is at least one particular section of his paper,** where to all practical purposes he iden-

⁽¹⁾ "Truth, Negation and Contradiction", *Synthese* 66 (1986), p. 1-13.

* He does say, for instance, that when a violation of LEM leads to that of LNC, the meaning of "true" also suffers a considerable shift. (Op. cit. p. 13).

** Section 10.

tifies the violation of LEM with that of LNC, as the passages to be quoted will show beyond a reasonable doubt. And even though he seems to be approaching this identification far more cautiously than Hegel himself ever did, he is equally far from being as critical as he should have been, either, towards this marriage of dialectics with three valued logic. So in a sense this paper will contain some of the criticisms which I am sure must have occurred to him but which he chose to withhold.

In the same collection an article of similar, if not indeed identical, perspective, by Mr. A.S. Karpenko may be found. Its very title is so suggestive that it practically tells the whole story in a single phrase: "Paraconsistent Structure Inside of Many-Valued Logic". ⁽²⁾ Reading through it one realizes that the marriage between non-bivalence and dialectics spoken of has several other match-makers involved in it. ⁽³⁾ The important differences between three-valued and many-valued logic notwithstanding, ⁽⁴⁾ one cannot but be struck by the thought that, when all is said and done, both approaches share a common inspirational background: the idea that in a logic where LEM no longer holds, LNC must be sacrificed also. (Though perhaps Mr. Karpenko would not endorse my particular choice of words, such as "sacrifice", in this connection given his self-avowed philosophical preferences: "Contradictions are *useful* essences" he says. ⁽⁵⁾ An idea with which Prof. von Wright also flirts: "Dialectical synthesis is a logically legitimate inference in certain cases" and "Truth-Logic provides a bridge between classical logic and that of Hegel's". ⁽⁶⁾)

Again it should be emphasized that Mr. Karpenko does in no overt fashion actually *attribute* the violation of LNC to a correlative violation of LEM. At least not in so many words. What he does do, rather, is to confine himself to "an extraction of the minimal means of constructing a paraconsistent logic out of a many-valued one" ⁽⁷⁾ and otherwise seems content to stay clear of any enquiry as to *why* is such a structure there

⁽²⁾ Synthese, p. 63-69.

⁽³⁾ See for this Mr. Karpenko's own reference list, p. 69.

⁽⁴⁾ In a many-valued system the range of intermediate values changes *continuously* while the values permitted by a three-valued system are *discrete*. This difference is of crucial importance, especially in view of the arguments to be expounded here.

⁽⁵⁾ Synthese, p. 63.

⁽⁶⁾ Synthese, p. 13.

⁽⁷⁾ Synthese, p. 63.

in the first place. In view of this Mr. Karpenko might justly wish to renounce a criticism which is inappropriately extended to a self-contained work of his, intended to tackle a different kind of problem. But that's just it. His endeavour is *not* self-contained, in spite even of his own view of it (assuming he has one such). First of all because I take it that a "paraconsistent" structure inside of many-valued logic, if it exists at all, is either a *necessary* part of this logic or else it shouldn't be there at all. Yet if a necessary part of it, then something which is there because of the very *nature* of this logic, that is to say, because of its non-bivalent character. Thus the fact that LEM is no longer valid in it must have a *bearing* on the fact that it is paraconsistent, viz. on the fact that LNC is not valid in it either.

Second, because in view of Mr. Karpenko's arguments it becomes all the more imperative to be able to *distinguish* between a many-valued logic which will *have* to meddle with LNC and one which will not, assuming there can be one such constructed. It is, I think, obvious, that whether such a distinction is possible or not, whether, that is, there can be non-bivalent logics which can simultaneously satisfy LNC, is a question to be settled primarily at the *conceptual* level, where the direct relation between LEM and LNC, if any, will be brought to surface once again. And Mr. Karpenko's exposition is not particularly helpful on this matter.

Any other part of Mr. Karpenko's work, not involved with the foregoing issues, should not be thought of as being held in dispute here or even referred to.

Now the contention that a violation of LEM must of needs lead to a subsequent violation of LNC, as I am sure many will grant, is not the sort of thing that one can afford to be too liberal about (as Prof. von Wright seems to be doing). Much less so a stand to be adopted *eagerly* (as Mr. Karpenko seems to be doing). For its consequences do not solely concern the field of formal logic, where the overabundance of systems in which LEM is not valid should by itself be adequate proof of their possibility and so, by the offered criterion, proof of the possibility of inconsistent logics as well. But as if this was not enough of a worry in itself (though to Prof. von Wright and Mr. Karpenko it does not seem to be), such consequences would no doubt extend to embrace the corpus of human knowledge as a whole, given also the *epistemological* content of LEM and LNC, which quite a few logicians are prone to neglect for the sake of "purely" formal considerations, as if it weren't really the necessities

of the former which dictated the adequacy of the latter or as if one could really get along exclusively with those. ⁽⁸⁾

Fortunately, however, there has not been lack of men of width and scope, who latter would not seriously entertain the thought of separating the two. It is just this sort of approach, in which the attendance to epistemological import is so harmoniously combined with formal considerations, that one encounters in Hans Reichenbach's logic for quantum mechanics. According to its author "the language of quantum mechanics is written in terms of a three-valued logic". ⁽⁹⁾ Now it is of great importance to capture the true spirit of this claim (with which I wholeheartedly agree despite the fact that I happen to reject Reichenbach's *specific* interpretation of QM – based on wave-particle duality – and consequently *his* version of quantum non-bivalence, as will become evident in due course).

Because Reichenbach starts his considerations from reality itself, which QM seeks to describe, and draws whatever conclusions are to be drawn regarding the adequacy of two-valued logic by having first settled in his mind the issue of the *physical* situation obtaining and whether the latter, thus conceived, would satisfy the prerequisites for rendering a QM statement singularly true or else singularly false. And if the physical conditions obtaining could not satisfy such prerequisites, as is indeed to be expected from quantum *indeterminacy* (in Reichenbach's sense or in that of Hooker's which will take its place in the present treatment), then LEM would certainly have to be abandoned, whatever consequences this was likely to imply. One could then proceed to construct one's logic by

⁽⁸⁾ This tendency is clearly illustrated in the usual protest of logicians, that a contradiction implies just any proposition and therefore "trivializes" the system. As if it was merely *this*, which is really wrong with admitting contradictions. This line of protest, which confines itself to the *formal* consequences inside a calculus when contradictions are admitted, to my mind resembles the case of a man who is shot in the stomach and all that he can think of are his hospital expenses. For surely there must be *other* (and better) reasons for considering contradictions undesirable, than that merely any proposition is derivable from them. Such a misplaced emphasis justly creates the impression that, once this consequence is removed, there will be nothing else left to complain about. These aficionados of purely formal adequacy truly get what they deserve in the end, just after this element is successfully removed, as is pointed out by Mr. Karpenko himself. (p. 63 and forward). What will they do then? Shrug perhaps and let contradictions take over.

⁽⁹⁾ "Philosophic Foundations of Quantum Mechanics", Univ. of Calif. Press, 1965, p. 43.

transcribing the properties of the physical situation into a formal system possessing the very same structural features as the physical situation itself or, at any rate, whatever features in the formal language would correspond to this situation. And do all this with as few sacrifices as possible. Then and only then can one go on to affirm whether or not the violation of LEM would subsequently entail the violation of LNC also (and in the situation to be discussed the contrary will prove to be the case). But one will learn absolutely nothing about the world itself by simply "extracting a paraconsistent logic out of a multi-valued one", unless the latter was itself dictated by the physical conditions obtaining, namely, unless such conditions rendered a particular sentence or group of structurally similar sentences *objectively* undecidable in the sense required by such logic. And when such (multi-valued) logic represented the actual structure of the physical situation involved, then and only then would it make sense to look into its internal structure and, upon discovering a paraconsistent element there, proceed to pronounce that, together with LEM, LNC itself has broken down in the real world.

Only confusion will result if one proceeds conversely, viz. *from* a multi-valued logic with no definite physical interpretation *to* its putative consequences on LNC, if any. QM is a theory about the structure of the *actual* world. Which structure, if Reichenbach is right, could well require a three-valued logic for being adequately handled in conceptual terms. If one then held the view that *any* system of three-valued logic whatever would on deeper scrutiny prove itself "paraconsistent", and hold this on the frequent but *confused* supposition that it doesn't really make any difference whether we decide to say that a sentence is neither true nor false or decide to say instead that it is both true and false (and Hegel with von Wright say precisely this, together with several others ⁽¹⁰⁾), then, on this supposition, QM could be indirectly employed as an argument that the Law of Non-Contradiction has collapsed in the actual world. For if LNC cannot

⁽¹⁰⁾ For instance, Fr. Waismann, who speaks of "a *competition* between different images" when one's sense data are called back in memory, leading to "a transfer of the indeterminacy to language". As a consequence, regarding irretrievable impressions of the past, "the assertion 'The patch is either white or not white' is undecidable". ("Are there Alternative Logics?", Proc. of Arist. Soc., Jan. 1946, p. 95 and 98 respectively). Notably, the patch is rendered *neither* white nor non-white because it could *equally* have been or not have been white. So that something which is just as much true as it is false is something which is neither true nor false.

be sustained in a three-valued logic, and if QM is a theory which requires such a logic, and if, finally, QM is a theory describing the actual world, then this theory contains as an implicit consequence the assertion that LNC cannot be sustained in the actual world.

One clearly sees the interests which dialectical philosophers have invested in projects of this kind. But theirs, I hope, is not the only word which deserves to be heard on this matter. First of all, there is still the fundamental question to be settled, a question which must receive absolute priority in any investigation of this kind, and this is whether or not LEM and LNC are *independent, primitive* laws of Logic, a question which, if answered in the affirmative, would render all attempts at linking their fate futile. Now there *are* arguments in existence which claim that they are not, and handbooks of Logic abound with them. They do, however, suffer from a fatal disease. They are one and all so incurably *circular*,⁽¹⁾ that they amply justify the suspicion that the job can't be done without circularity. And that consequently these two laws must be independent of one another. Whereupon, secondly, the specific claims that one says something self-contradictory in merely denying LEM. which

(1) First, there is the following text book favourite: $\neg(P \vee \neg P)$ entails $\neg P \& \neg \neg P$. $\neg \neg P$ entails P and so a contradiction is obtained. But unless $\neg(P \vee \neg P)$ is shown false on *other* grounds, the move from $\neg \neg P$ to P is circular. E.J. Lemmon proposes another (BEGINNING LOGIC, Nelson's Univ. Paperback 1971, p. 52) which however hardly fares any better. It is first assumed that $\neg(P \vee \neg P)$. Then P . Then P is turned into $P \vee \neg P$ by introducing a disjunction. Then $P \vee \neg P$ is shown to be contradictory with the initial assumption, $\neg(P \vee \neg P)$, and so assumption P is dropped. Then $\neg P$ takes its stead. $\neg P$ is turned into $\neg P \vee P$ by introducing a disjunction and a contradiction is obtained once more. What is remarkable here is that there is *not one* step in the syllogism, which is not circular. First, by jointly assuming $\neg(P \vee \neg P)$ and P , it is not thereby shown that $\neg(P \vee \neg P)$ *itself* contains a contradiction, but only that it does *if combined* with P , which latter is anything but peculiar, since this combination is surely contradictory, given the initial $\neg(P \vee \neg P)$. So the contradiction is forcefully introduced rather than obtained. Second, insofar as $\neg(P \vee \neg P)$ still holds, the usual rule of " \vee -Introduction" is no longer a *tautology*, so one cannot use it for obtaining $P \vee \neg P$ from P . Finally, when even by two successively circular moves Lemmon manages to reach a contradiction, and thus reject alternative P , he goes on to conclude from this that $\neg P$ should then be the case, forgetting once more that until $\neg(P \vee \neg P)$ has been shown false on *other* grounds, $\neg P$ cannot be derived at the exclusion of P , for there may be other possibilities, since it is this precisely which $\neg(P \vee \neg P)$ affirms. But if it is impossible to derive a contradiction from $\neg(P \vee \neg P)$ without reasoning circularly, and given that both the alternatives available to standard logic, i.e. P and $\neg P$, have been exhausted, it is impossible to derive a contradiction by assuming $\neg(P \vee \neg P)$. Therefore LEM and LNC cannot be interdependent.

Hegel handed down to his intellectual descendants and other distant relatives, must all, without exception, be entirely devoid of any substantial value, a fact which should be subsequently exposed in any careful, and non-dialectically polluted, examination of the arguments supporting them.

If this can be done, then three-valuedness, which may well represent the actual logic of the world we live in (and I will argue that it does), will in no way permit its "paraconsistent" interpretation, a fancy term, really, for saying simply that we no longer know where we are. And if it *can* be done then I would urge that it *should* be done. Because, I dare say, compared to what these other authors have to offer, it is still a *preferable* alternative.

2. *Revival of a Dialectical Argument*

The argument which follows has Hegel as its ultimate source, though Prof. von Wright does not refer directly to him. I shall begin with his version and then proceed to compare it with the Hegelian original:

A thesis is put forward, call it θ . It has an antithesis which is its negation, $-\theta$. It is then shown, one way or the other, that the thesis is not true. Thus we have $-T\theta$, where the symbol "T" stands for the phrase "it is true that". It is also shown that the antithesis is not true, $-T-\theta$. Thus neither the thesis nor the antithesis is true. From this is concluded that both the thesis and the antithesis *are* true. This is called Dialectical Synthesis. ⁽¹²⁾

For historical purposes only I think it should be mentioned that this description is not entirely accurate. As it happens, Hegel himself does not proceed the way just indicated, but rather argues *directly* that both the thesis and the antithesis are equally true (whatever "true" would come to mean in this connection) and not that they are both false in order that they can then be both true. According to him (and to Marx and Engels also) dialectical synthesis is a self-subsisting logical phenomenon, not in

⁽¹²⁾ Synthese, p. 5, italics in the original.

need of a roundabout support, which to them can be traced back to the fact that everything in the world contains its own negation, beginning from "Being" which contains its opposite. "Non-Being" as indistinct from itself. ⁽¹³⁾ However, a passage in Hegel, identical to the one quoted from Prof. von Wright does indeed exist. It is only a side-issue with him, an isolated piece of reasoning, which Hegel takes as an *instance* of dialectics rather than its foundation (as the same argument features in Prof. von Wright's version) and so in no way crucial to its fate. But it is there even so and although it may not be too crucial to dialectics, it surely is crucial to three-valued logic, if it is valid. Luckily, however, it is not. It is the following:

The Maxim of Excluded Middle is the Maxim of the definite understanding which would fain avoid contradiction but in so doing falls into it. *A* must be either *+A* or *-A* it says. It virtually declares in these words a third *A*, which is neither *+* nor *-*, and which, at the same time, is invested with *+* and *-* characters. ⁽¹⁴⁾

This is indeed an amazing conclusion. First we are told that something is *neither A nor -A* and immediately after we are being told that now it is *both A and -A*. First, *A* and *-A* are both *false*. Then, all of a sudden, they are both *true*! That such verbal jugglery is the constant companion of Hegel's style, a jugglery vital to him for establishing the presence of contradictions where none are to be found, is not exactly news and by now it should impress no one. Nor is the quoted sample amongst the best of his tricks. But that Prof. von Wright should present it without so much as a word of scepticism is rather surprising. All the more so when absence of criticism is accompanied by almost unreserved endorsement, as can be seen from the passage which follows:

Consider a process such as rainfall. It goes on for some time and then it stops. It does not stop suddenly, let us assume, but gradually. (Then) during a certain stretch of time it is first definitely raining, (*p*), later definitely not raining, (*-p*), and between these two states in time there is a "zone of transition", when a few drops may be fall-

⁽¹³⁾ LOGIC, § 86-7 (transl. W. Wallace, Oxford Clarendon Press, 1975), p. 126-127.

⁽¹⁴⁾ Op. cit. § 119, p. 172, my italics.

ing – too few to make us say that it is raining then but too many to prevent us from saying that rain has definitely stopped. In this zone the proposition that (p), is *neither* true *nor* false.

One could, however, also take the view that as long as some drops of rain are falling it is *STILL* raining – but also the view that when there are only a few drops of rain falling, then it is *NO LONGER* raining. Then instead of saying that it is *neither* raining *nor* not-raining, one would say that it is *both* raining and not raining in this area. ⁽¹⁵⁾

This, then, is the identification of a “violation” of LEM with a correlative “violation” of LNC earlier spoken of, which Hegel’s dialectics have directly or otherwise inspired to Prof. von Wright. With the difference, of course, that both these violations have occurred only in Prof. von Wright’s own imagination and not in actual reality. First of all, even if it be assumed together with Prof. von Wright that two all too fundamental principles of thought, LEM and LNC, actually depend on mere *perspective*, i.e. on “what view one can take” as he puts it, still I would be inclined to suggest that, insofar as one *can* take the view which would satisfy those two principles, then that would be the view to try out first. Whereupon, if priority was acknowledged to LEM and LNC, one would say that, so far as there is *any* rain falling, it is *still* raining in this case, rather than say that it is neither raining nor not-raining or that it is both raining and not-raining in this case.

Prof. von Wright, however, given a choice between: (a) saying that it is still raining, and thereby salvage LEM and LNC, or (b) not saying this, and thereby sacrifice both LEM and LNC, would just as soon opt for the second alternative. This, so far as choices are concerned, regarding which Prof. von Wright is hardly offering us any good reason for having thus made his own. But in actual fact, there are no choices involved at all in this case and the previous concession that there were only served in verifying how would Prof. von Wright himself exactly choose, were he actually given one. But he is not and neither are we. Because when only a few drops of rain are falling, in which case one may say that it is neither raining nor not-raining and whatever else goes with it, on a different ver-

⁽¹⁵⁾ Synthese, p. 12-13. Capital letters for the author’s italics. The rest mine.

sion of the incident it is simply *drizzling*, and once one has said that, then the proposition that "it is drizzling" is true and all other descriptions simply false, so that no one can then say that it is neither drizzling nor not-drizzling. For that would be meaningless. Likewise, when there is a continuous transition involved, such as that between tall and short, one can very well say of someone that he is neither tall nor short, the man in question often being only of medium height. But one cannot equally well say of him subsequently that he is neither of medium height nor, however, not of medium height also. For that *would* be meaningless. And when it comes to transitions which are not continuous, e.g. "alive/dead", then, as I am about to argue, the consequences are anything but what Prof. von Wright would have us believe.

Finally, when it is still raining there is yet another, even if less literary, description at our disposal, which latter is perfectly independent of what we or anybody else might choose to say. This is that "the amount of rain now falling is x cm³/sec" where x is a variable capable of receiving any value ranging from zero to whatever else the case at hand may require, including among those all these instances during which, according to Prof. von Wright, it would be said that it is neither raining nor not-raining. Now, at any instant of time or interval t , taken within the transition period, during which, according to Prof. von Wright, there are too few drops falling to make us say that it is still raining and too many for preventing us from saying that it is not, there will always correspond a *single* value to x , rendering the description which contains this specific assignment true and all the others uniquely false. That would show that at any time during the rainfall any specific value assignment to x is either true or, if not, then false. Which conclusion proves that LEM is still valid in this incident (if, indeed, a proof is at all required).

Yet in Prof. von Wright's reconstruction of the incident, at the very same time t , i.e. during the transition zone, it would be said that it is neither raining nor not-raining and, by the very same token, *both* raining and not-raining. Starting by what seems to be a perfectly ordinary, indeed a trivial, case of two-valued logic, Prof. von Wright manages to contradict both LEM and LNC. One can hardly avoid the remark here, that if the inconsistent logics currently in fashion rely on arguments of *this* sort, they had best be left alone.

But it is not simply that the passage is burdened with an almost playful dismissal of LEM. What is more, there is also the issue which is raised

upon its alleged violation. The subsequent claim, namely, that once we have tucked the violation of LEM safely in our pocket, then "*instead of saying that it is neither raining nor not-raining, we would say that it is both raining and not-raining*". So there is much more at stake involved. But why *would* we say that? Because suppose that we do concede Prof. von Wright the violation of LEM which he seems so intent on establishing and humour him a while by admitting that it *is* neither raining nor not-raining in the transition zone. Yet why suppose the rest of it as well? Why should, from the putative – and all so frivolously supported – situation that it is neither raining nor not-raining, follow that it is *both*, raining and not-raining? As presented, and all other objections aside, the argument still can only have the very *opposite* conclusion than the one which Hegel and Prof. von Wright* actually draw from it. Because when it is admitted that LEM is shown to be false and when, consequently, two contradictory alternatives are both false, then, it goes without saying, *neither* one is true. But if neither one is true, then it is no longer possible for us to say that they are *both* true. And so if it is neither raining nor not-raining it is impossible to say that it is both raining and not-raining, which, however, is precisely what Prof. von Wright does say.

To make this general, if any two contradictory alternatives can be shown to be both false, and so LEM to be false, the two contradictory alternatives which falsify LEM *will never on that assumption* falsify LNC, because the latter will have been shown false when and only when two contradictory alternatives are both *true*, something which (presumably) can never be done so long as, according to the previous claim itself, they are already proven both *false*. Strange though this may sound, the case which would show that LEM is false would *ipso facto* show that LNC is true. What would indeed threaten LNC, would have to be an argument establishing that two contradictory alternatives are somehow both true, (e.g. dialectic-

* Prof. von Wright misses complete identification of the two "violations" by an extremely narrow margin. For he adds that the sense of "true" in which it is affirmed that it is neither raining nor not-raining is not the *same* as that in which it is affirmed that it is both, raining and not-raining. (p. 13) But I do not see how this can help. For either the two cases are near equivalent, in which case the shift in meaning is insignificant, or the shift in meaning *is* significant, in which case the two cases are no longer equivalent and then "neither (p) nor (–p)" cannot be turned into "both (p) and (–p)". So far as I can see, this "explanatory" addition merely contradicts the reasoning of the whole passage.

tics), and this is the exact *opposite* of anything that anyone can possibly extract from a violation of LEM! If this is so, then it appears *prima facie* that in a logic where LEM is not assumed as valid, LNC should be all the more strengthened for it. And, as I am about to argue, this is precisely the case. A result of this kind, if tenable, is bound to have some importance for the conceptual foundations of three-valued logic.

3. *Transitions of a Different Nature*

I shall now argue that when LEM is shown to break down in a certain case LNC must *necessarily* be upheld in the very same case and *by the very same standards*. In order to do this I too, like Prof. von Wright, will discuss a process of transition from one state to another and of a "zone" separating them. But it will prove to be a radically different sort of transition with a radically different zone. Fortunately a considerable part of the ground has already been ingenuously covered by a series of works which, starting from those of Niels Bohr himself, and proceeding through the works of Prof. P.K. Feyerabend, ⁽¹⁶⁾ culminate in an argument by Prof. C.A. Hooker, on the final form of which I will rest my own case. None of the works mentioned is concerned with the foundations of Logic. They all confine themselves to the question of the true nature of a quantum mechanical transition. Yet, if conceded, they can easily be proved to entail, in their own right, certain crucial consequences for Logic itself and do so, in fact, in a rather obvious way. This will become especially apparent from the structure of Prof. Hooker's version of the argument, consisting of Six Postulates and a Lemma. So what I will be doing, essentially, is to put the finishing touch to it:

- P. 1: Time is a continuum.
- P. 2. (i) No physical system can be in two discrete states at the same time, and,

⁽¹⁶⁾ Prof. Feyerabend has formulated the original version of the argument which follows in two of his works; "N. Bohr's Interpretation of the Quantum Theory" (CURRENT ISSUES IN THE PHILOSOPHY OF SCIENCE, Holt, Rinehart & Winston, N. York 1959, p. 372-3) and "Problems in Microphysics" (FRONTIERS OF SCIENCE AND PHILOSOPHY, Univ. of Pittsburgh Press 1962, p. 195-6). But of course the roots of these ideas can be traced back to Bohr himself. (See especially his ATOMIC THEORY AND DESCRIPTION OF NATURE, Cambridge Univ. Press 1934, p. 94).

- (ii) For every change of state there is a *first* temporal instant at which the succeeding state is occupied, and,
 - (iii) For every change of state there is a *last* temporal instant at which the preceding state is occupied.
- P. 4: Energy exchanges involve finite, indivisible quanta of energy only.
- Lemma: P. 1 to P. 4 jointly imply:
- P. 5: The energy exchanges must take a finite amount of time.
- Lemma: P. 1 to P. 6 imply:
- P. 7: During the finite transition time *no definite energy* can be assigned to the systems. ⁽¹⁷⁾

Take now the case of a physical system possessing energy equal to $1E (= h\nu)$. As P. 4 states, the system can only be found in *discrete* energy levels, all the *intermediate* levels being thereby excluded. This is Bohr's famous "quantum postulate". According to this postulate, if the energy of the system is to decrease from $1E$ then, just as the late Prof. Alfred Landé has put it, the energy would fall *abruptly* to zero. ⁽¹⁸⁾ Now while the system is still in $1E$, the proposition "the system has energy" is the case. And when, some time later, it falls to zero, the proposition "the system does not have energy" is the case.

But what is the case when the system is *no longer* in $1E$ but *not yet* in zero? According to Hooker's P. 2 (ii) and (iii), there must be a *last* temporal instant, when the preceding state is occupied, and a *first* temporal instant, when the succeeding state is occupied. Nor could the time required for the transition from the former state to the latter be *zero*, because then we would have to suppose that a physical system *can* be in two distinct states at the same time, which P. 2 (i) forbids. Consequently the time required for the system to pass from $1E$ to zero must be a $t > 0$. And there must be a last instant, t_1 , when the preceding state is still occupied, and a first instant, t_2 , when the succeeding state is entered, which are *separable* from one another. Now take any t' such that $t_1 < t' < t_2$. At t_1 (and backwards) the case is that the system *has* energy. And at t_2

⁽¹⁷⁾ "Energy and the Interpretation of Quantum Mechanics", Austral. Journ. of Phil. 49, n. 3, 1971, p. 263, italics mine.

⁽¹⁸⁾ "The Logic of Quanta", Br. Journ. for the Phil. of Sc. Feb. 1956, p. 300.

(and forwards) the case is that it does *not*. But, to repeat the question, what is the case at t' ? Since t' was taken as *later* than t_1 , that "the system has energy" is not the case. And since it was taken as *earlier* than t_2 , that "the system does not have energy" is not the case either. Thus, during the finite transition time, when due to the discreteness postulate no definite energy can be assigned to the system, (P. 7), it can *neither* be said that the system has energy *nor*, however, that it hasn't. So of two contradictory alternatives neither one is the case. Then LEM must be false. This is the *only* possible conclusion which Hooker's table of Postulates allows. And since his Postulates are arranged in accordance with the principles of QM, the physical theory currently accepted as a description of the actual world, then the only possible conclusion which such principles allow in respect to this world is that LEM cannot be upheld in it, at least in its present form* and at least so far as discontinuous transitions are concerned. (Provided, of course, that Hooker's Postulates do represent the physical content of QM).

This, I contend, is the absolutely *inescapable* conclusion of Prof. Hooker's Postulates, so inescapable in fact that to affirm them and yet deny that LEM is violated as a consequence, would be to affirm something contradictory. The question, however, is, which Postulate *exactly* is the one responsible for its violation. Undeniably, all are involved. But only one of them plays the key role. And this is Postulate P. 2 (i), which says that no physical system can be in two discrete states at the same time. But why can it not? Well, if at all, then for one reason and that reason alone. Because of the *assumed validity of LNC itself*, which is presupposed. For it is this Law which supports P. 2, in itself a Postulate which has nothing specific to do with QM, as Prof. Hooker also remarks. ⁽¹⁹⁾

LEM is violated because the transition time is greater than zero. For only then can it be maintained that during this finite time, which separates the two contradictory alternatives, the only ones which really seem to be available, LEM can no longer be satisfied, since during such time the

* It must be stressed that in the case examined LEM has been falsified only in its *initial* formulation, viz. the one which says that every proposition is true or else *false*, the so-called "Principle of Bivalence" (which, it may be said, preceded the construction of non-bivalent logics). But it can still be maintained in a different form, this being that every proposition is true or else *not-true*. Nothing has come up in our analysis which violates *this* formulation.

⁽¹⁹⁾ Op. cit. p. 264.

system cannot but be *in-between* two contradictory states and therefore answering to neither. But that there is a finite time at all, separating the two incompatible states, can only be affirmed by force of LNC or not at all. Thus, LEM is in this instance violated *because* LNC is retained. A mere look at Hooker's P. 2, itself but an expression of LNC, suffices for verifying that none of the other conclusions would ever follow without it. Far from being interdependent, LEM and LNC prove *incompatible* in Prof. Hooker's account of discontinuous transitions. It almost seems miraculous that Nature, the structure of the operations of which makes the collapse of LEM inevitable (if QM is true), and so greatly upsets the foundations of our Logic, should be so benevolent as to warrant that LNC will come out all the stronger in the way of doing it, thus minimizing the impact of the consequences.

One can, of course, suggest instead that if QM does indeed entail a violation of LEM of the sort argued here, then QM is itself false rather than the other way round. For after all, QM is nothing but a *theory*. And, one might observe, Laws of Logic, which predate all physical theories (in the logical as well as in the temporal sense), are *themselves* the background by which physical theories should be judged rather than the converse. Instead of being branded as conservative, reservations of this sort are perhaps to be attended to with deserving care and, at any rate, are far more commendable than analyses which so casually spell the doom of LEM in processes as familiar as rainfalls. However, in this particular context they won't do. And they won't because transitions of the kind discussed by Prof. Hooker do not only occur in QM, which incorporates discontinuity as a physical, therefore expendable, hypothesis but have actually occurred already in areas which have nothing in common with QM and consequently do not depend on its specific premisses.

I am referring to a case of no small significance, which has experimentally disputed the validity of LEM more than 150 years ago, although this may have escaped the notice of logicians. The predicates "organic"/"inorganic" are *contradictories*, i.e. they normally can neither be both true nor, however, both false, and not mere *opposites* (e.g. "tall/short") which latter cannot both be true but can be both false (when someone is of medium height). Upon this it would follow that for every sample of matter in existence there exist but two possibilities; either that it is organic or, if not, then inorganic. Yet as early as 1828 an organic compound, urea, was experimentally created out of inorganic substances. By force of

Hooker's Postulates P. 1, P. 2 (i), P. 2 (ii) and P. 2 (iii), during the process of such transformation there must have been a finite interval of time $t > 0$, at which the chunks of matter employed by the experimenters must have been neither organic nor inorganic. Again, no other conclusion is possible in regard to this experiment, if the afore mentioned Postulates apply. And the transition which is deemed to have taken place, if the experiment was ever succesful, will then reveal properties identical to those of the quantum mechanical transition discussed by Prof. Hooker. But that the predicates "organic/inorganic" are contradictories rather than opposites is no longer a specific physical hypothesis but simply a convention adopted in *ordinary language* itself. And though one can meaningfully say of a physical hypothesis that it is either true or false, and dismiss it as being the latter when it contradicts a law of logic, one cannot meaningfully say of a *language* that it is either true or false, and dismiss *it* as "false", when its structure is found to contradict a law of logic.

Indeed, what is to be done if the semantic relation of the predicates "organic/inorganic", is, as a matter of linguistic fact, so arranged as to treat them as clear cut contradictories, as such, that is, that there is no intermediate state, in other words *no middle* allowed in-between them? And to propose that we are still at freedom to re-arrange our stipulations in such a way that we will from now on be able to treat the predicates "organic/inorganic" as opposites rather than contradictories, a strategy which would re-establish the continuity of their transition and therefore the validity of LEM, would not only sound more like the expressing of a pious hope and less of offerring a practical (or practiceable) advice. It would, moreover, amount to forgetting that LEM, whose salvation this manœuvre aims at, *itself depends* on existing use. If one is seeking to salvage LEM, which the transition from the inorganic state to the organic seems to violate, one should always bear in mind that, if LEM is at all expected to hold in the case at hand, then this is *because* we have decided to employ these predicates in our discourse in just *this* sort of fashion instead of in another. Having fixed them as contradictories rather than as mere opposites simply reflects our decision to use words in a certain fashion and not in a certain other. Whatever credit is then due to LEM, it just springs from this source.

One cannot properly seek to salvage a direct *consequence* of existing usage, this being LEM itself, implicitly accepting thereby that if the consequences of such usage are to be respected then so is to be this usage

itself, while seeking, on the other hand, to *undermine* the usage itself in order that its consequences be not allowed to perish. This would surely be a very odd way of exercising one's sense of priorities. And lest I myself be accused of actually *reversing* this mistake when I, instead, seek to retain the existing usage itself but reject LEM, which is its direct consequence, I hasten to add that my specific anchoring onto the former does in no way commit me to the *validity* of the latter. Because in admitting the contradictoriness of the predicates in question, I am only affirming that, as a consequence of established usage, there is no other *describable* state in-between them. But I am not thereby affirming in consequence that, since language lacks any other means of predication for characterizing the evolution of physical systems, the only two which do happen to be available in it and which do correspond to definite descriptions of the system must actually and *necessarily* always be the case. In affirming, in other words, that the contradictoriness of these two predicates is an irreversible linguistic fact, I am merely contending that these are the only ones *available*. But I am not thereby affirming that either one of them must always be *true*. Whereas he who contends the reverse of this, viz. that either one of these two and nothing but these two must always be true, *would* be committed to the view that these are the only ones which are really available. On the contrary, what the present contention amounts to is, that a physical system which is being successively characterized by the pair of contradictory predicates in question will either be organic, or inorganic, or, finally, in a completely *indeterminate* state. That is to say, Prof. Hooker's P. 7.

But when in-between a pair of contradictory (and definite) predicates one is forced to postulate the insertion of an indeterminate state, one does not thereby decry the (linguistic) fact that the two predicates already in existence are still the only ones that we are left with. Because an *indeterminate* state is not the sort of thing to which a third (intermediate) predicate could ever correspond, in fact not the sort of thing to which *any* predicate could correspond. Thus its postulated insertion in no way amounts to the introduction of any *new* predicate. And this is perfectly consistent with established usage. It is not however consistent with LEM, as we have already seen.

One can always demand, of course, to examine the credentials of the notion of indeterminacy itself. Is such a notion at all legitimate, for which it can neither be said that it is nor that it isn't? But a questionnaire of

this kind would place our faith in LEM in advance of the very test which could cast doubts on its validity. Our only other hope is to seek refuge in LNC itself. But is the notion of indeterminacy *self* contradictory? I would certainly say not, since if nothing *can* be positively said about a state of this kind, then a fortiori nothing *self contradictory* could be said about it.*

Nor, for that matter, does the contrary assertion, "all states in the world must be determinate" seem to express a *logical* truth. Much less in turn does the assertion "all changes of state must be continuous", from which the former can be deduced, seem to be able to qualify as one. In point of fact it was included among Kant's Table of *Synthetic* Principles,** much as he would desire it to be analytic. The eventuality which the above two universals seek to rule out in advance of experience is simply the admittance of a *void* in the world (the traditional vacuum). But the notion of the void (it is almost impossible to say what this notion is – at any rate it must be utterly negative), is notorious for its elusiveness, whenever efforts to pin it down as self contradictory were undertaken. What these two universals do express, together with LEM which is intimately connected with them, are much rather important *conceptual* truths, or near-truths, of the kind which Kant entitled "synthetic but a priori". They do not express self evident truths and therefore cannot be regarded as self supporting but they still express universal principles of empirical investigation that we simply cannot do without. To the extent that the validity of LEM depends on them, the latter too expresses a conceptual truth, or near-truth, of comparable importance. And I say "near-truth" for the simple reason that, even in the cases where LEM cannot be upheld, there is still nothing better that we can think of which might replace it. Nor can there ever be such a thing as its *total* invalidation. (The present argument certainly is not). For although not self contradictory, it is still *inconceivable* that two contradictory alternatives can be both false. *What* the case is, when neither of them is, is something for which our language has no words and our intellect no representation. (It is for this reason

* This remark is not essentially different from the one made in Section 2, where it was maintained that one can never say something self contradictory in simply denying LEM. One obviously cannot, since in denying it one is not really *saying* anything at all. But to say something self contradictory, one must say something.

** For a somewhat fuller treatment see note N. 22.

mainly that we dismiss this possibility). That we have chosen to sacrifice it in the two cases discussed, was not the product of care-free considerations dropped casually about while having our tea in a rainy evening. It was only a strict sense of *priority* which dictated this choice, when it became evident that a choice was inevitable. For in the case of a conflict between two equally indispensable principles of thought, one of which is self evident and the other not, the choice should normally favour the former. (Though abnormal choices are by no means unfrequent). The point, however, is to realize that a choice between them *may* arise. Because these two principles are not necessarily compatible and we have just examined two instances of structural similarity in which they are not. And if LEM fails in QM as a consequence of a physical hypothesis which could well turn out false, and so relieve us of the obligation to dismiss it, in ordinary language it seems to have failed long ago. So we really have no reason for retaining it for either. It is imperative, however, to always bear in mind just *why* we were forced to do this.

4. *Why Must a Non-bivalent Logic Always Be Consistent*

In Sections 1 and 2 I gave an account of the thesis proposed by Hegel and Prof. von Wright (and, indirectly but clearly, by Mr. Karpenko), according to which in denying LEM one must eventually but inevitably deny LNC in the process. I then offered a number of reasons for thinking that their claims are unfounded and their arguments invalid. (This charge, of course, does not concern Mr. Karpenko, who does not advance any). At the end of these sections I indicated, moreover, that Hegel and Prof. von Wright are not simply mistaken in their specific choice of arguments but rather *have* to be mistaken in any argument to this end, as a matter of principle. This was founded on the general, preparatory remark that, if two contradictory assertions are both false then, above all else, they cannot both be true. On the force of this remark alone it would follow that if LEM is false LNC must be true.

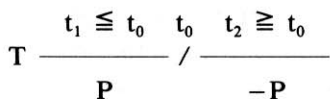
With the purpose of demonstrating this very point I presented in Section 3 an argument by Prof. Hooker which, to my mind, gives a perfectly definite content to this general, preparatory remark. Prof. Hooker himself intended his (discreteness) argument to mainly depict the conceptual problems raised by the quantum situation but, in accomplishing this he at

the same time offered us a model for physical change which, due to uniqueness of its structure and range of applicability, was found to also have important consequences for the foundations of Logic itself as well as of the specific problem at hand. Prof. Hooker's model for transitions, as even the quickest of comparisons will reveal, is the utter negation of everything which Prof. von Wright has claimed in his own. In Prof. Hooker's model LEM is shown to fail during *dis*-continuous transitions and *not* during continuous ones, as it (allegedly) does in Prof. von Wright's own. In Prof. Hooker's model LEM is violated during a certain process *on condition that LNC is retained* for the same process, contrary to Prof. von Wright's model, where (again allegedly) both are violated in the same process. In Prof. von Wright's model LEM and LNC are treated as *interdependent*, in Prof. Hooker's model they are mutually *exclusive*. In short, there is the greatest possible contrast between these two models. And it has been made quite clear, I think, which of the two models I consider nearest to the truth.

Still, my own development of Prof. Hooker's model has not yet succeeded in showing *precisely* what it set out to prove. I had wanted to indicate that if LEM is assumed false, LNC would as a consequence emerge true. But what I have been able to extract from Prof. Hooker's model is, that during a discontinuous transition, if LNC is assumed true, then LEM will as a consequence emerge false, viz. the *converse* of what I set out to defend. So the task is not yet fulfilled. Yet Prof. Hooker's argument has really prepared the way towards this end, since it *has* been shown in his model that LEM and LNC can be mutually exclusive and mutual exclusion is a *symmetric* relation. Consequently, the converse should also hold.

In his model it can be shown that if LNC is assumed as valid during a discontinuous transition, LEM must be violated within the very same process and by the very same reasoning. Now if the converse of this must also hold, then, conversely, if LEM is to be violated, then this must involve a discontinuous transition, during which LNC will emerge valid, for the very same process and by the very same reasoning. Let us see then whether this really follows.

FIGURE 1



Assume that an event, such as, say, the death of X , occurs at t_0 . We may designate as t_1 any instant of time before or up to t_0 and as t_2 any instant of time from t_0 and later than. As the design shows, t_0 divides *TIME* in two parts and

does so exhaustively, since for any time t_1 X is alive and nothing but, and so P is true and $-P$ false, and for any time t_2 X is dead and nothing but, so $-P$ is true and P false. Either way LEM is so far satisfied.

It should be noted that the specifications presently laid down allow of at least one instant in time such that both P and $-P$ may be simultaneously true, as the dual stipulation $t_1 \leq t_0$ and $t_0 \geq t_2$ seems to entail. Arranging our premisses so as to avoid this possibility would be premature, because it is this issue chiefly which is to be resolved below.

Our concern is to determine which are the steps that must be taken for achieving a violation of LEM in a case such as the one depicted in FIGURE 1. Naturally such an aim can be achieved only if P is assumed as false and also $-P$ *simultaneously* assumed as false, if at all. But as FIGURE 1 shows there is yet no room for such eventuality, since for any $t_1 \leq t_0$ P is true and $-P$ false and for any $t_2 \geq t_0$ $-P$ is true and P false. What adjustments must be introduced in the conditions depicted by FIGURE 1 in order to make *both* false?

Let us see about P first. If P is to be false, it must be so at a time t' , such that $t_1 < t'$, since up until t_1 P is still true. But at a time $t' > t_1$ FIGURE 1 shows that $-P$ is now true and we don't want that either. Conversely, if $-P$ is to be false, this must be so at a time t' , such that $t' < t_2$, since from t_2 and forward $-P$ is now true. So if P and $-P$ are to be made both false, we must postulate a time t' such that t' will always be *later* than t_1 , the last moment when X was still alive, and so P still true, and *earlier* than t_2 , the first moment when X was dead, and so $-P$ then true. So if there is to be a time t' at all, during which neither of the two contradictory assertions, P and $-P$ is true, then this must be such that $t_1 < t' < t_2$, since there can be no other time anywhere within T , when at least one of P or $-P$ will not be true.

But if so, then a stretch of time of duration t' *must always intervene* between t_1 , the last moment when X was still alive, and t_2 , the first moment when he was dead. And unless the insertion of such an interval is postulated, LEM will not have been shown false anywhere within T .

However, once such an interval *is* inserted, it drives a *wedge*, so to speak, between t_1 , the last instant when P is true, and t_2 , the first instant when $\neg P$ is true. But if a wedge of this kind, viz. a temporal slice of duration $t' > 0$, must always be inserted between t_1 , the last instant when P , and t_2 , the first instant when $\neg P$, then it should never be possible for t_1 and t_2 to be co-temporal. Because the insertion of t' *pushes apart*, as it were, t_1 and t_2 and so does the two contradictory states P and $\neg P$ correlated with them, thus perpetually preventing their contact. As a consequence there will never be a moment in time, when both P and $\neg P$ are true. It follows that if LEM is to be rendered false anywhere within T , this cannot be done but by a manner which will, in the very same process, satisfy LNC for the very same period of T .

The new (and final) situation is depicted in FIGURE 2.

FIGURE 2

	t_1	t'	t_2
T	_____	_____
	P	$\neg(P \vee \neg P)$	$\neg P$

As FIGURE 2 shows, during the interval t' , taken always later than t_1 and earlier than t_2 , neither of the two contradictory assertions is true. Yet it does show, at the same time, that in the conditions specified, they cannot ever be both

true. (In fact, if it ever showed anything *other* than that, it should be very odd indeed). If LEM is therefore to be violated anywhere within T , then I submit that this can be accomplished in no other manner, save by introducing an assumption which will in the very same process and at the very same period of T *automatically* satisfy LNC. Either the job can be done in this way or it cannot be done at all. If correct, this result must be of some significance for the foundations of three-valued logic and for securing its *rational* character.

It is, essentially, the very same result as the one obtained by the converse method in Prof. Hooker's model, which just as clearly suggested what the present investigation also confirmed, viz. that LEM and LNC can sometimes be *mutually exclusive*. "Sometimes" here means, of course, during discontinuous transitions. Yet contrary to Prof. Hooker's method, the validity of LNC was *not* presupposed in this approach. The only thing which the present enquiry focused over was to determine whether, and in what circumstances, could LEM be violated within T , if at all. That LNC should necessarily be satisfied in the process was merely obtained as an *emerging consequence* of such an attempt.

The truly remarkable thing in this whole affair is the impasse which we are being faced with, an impasse which is already amply exemplified in Prof. Hooker's own set of premisses. The system has $1E$ at t_1 and $E=0$ at t_2 . Now we can either *deny* that there is an interval $t' > 0$ separating them, in which case t_1 and t_2 will coincide, thus violating LNC but not LEM, ⁽²⁰⁾ or *affirm* that there is such an interval separating them, in which case t_1 and t_2 will no longer coincide, thus now violating LEM but not LNC. This indicates that LEM and LNC cannot be jointly satisfiable in the case at hand, no matter what our choice may be. This situation, however, will emerge on *two conditions*.

C.1. That alterations are discontinuous, and

C.2. That time itself is *not*. That is to say, Hooker's P. 1.

Indeed, a transition from a certain state to another, when these states are defined by *contradictory* predicates, cannot but take place discontinuously. This can be seen clearly from the argument regarding the transition from "inorganic state" (at t_1) to "organic state" (at t_2). It is this precisely which is meant when it is affirmed that two predicates are contradictory *rather than* opposites. That there is no other state allowed in-between the former, no third possibility or "middle", which however is always allowed for the latter. And if there is a transition in process from a state defined by a certain predicate to a state defined by its proper contradictory, then during the transformation there will be times when neither is applicable. But if neither is then *no other* ever is. And so there can be no definite description which could ever fit the facts.

Yet observe how utterly different the situation is, if we now turn to consider a transition between states described by *opposite* predicates instead, such as those of "hot" and "cold". When the coffee in my cup turns cold after a while, there will always be a definite intermediate state between these two predicates for my coffee to have been in, and it is just this sort of state which the predicate "lukewarm" aims at characterizing. There will therefore always be a particular description, namely a specific quoted temperature, which will fit the facts. Because opposition between predicates merely states a difference of *degree* (e.g. "tall/short", "dark/bright",

⁽²⁰⁾It is evident, I believe, that LEM is not affected, *even* if $t_1 = t_2$. For if $P \& \neg P$ is the case, then a fortiori $P \vee \neg P$ should be the case, since this disjunction is false only if both its disjuncts are false, whereas now neither one is. This simple fact suffices for confirming that LEM and LNC are not necessarily co-satisfiable.

"hard/soft"), i.e. a variation between states which are one and all measurable *by the same metric unit*. While contradictory predicates convey a difference of *kind*. This is why contradictoriness of predicates implies discontinuity while opposition does not. Because between any two degrees one can always think of an *intermediate* degree. And since there will always be one such degree between any other two, there will always be one particular description fitting the facts, viz. the one quoting this degree, which will be true, all others being simply false. But in such a process LEM will be constantly satisfied. Only in transitions between states of a different *kind* should it fail.*

It is therefore nothing short of astonishing that Prof. von Wright should seek to violate LEM in a *continuous* transition, ending up as a consequence of this with a case in which LEM itself was perfectly sound and his own argument not. LEM can never be violated in a continuous transition because in a transition of this kind between any two successive states there will always be a definite state for the thing to be in and therefore always something which *is* the case. Only during discontinuous transitions will there be times when *nothing* can conceivably be the case and all available descriptions, the two contradictory ones including, equally false.

On the other hand, LEM can apparently be salvaged even at the face of discontinuous transitions, provided that we choose to adjust C.2 and assume a *discrete time*. Clearly C.2 is a premise crucial to the argument and this is why Prof. Hooker gives it priority in his Table of Postulates as P.1. Because only if between any two instants of time there is always a *time*, can we proceed to enquire "what is the system's state now?", referring to that particular time. In other words, only if there is a time between t_1 and t_2 , i.e. a t' , can the foregoing question legitimately arise. So if we quantize time in this particular case, we can hope to co-ordinate discrete alterations with discrete instants and then deny that there is any *real* time between any two of those. So then the times at our disposal for asking "what is the system's state now?" will no longer be unlimited (as they would be if time is continuous), but will be found to correspond to the definite, discrete states of the system in question. This strategy would allow us to retain both LEM and LNC, for at the discrete states themselves, which are definite, they can both be satisfied.

* This is true for *energy* also. Because the transition from 1E to 0 is also a transition between states of a different kind. Existence and non-existence.

But this manoeuvre seems just too convenient to be true. Not only will it introduce the notion of *many times*, as many in fact as there are systems in transition, which will then be only accidentally co-relatable with one another, if at all, since there is absolutely no guarantee that all physical systems will begin changing at the *same* time. So that if a second system goes in transition just after the first one has begun its own, it will be changing in a *different* and *independent* time. Moreover, it is highly mysterious per se to say that between two *separate* instants of time there may intervene *no time*. If not, then on what grounds do we call them separate? On the other hand, they have to be separate, if they are to match the separate, discrete states. In any case, quantization of time is hardly a manageable hypothesis and has not been adopted even by QM itself. There is no reason why we should.

Such alternative proposals being more problematic than the problem itself, we had better come to terms with the initial contention, that if time is continuous and alterations are not, LEM and LNC are no longer co-satisfiable, and get on as best we can. (But if not co-satisfiable, they cannot possibly be interdependent. Because then they would either hold together or go down together). Still, a number of fierce objections have been raised in the past regarding the *compatibility* of these two premisses, i.e. C.1 and C.2. Kant and Mendelssohn have argued ⁽²¹⁾ (from different standpoints) that if time is continuous, so must be alterations. But this doesn't really follow, as I'm inclined to think, unless we also assume that events have instants as *parts*. ⁽²²⁾ Then it could indeed be maintained that

⁽²¹⁾ For Kant's position see his CRITIQUE OF PURE REASON (transl. N.K. Smith), B254-A209. For Mendelssohn's intriguing argument see J. Bennett's KANT'S ANALYTIC (Cambridge Univers. Press 1977), p. 178-180.

⁽²²⁾ This is not very far from the truth, concerning Kant. Because by his standards nothing can become an object of my perception unless it has satisfied the requirements of my a priori intuition, in this case the intuition of time. In accordance, any temporal experience of mine will be of entities sharing the very fabric of time itself. And so they must themselves be continuous, if time, to which they are subjected, is. This claim of the Transcendental Aesthetic is rediscovered in the Anticipations of Sense Perception. (The Categories of Quality "schematized"), where now the infinite divisibility of time is categorially injected within the very structure of sensibility, as an a priori rule for its operation. It would then seem, on this view, that our sensory apparatus is blind to discontinuous alterations *in advance* of experience. But this should not be taken to mean, on Kant's terms, that the Anticipations are *analytic* truths. For what these Anticipations do furnish is a kind of knowledge which, according to Kant, is both; synthetic *and* a priori.

if events evolve discretely, so must the parts comprising them and therefore so must be instants. Yet to this suggestion I cannot but reply in Prof. Hooker's way, who clearly saw ahead in anticipating it and who insists on distinguishing between time itself and "physical *states*, i.e. changes in physical quantities *other* than time".⁽²³⁾

Besides, regarding QM itself, it is precisely this combination of a continuous time with discontinuously changing energy levels which is responsible for the *complementarity* between energy and time and the Uncertainty Relation itself, which links these two variables and which could not be obtained unless *both* these premisses are present.⁽²⁴⁾

On the whole, I submit that premisses C.1 and C.2 clearly express a logical possibility and, moreover, seem to be equally required by the scientific theory currently employed for the description of the dynamics of physical systems, and thus they might even represent the actual structure of the world we live in.

5. Closing Remarks

If this is true, then it is important. In QM the key-word is *discreteness*. Namely, the postulate according to which the energy levels admissible for a physical system must be sharply *separable*. It is a key-word for the present argument also. For if two successive states are to be kept sharply separate there must exist a *time*, i.e. a $t' > 0$, which *keeps* them separate. This is but a natural part of the conception of discreteness. Yet miraculously it is just this sort of discreteness (and, as we have seen in the case of urea, not only quantum discreteness), which by a *single* move violates the requirements of LEM but *satisfies* those of LNC. If $1E$ and $E=0$ are (directly) successive, *and* if a certain lapse of time is to keep them separate, then: (A) When, during $t' > 0$, the system is no longer in $1E$ and not yet in $E=0$, it answers to neither of those descriptions and so, those being contradictory, it violates LEM. However, (B), if a lapse of time *does*

⁽²³⁾ Op. cit., p. 264, italics in the original.

⁽²⁴⁾ I have presented this argument in two earlier works, "Discontinuous Alterations of State and the Question of Determinism", (DETERMINISM IN PHYSICS, Gutenberg, Athens 1985) and in much greater detail in "The Uncertainty Relations of Energy and Time and the Conflict between Discontinuity and Duality" in the series FUNDAMENTAL THEORIES OF PHYSICS, vol. MICROPHYSICAL REALITY AND QUANTUM FORMALISM, Reidel-Kluwer Academic Publishers, Dordrecht 1988.

separate the two contradictory descriptions, as (A) requires, then LNC is automatically satisfied, because to say that a lapse of time separates two contradictory descriptions is simply to say that they cannot both be true at the same time, which is what LNC demands. To put it somewhat differently, and more directly, if something is to violate LEM, it must keep an equal distance from both of its contradictory descriptions. But in order to keep such a distance, a *distance there must be!* So, to say it once more, the very argument which must be employed for falsifying LEM is ipso facto an argument which will satisfy LNC. Which, really, is anything but strange. For when it is *neither* raining *nor* not-raining, it should never be *both* raining and not-raining.

QM, in Prof. Hooker's model, seems to respect this simple and self-evident truth by drawing a picture of reality ideally suited to give it definite content. (As indeed does the transition from the inorganic state to the organic). So reality appears to require this particular form of logic, in which LEM is not a valid law but in which, because of it, LNC still is. There should therefore be a *specific* type of non-bivalent logic corresponding to reality thus conceived. One in which LNC must be expressly valid, a fact which, if the physical argument is faithfully transcribed in formal language, should then also be an inseparable element of the formal structure of the ensowing calculus. And thus constructed, this logic should be both non-bivalent *and* consistent. And the one best suited to the structure of physical reality as presently conceived.

This result leaves very little for Mr. Karpenko and Prof. von Wright to go on, in their endeavour to reduce non-bivalent logic to dialectics. It would leave too little to Mr. Karpenko, because the *uninterpreted* logics from which he "extracts a paraconsistent structure", since divorced from a definite and concrete model of physical application, have absolutely nothing to teach us about the world and can at best only instruct us about their *own* properties. I have no argument against Mr. Karpenko on this score but then again I do not *need* to have one either. An uninterpreted logic makes no assertion of any kind about the world and so it is perfectly inconsequential from an epistemic point of view, regardless of whether or not it is paraconsistent. It will not make the slightest difference either to this argument or to dialectics, so I have no quarrel with it. Unless, of course, it should pretend that it does. But then it could no longer afford to non-comittally mask itself behind the paraconsistency of a *purely formal* system and, as if nonchalantly, drop the issue right there, leaving it

to others to decide just what it is exactly which this paraconsistency means. This will now have to be explained in concrete, physical terms. And then come directly face to face and ultimately be *contradicted* by the scientific theory best qualified for handling the issue at hand, rather than play it safe behind an empty formalism.

It leaves even less to Prof. von Wright. Of course the latter does indeed have a physical model to which to apply his Truth-Logic, ⁽²⁵⁾ an obligation which he is too good a philosopher to neglect. Yet although this Logic in itself is a paradigm of elegance and simplicity, still the physical model chosen for its application is totally unsuitable for the task assigned to it. For either Prof. von Wright's rainfalls are continuous processes, in which case LEM is always satisfied and his argument fails, or they are discontinuous (and they may well be, since raindrops are discrete), in which latter case LEM will certainly fail, but then in a way which is hardly compatible with his own requirements, and so again his argument fails.

There may be cases of non-bivalence discovered, which might suit his Truth-Logic, and some of these he points to himself. Most spectacular among those are the classes of sentences which name *non-existent* subjects, those which refer to *future* events and those, finally, which refer antinomally to *themselves*, all of which, admittedly, are at one time or another known to have created acute problems to LEM. ⁽²⁶⁾ But it would

⁽²⁵⁾ Prof. von Wright calls his system "Truth-Logic" because in it he introduces the operator "T" within the object language of the calculus in front of any propositional variable, thus obtaining four separate cases, as opposed to the two of ordinary logic. In this way we have Tp (= it is true that p), $T-p$ (= it is false that p), $-Tp$ (= it is not true that p), and $-T-p$ (= it is not true that not $-p$).

⁽²⁶⁾ He discusses all of these in p. 3-4 and 6-7. Regarding the statements referring to the future, in particular, epistemologically the most respectable case of the lot, Aristotle's memorable argument against their having a definite truth-value perhaps deserves a brief hearing. It goes like this: Suppose that a (true/false) statement referring to a future event is true on being uttered. Then it will be impossible for the event not to occur, which would make the statement *necessarily* true instead of true *or* false. Suppose then that the statement is false on being uttered. Then it will be impossible for the event to occur, which would make the statement *necessarily* false, instead of true *or* false. Either way we obtain a contradiction. So we cannot consistently assign either the value "true" or the value "false" to such a statement. (DE INTERPRETATIONE, ch. 9, transl. by J.L. Acritll, Oxford Clarendon Press 1963, p. 50-53 and 132-142 for Acritll's own remarks.)

Jan Lukasiewicz makes it quite clear that his investigations are inspired by this very argument of Aristotle's. (SELECTED WORKS, North Holland-Amsterdam & London 1970, p. 175).

be rash to assume that such instances of its violation would seriously threaten the present argument or, for that matter, succeed in showing that LEM is violated in *reality*. All the afore mentioned categories of neither-true-nor-false sentences, in spite of their separate peculiarities, still manage to share one thing of paramount importance in common.

The fact that they one and all have *exceptionally weak links with reality*. Sentences with non-existent grammatical subjects, because they designate subjects which *in principle* are said not to exist. (If the subjects in principle do exist, then the sentences become two-valued). Self-referring sentences for the very simple reason that they are *self*-referring. So that their truth-or-falsity, or rather their neither-truth-nor-falsity, make not the slightest difference to the history of the world. And sentences referring to future events, because they are referring to events which *logically* (though not causally) might just as much turn out to be true as they might turn out to be false, so that the future to which they refer (logically, though not causally) is not yet *formed*. (On the contrary, if such sentences *are* considered causally, then the future *will* be formed, but then these sentences will no longer be just as much true as they are false, but only true *or* false).

Due to the fact that these types of sentences are practically *severed* from reality, they lack a definite truth value not as a matter of *fact* but as a consequence of their own *logical content*. The very claim that they lack a definite truth-value is founded upon arguments ascertainable *a priori*. As a consequence, the failure of LEM in their case cannot be consistently attributed to conditions obtaining in reality or to the physical hypotheses depicting them. But the non-bivalent model expounded here (as well as that of Reichenbach's for that matter) *is* attributed to such conditions and the emerging non-bivalent structure, instead of being ascertainable *a priori*, is *contingent* upon the presence or absence of such conditions. Therefore the other three cases of non-bivalence just mentioned are too dissimilar to concern it.

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