A BRENTANIAN BASIS FOR LEŚNIEWSKIAN LOGIC

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1. Brentano's Analysis of Categorical Propositions

In his *Psychologie vom empirischen Standpunkt* of 1874 Brentano created something of a stir among philosophers and logicians by maintaining, against practically the whole tradition back to Aristotle, that all categorical propositions are reducible to propositions asserting or denying existence. (1) The claim was soon recognised by him as too ambitious, and his later theory of judgment, while still highly reductionistic in tone, is more complex than that of the *Psychologie*. The core of his claim lay in the reduction of the four traditional categorical forms to existential form. We quote his words here in full: (2)

> The categorical proposition, "Some man is sick," means the same as the existential proposition, "A sick man exists," or "There is a sick man."
> The categorical proposition, "No stone is living," means the same as the existential proposition, "A living stone does not exist," or, "There is no living stone."
> The categorical proposition, "All men are mortal," means the same as the existential proposition, "An immortal man does not exist," or, "There is no immortal man."
> The categorical proposition, "Some man is not learned," means the same as the existential proposition, "An unlearned man exists," or, "There is an unlearned man."

(1) Brentano uses 'existential proposition' for both affirmations and denials of existence. Reducibility is not symmetric for Brentano: it is therefore stronger than logical equivalence. Brentano's reasons have to do with his theory of judgment, which we here leave aside; we shall consider only the symmetric relation of logical equivalence.

(2) Franz BRENTANO, *Psychology from an Empirical Standpoint*, London: Routledge & Kegan Paul, 1973, p. 210f.

So for Brentano universal propositions are devoid of existential import, and, presupposing his own theory of the basic nature of existential propositions, Brentano claims universals are (inherently) negative. Brentano's ideas here are no longer as foreign to us as they were for his contemporaries. In the usual treatment of categorical forms in post-Fregean predicate logic, the lack of existential import for universals and the potential ubiquity of existentials (positive and negative) is conceded, in that we may take the following as translations of the vernacular forms into forms of first-order predicate logic:

Some man is sick	:	$\exists x(x \text{ is a man} \wedge x \text{ is sick})$
No stone is living	:	$\sim \exists x(x \text{ is a stone} \wedge x \text{ is living})$
All men are mortal	:	$\sim \exists x(x \text{ is a man} \wedge \sim(x \text{ is mortal}))$
Some man is not learned	:	$\exists x(x \text{ is a man} \wedge \sim(x \text{ is learned}))$

This much is known to any logic student. But while the treatment shares some features with that of Brentano, it differs from the latter in at least the following four important respects:

(A) Existence is expressed by a quantifier, not by a verb, such as 'exist', or a verbal construction such as 'there is (are)' or '*es gibt*'. Of course Brentano is well aware that these are no ordinary verbs. Quoting Kant with approval, he notes that they do not serve to express a predicate, but rather represent special judgmental acts or operators of affirmation and denial of a presented content. Unlike Kant and his modern imitators Frege and Russell Brentano does not go on to hold that existence is nevertheless a *second-order* predicate. For Brentano it is not a predicate *at all*.

(B) Where Brentano uses negative and conjunctive *terms*, the modern treatment uses only negative and conjunctive *propositions*. Of course it is possible to define negative and conjunctive predicates in classical logic in terms of propositional negation and conjunction, but where it is a question of priority, as here, that may not be so important. And while a treatment of a wider class of propositions will certainly require propositional connectives, widening our interest may also put in jeopardy the assumed equivalence between propositional and predicate operators.

(C) The normal analysis takes as basic the propositional from aF, where a is a singular term and F a (monadic) predicate, whereas it is precisely Brentano's claim that the existential form neither has nor

presupposes a subject-predicate analysis.
(D) Where the *aF* form does not have a simple intransitive verb in place of the *F*, it builds this predicate up out of a non-verbal complement with the help of the copula. Brentano's proposals aim on the other hand at showing the *dispensability* of the copula in favour of affirmation and denial of existence. Of course predicate logic also shows the copula to be for many purposes dispensable in favour of intransitive verbs, although there are obviously good pragmatic reasons why the copulative form is so widespread in natural languages, even if it is not always a verb equivalent with the Indo-European 'be' which performs the copulative role. In practice, logicians paraphrasing natural language sentences into predicate logic do not hesitate to use the copula freely, and in a sense it is *more* important for them than for natural language, in that propositions like those paraphrased above require the common nouns and adjectives like 'man' and 'sick' *always* to be accompanied by a copula, whereas such expressions are precisely most useful (that is part of their point) in verbless phrases like 'this sick man'.

Thus the usual treatment of existentials like 'There is a man' presupposes both copula and subject-predicate form: There is a man = Something is a man = $\exists x$(x is a man). (3) For Brentano this has things precisely the wrong way round. What we shall show is that within certain quite wide limits Brentano's position is *no worse* than the orthodox one, which is itself a revisionary position, albeit one weaker than Brentano's.

Apart from its not applying to various kinds of proposition not equivalent to one of the categorical forms, a modern logician might criticise Brentano's reduction somewhat along the following lines:

> "It is claimed that the existential 'is' can replace the copulative 'is'. But what is replaced in these reductions is not the copulative 'is' alone but rather the various binary sentence-frames 'Some ... is ---', 'All ...s are ---', 'No ... is ---', 'Some ... is not ---'.
> It is true that 'is' (or 'are') occurs in these, but not alone. The

(3) Cf. A.N. PRIOR, *The Doctrine of Propositions and Terms*, London: Duckworth, 1976, p. 115, where this is called "an answer to Brentano". What follows is effectively an answer to Prior.

crucial use of copulative 'is' is in atomic sentences of the forms
Singular Term + is + Adjective Phrase, e.g. 'Socrates is clever'
Singular Term + is a(n) + Noun Phrase, e.g. 'Socrates is a man'
Singular Term + is + Prepositional Phrase, e.g. 'Socrates is in the Lyceum',
which fit at best unhappily into the Aristotelian picture.
It is true that the copula does not appear in isolation here either, but at least it is in its right place, namely as part of the predicate, something which Brentano's contemporary Sigwart, and later Frege were to emphasise. In any case the copula is logically unimportant: it is a mere syntactic dummy demanded by the grammar of Indo-European languages."

There is some point to the criticism, but Brentano's idea is more versatile than this critique makes it appear. Brentano can effectively handle, if not the singular terms of natural languages, then at least something *very like* them, the singular names of a logical language which boasts a form of singular copula and is closer to traditional and Brentanian logic than is Frege-Russell predicate logic, namely the language of Leśniewski's Ontology.(4) What we show is that it is possible to base Ontology jointly on two primitives employed by Brentano in his reduction: an expression for existence and nominal conjunction. This not only provides (yet) another basis for Ontology: it shows that Brentano's claims for the existential form are considerably stronger than orthodox predicate logic is able to admit. We then sketch how a system of Ontology with extensionality allows even existence to be defined using conjunction, making this the sole undefined notion.

2. Language and Symbolic conventions of Our Version of Ontology

Like Leśniewski we here consider a logical system to be constituted by a finite basis of axioms and rules according to which at any stage in

(4) Cf. my "On Understanding Leśniewski", *History and Philosophy of Logic* 3 (1982), 165-191.

the development or growth of the system only finitely many theses have been stated. (5) Theses comprise axioms, theorems and definitions, all of which conform to suitably formulated conditions stated in a metalanguage. (6) The language here employed presupposes Protothetic, Leśniewski's logic of sentences and sentential functors, which introduces not only the truth-functional connectives but also quantifiers and rules for their manipulation. These rules, which are akin to the axioms or rules for quantifiers in predicate logic, may be applied, *mutatis mutandis* (in this case changing the syntactic category of variables bound) in Ontology. We shall need in fact to consider only nominal variables. Since Leśniewskian quantifiers are both free of existential import we write them 'Π' and 'Σ' to distinguish them from the more usual '$\forall$' and '$\exists$'. (7)

Symbols and Conventions of the Language

Nominal Variables are taken from the beginning of the lower case Italic alphabet as required.

Predicate Variables – we shall need only one: ϕ.

Nominal Constants are defined as we come to them. We shall use '$\wedge$' and '$\vee$'.

Predicate Constants – we use the following, where their intuitive meaning is given in parentheses: Unary (written before their argument): E (existence), ! (uniqueness), E! (singular or unique existence); Binary (written between their arguments): $\triangle$ (overlapping, partial inclusion), $\subset$ (inclusion), ε (singular inclusion), $\not\subset$ (exclusion), $\mathbin{\triangle\!\!\!|}$ (partial exclusion), $\simeq$ (identity or coextensionality), $=$ (singular identity).

Sentential Connectives are $\sim \wedge \vee \supset \equiv$. To avoid unnecessary parentheses these are assumed to bind in decreasing order of strength

(5) 'Development' and 'growth' can be understood either literally, or as referring to an abstract ordering. Leśniewski preferred the former reading, and with his aversion to ideal entities, Brentano would probably have sympathised.

(6) For an indication as to how these conditions are stated in a nominalistically and finitistically acceptable way, cf. E. Luschei, *The Logical Systems of Leśniewski*, Amsterdam: North-Holland, 1962.

(7) On the relationships between Leśniewskian and classical quantifiers cf. my "Leśniewski's Logic and its Relation to Classical and Free Logics", in G. Dorn & P. Weingartner, eds., *Logic and Linguistics*, New York: Plenum Pr., 1984, forthcoming.

as stated, with '$\sim$' binding strongest and '$\equiv$' weakest, unless overridden by parentheses. A left parenthesis may be replaced by a dot in the manner of Church, and where a string of like connectives appear without parentheses the subformulae associate to the left.
Quantifiers are Π (universal) and Σ (particular). They may bind any number of distinct variables of any syntactic category without regard to order. Binding is shown simply by placing a quantifier before a string of tokens of the variable bound. *Note*: In writing sentences we conventionally omit quantifiers whose scope is the whole remainder of the sentence. The variables which thereby appear to be free are *only* apparently so. Leśniewski's treatment of quantifiers makes it unnecessary to introduce a special class of free variables, but our convention retains their convenience.
Quantifier Scope is marked by upper corners $\ulcorner$, $\urcorner$ following Leśniewski.
Nominal Functor Constants. Nominal conjunction is represented by bracketed juxtaposition: '(ab)' may be read as 'a and b'. In practice we omit outermost parentheses and use the same dot-and-association convention as for sentential connectives: thus 'abc' abbreviates '$((ab)c)$', and '$a.bc$' abbreviates '$(a(bc))$'. Thus parentheses are kept to a minimum. The syntactic category N/NN of nominal conjunction makes it clear that a conjunction of names is a name, so conjunction may be iterated arbitrarily often. We also make use of the unary functor N of nominal negation, which may be read 'non-'.

3. *The Brentanian Basis of Ontology*

This consists of three proper axioms and a rule schema for introducing new nominal and quasi-nominal expressions by definition. The rule schema for introducing sentential and quasi-sentential expressions (8) is as found in other formulations of Ontology (9) and is not here given. Also not stated here are the rules of substitution and quantifier manipulation.

(8) An expression other than a sentence is quasi-sentential if its syntactic category in the usual quotient notation begins with an 'S'. Predicates are quasi-sentential. The category index of names and quasi-nominal expressions begins with 'N'.

(9) Compare C. Lejewski, "On Leśniewski's Ontology", *Ratio* 1 (1958), 150-176.

Axioms

$$
\begin{array}{ll}
\text{BA1} & \mathrm{E}a \equiv \mathrm{E}aa \\
\text{BA2} & \mathrm{E}abc \equiv \mathrm{E}a.bc \\
\text{BA3} & \mathrm{E}ab \equiv \Sigma c \ulcorner \mathrm{E}ca \wedge \mathrm{E}cb \wedge \Pi de \ulcorner \mathrm{E}dc \wedge \mathrm{E}ec \supset \mathrm{E}de \urcorner \urcorner
\end{array}
$$

Rule of Definitions

A definition is an equivalence without free variables of the form

$$
\text{BR} \quad \mathrm{E}a\mathrm{X}[\ldots] \equiv \Sigma b \ulcorner \mathrm{E}ab \wedge \Pi cd \ulcorner \mathrm{E}cb \wedge \mathrm{E}db \supset \mathrm{E}cd \urcorner \wedge \mathrm{A}(\ldots) \urcorner
$$

where X is the name or quasi-nominal expression to be defined, where if it is a quasi-nominal expression (i.e. not a name) the brackets contain variables other than a and b, all of which are bound by the (tacit) quantifier preceding the whole formula, each occurs only once in the brackets after X, and all belong to categories already made available in the development of the system. A is a sentence or open sentence containing as free variables none which are distinct from a,b and those following X, and containing no constants which have not occurred previously in the development of the system, either as primitive or as defined.

The definitions in terms of existence and nominal conjunction are now given for a series of predicates which should already be familiar from other expositions of Ontology, and whose intuitive meaning was given in the previous section:

$$
\begin{array}{ll}
\text{BD1} & !a \equiv \Pi bc \ulcorner \mathrm{E}ba \wedge \mathrm{E}ca \supset \mathrm{E}bc \urcorner \\
\text{BD2} & \mathrm{E}\,!a \equiv \mathrm{E}aa \wedge \Pi bc \ulcorner \mathrm{E}ba \wedge \mathrm{E}ca \supset \mathrm{E}bc \urcorner \\
\text{BD3} & a \triangle b \equiv \mathrm{E}ab \\
\text{BD4} & a \not\subset b \equiv {\sim}\mathrm{E}ab \\
\text{BD5} & a \subset b \equiv \Pi c \ulcorner \mathrm{E}ca \supset \mathrm{E}cb \urcorner \\
\text{BD6} & a \mathbin{\triangle\!\!\!\mid}\; b \equiv \Sigma c \ulcorner \mathrm{E}ca \wedge {\sim}\mathrm{E}cb \urcorner \\
\text{BD7} & a \simeq b \equiv \Pi c \ulcorner \mathrm{E}ca \equiv \mathrm{E}cb \urcorner \\
\text{BD8} & a \,\varepsilon\, b \equiv \mathrm{E}ab \wedge \Pi cd \ulcorner \mathrm{E}ca \wedge \mathrm{E}da \supset \mathrm{E}cd \urcorner \\
\text{BD9} & a = b \equiv \mathrm{E}ab \wedge \Pi cd \ulcorner \mathrm{E}ca \wedge \mathrm{E}da \supset \mathrm{E}cd \urcorner \wedge \Pi cd \ulcorner \mathrm{E}cb \wedge \\
 & \qquad\qquad \mathrm{E}db \supset \mathrm{E}cd \urcorner
\end{array}
$$

followed by definitions of two constant names and nominal negation:

$$
\text{BD10} \quad \mathrm{E}a\vee \equiv \Sigma b \ulcorner \mathrm{E}ba \wedge \Pi cd \ulcorner \mathrm{E}cb \wedge \mathrm{E}db \supset \mathrm{E}cd \urcorner \urcorner
$$

$$\text{BD11}\quad \mathrm{E}a\wedge \equiv \Sigma b\ulcorner \mathrm{E}ba \wedge \Pi cd\ulcorner \mathrm{E}cb \wedge \mathrm{E}db \supset \mathrm{E}cd\urcorner \wedge \sim\mathrm{E}ba\urcorner$$
$$\text{BD12}\quad \mathrm{E}aN[b] \equiv \Sigma c\ulcorner \mathrm{E}ca \wedge \Pi de\ulcorner \mathrm{E}dc \wedge \mathrm{E}ec \supset \mathrm{E}de\urcorner \wedge \sim\mathrm{E}cb\urcorner$$

In view of BD8 the axiom BA3 is equivalent to the formula

$$\text{BT1}\quad \mathrm{E}ab \equiv \Sigma c\ulcorner c\,\varepsilon\, a \wedge c\,\varepsilon\, b\urcorner$$

which is the definition of 'Eab' one would expect to find in a system of Ontology based on 'ε' as primitive. However in view of BD3 it is easier to prove equivalence of the Brentanian system B with a system of Ontology based on partial inclusion △. Lejewski has shown that such a system, based on the sole axiom (9)

$$\text{LA}\quad a \triangle b \equiv \Sigma c\ulcorner c \triangle a \wedge c \triangle b \wedge \Pi de\ulcorner d \triangle c \wedge e \triangle c \supset d \triangle e\urcorner\urcorner$$

with the following rule schema for nominal and quasi-nominal definitions

$$\text{LDS}\quad a \triangle \Psi[\ldots] \equiv \quad \Sigma c\ulcorner c \triangle a \wedge \Pi de\ulcorner d \triangle c \wedge e \triangle c \supset d \triangle e\urcorner \wedge \phi(\ldots)\urcorner$$

is equivalent to the systems based on singular inclusion. Since LA and LDS look like mere rewrites of BA3 and BDS (or vice versa!) the Brentanian system appears at first sight to be a trivial notational variant of Lejewski's. In fact the matter is a little more complicated – and interesting – than at first appears, because the Brentanian system, unlike that of Lejewski and most other systems, is simultaneously based on *two* primitive notions. If a system *can* be based on a single primitive, it is an aesthetic defect if it is based on two, but in this case our interests are directed by the historical comparison under consideration, and not by purely logical considerations. We shall in any case see later how to remove the defect in a surprising way. We now see why axiom BA3 alone is insufficient axiomatic basis for B. For we need to know what 'Ea' means even for simple a, which is the point of BA1. BA2 is more interesting. The other two axioms alone are insufficient to generate a system as strong as Ontology. If in Lejewski's system based on △ we define nominal conjunction and existence respectively as follows:

$$\text{LD1}\quad a \triangle bc \equiv \Sigma d\ulcorner d \triangle a \wedge \Pi ef\ulcorner e \triangle d \wedge f \triangle d \supset e \triangle f\urcorner \wedge d \triangle b \wedge d \triangle c\urcorner$$
$$\text{LD2}\quad \mathrm{E}a \equiv a \triangle a$$

then the formula BA2 can be shown to be a theorem. But BA2 is independent of BA1 and BA3, as the following finite arithmetical model shows. We interpret all nominal expressions as designating one of the numbers 1, 2, 3 and 6, and interpret '$\mathrm{E}ab$' as meaning 'the highest common factor of a and b is not 1'. It is easy to see that '$\mathrm{E}a$' then simply means '$a \neq 1$', that BA3 means 'two numbers have a highest common factor greater than 1 iff they have a common prime factor', and is obviously true. We now let '$\mathrm{E}abc$' mean 'the highest common factor of the *greater* of a and b with c is not 1' while '$\mathrm{E}a.bc$' means 'the highest common factor of a with the greater of b and c is not 1'. Then while 'E233' is true, 'E2.33' is false.

BA2 is needed to show that the definition LD1 of conjunction in Lejewski's system may be derived in system B. Since B contains two primitives, *both* their definitions in Lejewski's system as well as the axiom must be shown to be derivable in B if we are to show the two systems equivalent.

Definitions BD3-4 are our counterparts of two of Brentano's reductions, those for the I and E form respectively. The A and O forms in Brentano's reductions may be derived as theorems:

BT2 $a \subset b \equiv \sim \mathrm{E}aN[b]$
BT3 $a \triangle b \equiv \mathrm{E}aN[b]$

We may thus fulfil Brentano's intentions within the context of a powerful modern system of logic.

4. *Beyond Brentano*

While Brentano has been given his due, it must be pointed out that the context in which his idea can be seen to be correct makes use of devices of which he was totally unaware in 1874, notably propositional logic and the quantifiers, as well as the scheme for nominal and quasi-nominal definitions and the definition scheme for sentential and quasi-sentential constants which we simply took over from other systems of Ontology.

While it is quite clear that Brentano's basis cannot be narrowed to that of existence alone, (10) by availing ourselves of Leśniewski's rule

(10) Cf. my "A Note on Leśniewski and Free Logic", *Logique et Analyse* 24 (1982), 415-20.

of extensionality, which we have so far not employed, we can effect a further economy, of which Brentano would hardly have dreamed – disposing of the concept of existence from the basis!

With extensionality and predicate quantification at our disposal, we can give the classical Leibnizian definition of identity (coextensionality):

$$\mathrm{B}^{+}\mathrm{D1}\quad a \simeq b \equiv \Pi\phi \ulcorner \phi(a) \equiv \phi(b) \urcorner$$

Note that this is a quasi-sentential definition and employs no constant *concepts* peculiar to Ontology in the definiens, simply variables from the syntactic categories made available once we have the category of NAME. Note however the following definitions which may be made using nominal conjunction:

$$\mathrm{B}^{+}\mathrm{D2}\quad a \subset b \equiv ab \simeq a$$
$$\mathrm{B}^{+}\mathrm{D3}\quad \mathrm{E}a \equiv \sim\Pi b \ulcorner ab \simeq b \urcorner$$

i.e. we have

$$\mathrm{B}^{+}\mathrm{T1}\quad \mathrm{E}a \equiv \sim\Pi b \ulcorner a \subset b \urcorner$$

Since Ontology can be based on $\subset$ alone,(11) $\mathrm{B}^{+}\mathrm{D2}$ and $\mathrm{B}^{+}\mathrm{D1}$ show that it can be based on conjunction alone as the sole Ontological primitive, with $\mathrm{B}^{+}\mathrm{D3}$ giving the definition of existence. We do not here attempt to set out a reasonably short axiom for Ontology based on conjunction alone: a long and unpretty axiom may be obtained by simply taking the axiom of a suitable system based on '$\subset$' and mechanically rewriting it using $\mathrm{B}^{+}\mathrm{D1}$-2.

5. *Cautionary Notes*

This final reduction leans on the extensionality of Ontology, and so it would not be so likely to meet with general acceptance as Brentano's own less extreme reduction. However Brentano himself was favourably disposed to extensionalism, so perhaps he would have

(11) Cf. Lejewski (*op. cit.*), pp. 165ff., also C. Lejewski, "Systems of Leśniewskis's Ontology with the Functor of Weak Inclusion as the Only Primitive Term", *Studia Logica* 36 (1977), 323-49.

approved. In view of the various historical links (principally Twardowski) and doctrinal similarities between the late Brentano and Polish philosophers and logicians, perhaps the affinity is not so surprising after all.

Our result however does *not* endorse Brentano's contention that all propositions, or even all categorical ones, are "really" existential. For since the Brentanian system is *equivalent* with one based on 'ε', Leśniewski's nearest equivalent to the singular copula of ordinary language, it can equally well be argued in the other direction that existence is "really" given by the old Aristotelian formula, "To be is to be something":

$$\mathrm{E}!a \equiv \Pi b \ulcorner a\, \varepsilon\, b \urcorner$$

with 'E' defined by

$$\mathrm{E}a \equiv \Pi b \ulcorner b\, \varepsilon\, a \urcorner.$$

From a purely logical point of view there is no reason to prefer one basis of Ontology to another, though there may be aesthetic, psychological or pragmatic reasons favouring one or another choice of basis. In any case, our arguments apply not directly to natural language, but only to Ontology, which is more limited in its linguistic devices and syntactic categories than natural language. As Brentano's own later efforts at *Sprachkritik* testify, the attempt to account for the whole gamut of natural linguistic phenomena within a limited framework throws a considerable burden on the would-be reductive logician or linguist and presents a stern challenge to his theory of language.

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