ON THE NATURE OF THE TIME

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What is the time? This is a question which has profoundly puzzled philosophers and scientists for centuries. It has not yet received a generally accepted answer. Moreover, as St. Augustine noted long ago, we have no problems with time in our ordinary activities, but only in trying to understand it. This is both peculiar and ironic. The copresence in the concept of time of everyday self-evidence with profound mystery is perhaps behind the judgement of many philosophers and scientists that the problem of the nature of time is the most exciting and important one in all of philosophy. (1)

Since our ordinary dealings with time are unproblematic, it is easiest and most natural to begin a search for an understanding of time with these dealings. They involve fundamental notions of duration, additional and partially derivative notions temporal order, and on top of these partially derivative notions of date or temporal position. Besides this basic complex of temporal notions, there is that of the shifting temporal modalities of past, present, and future and the unique dynamism of the now. Thus, time is not a simple entity, but a complex of interconnected constituents of at least two basic kinds. An attempt is here made to investigate the nature of these constituents and their interconnections.

1. *Fundamentals of measurement*

The basic notion required for measurement in general can be taken

(1) Many of the best known philosophical texts on time like that of St. Augustine are to be found in [3]. The more recent ideas and issues dealing with time are taken up in the collection [2].

to be that of a strict ordering. This is a transitive and asymmetric relation P such that the relation C={xy:x and y ∈PF=the field of P, but neither xPy nor yPx} is transitive and so an equivalence relation. The relation C is the equivalence relation determined by P. A positive joining operation from P is a 2-place function o such that x through z∈PF if xoy = z. Also, the following conditions must hold for v through z∈PF.

1) If xoy and yox ∈PF, then xoyCyox.
2) If xo(yoz) and (xoy)oz ∈PF, then xo(yoz)C(xoy)oz.
3) If xCv, xoy and vow ∈PF, and Q=P or Q=C, then xoyQvow just when yQw.
4) xPy just when there are v,w, and a zCx such that z in an o-part of y. That is, y is one of zov, voz, and (voz)ow.

That is, o has the commutative, associative, and cancellation properties in PF with respect to C and P that addition has among numbers with respect to = and <. Also, an o-join determines order among objects like + does among positive numbers. Hence, xPxoy and all objects behave like positive numbers.

A one-unit system of measurement can be thought of as a 5-place sequence s=<Poukd> with P a strict ordering, o a positive joining operation from P, u∈PF, k a positive real number, and d a positive integer. Let xSiyj be the recursively definable notion that x stands in C to (i times y)o(j/d times y) for natural numbers i and j one of which is positive and such that j≤d. Then $C_1(s)$ = the concrete one-unit measure determined by s = {xy: there are natural numbers i and j one of which is positive such that j≤d, xSiuj, and y=(i.k)+((j/d).k)}.

To understand how all of this works, it is useful to think of P as the shorter that relation among some rods, o the operation of joining rods end to end, u a meter rod, k=1, and d=1000. Then $C_1(s)$ assigns positive rational number lengths to rods in meters and/or millimeters.

It is sometimes useful for a measure to assign even irrational real number values by means of infinite sets of previously assigned values. The fundamental one-unit measure determined by s=$F_1(s)$ which does so in the needed cases is to be $C_1(s)$ completed in the obvious mathematical manner. That is, $F_1(s)$ is to be the intersection of all functions K from a subset of PF into the positive real numbers for which the following hold.

1) $C_1(s) \subseteq K$.
2) Assume that $S \subseteq PF$ is not empty and that y is a P-least upper bound of S. That is, $x \in S$ only if not yPx for all x and there is no z such that zPy and $x \in S$ only if not zPx for all x. Also, let r be the mathematical least upper bound of $\{K(x): x \in S\}$ and assume that K(x) exists for any $x \in S$. Similarly, assume that $S' \subseteq PF$ is not empty, y′ is a P-greatest lower bound of S′, r′ is the mathematical greatest lower bound of $\{K(x): x \in S'\}$, and K(x) exists for any $x \in S'$. Then the pairs y,r and y′,r′ are members of K as well if y and $y' \in PF$.

In other words, $F_1(s)$ is more inclusive than $C_1(s)$ only in having P-bounds assigned their proper mathematical bounds as values when enough values to do so are available. Notice that, if PF is finite, then, since the bounds of non-empty finite sets are their maximums and minimums, $F_1(s) = C_1(s)$. [2]

2. *Duration*

Strange as it may seem, temporal length or duration is measured in the same sort of way as length. The major differences are that the objects measured are processes and their strict ordering is determined not by events of object juxtaposition, but of process concurrence. It is of interest here that events differ from objects such as rods in that they have a directional aspect. Given any event e, let In e be the unique initial part of e and let Tm e be the unique terminal part of e. A point event or instant e can be characterized by that In e = Tm e. All other events are processes. Notice that processes differ from rods in that rods can be assigned initial and terminal parts in two ways while processes cannot. Since In e = In In e = Tm In e, In e is a point event. The same holds for Tm e. Clearly, e is the case only if both In e and Tm e are.

Using processes instead of elongated objects, a strict ordering P of processes according to their durations is to be thought of in a way which makes xPy mean that x and y are processes and every part of x

[2] This treatment of measurement is from [4].

corresponds to a part of y, but not vice-versa, if x and y are in concurrence. That is, x and y are run at one place so that one overlaps the other. Disturbing influences of arbitrary factors are here to be retained. If x and y are not run in one place so that one overlaps the other, they are in concurrence if perhaps imaginary duplicates x′ and y′ of them are run in this way. A part of x or y then corresponds to another of y or x if the duplicate of the first part does to the duplicate of the second one. (3)

Again using processes instead of elongated objects, that o is a positive joining operation from P should imply that, if x and $y \in PF$ and $z = xoy \in PF$, then In x = In z, Tm x = In y, and Tm y = Tm z. That is, o joins the initial part of y to the terminal part of x. Also, if In x = In y, Tm x = Tm y, and y has an o-part z such that zCx, then In z = In x and Tm z = Tm x. The observation of natural processes indicates that there are no processes x and y such that a join xoy of them in this way is a point event. That is, all natural processes have distinct initial and terminal parts and so point events are really dimensionless. We seem to only experience processes, but some (such as the blinking of an eyelid) run by so quickly that they appear to be point events. If e is experienced as a process, then In e differs from Tm e in that we have a memory trace of In e without any experience or trace of Tm e, but not vice versa. Also, our various biological clocks provide a clear feeling of duration or waiting between In e and Tm e. These observations are, of course, not subjective or illusory, but the effects of differences between In e and Tm e on our material constituents. In addition, if a process e is reversed as e′ (such as when a car is driven up and then backed away), we do not experience e and e′ as the same event run in opposite directions since In e ≠ Tm e′ by these same cues although In e′ = Tm e. Besides the cues and their material foundations, there are the differences in the relationships which In e and Tm e′ bear to the relative configurations of heavely bodies, watch components, and so on.

Once the strict ordering P and positive joining operation o have

(3) Those who manage to swallow the thesis of relativity that xPy can hold via one coordinate system while yPx or xCy holds via another must assume that only coordinate systems ar rest with respect to the fixed starsare used in the determination of P and C. Without the thesis, P and C are independent of coordinate systems.

been specified in the same sort of way as in the case of length, but now on the basis of processes and their concurrences, a unit must be selected. An easily transportable repeating process is most useful as a unit u. This can be the repetition of a pulsation, the burning of a candle of standard composition and size, or the completion of a circuit of an object moving about a closed path. If u is the completion of the circuit of the second hand of a pocket watch, it is convenient to have k=1 and d=60. The system s=<Poukd> specified in this way is a system of fundamental measurement of duration or temporal length. That is, the fundamental one unit measure $F_1(s)$ is one which assigns to some processes their durations in minutes and seconds.

3. *The direction of time*

If a,b, and p are events, then p is a process from a to b just when p is a process, In p = In a, Tm p = In b, and p is the case only if both a and b are.

If p is a process, then p is clearly a process from In p to Tm p. Let P be a strict ordering, C the equivalence relation determined by P, and o a positive joining operation from P with the mutual characteristics mentioned in the discussion of duration. Assume that a through c are events, p is a process from a to b, and q is a process from b to c. Then there is clearly a process from a to c. If p through c and poq $\in$PF, then poq is the needed process. Similarly, if p is a process from a to b, then there are processes from In a to In b and In a to Tm b. These are in general p and pob. If not pPa, then there are also processes from Tm a to In b and Tm a to Tm b. These are in general q and qob if p = aoq.

It will now be shown that time has a unique direction. That is, if p through b are events and p is a process from a to b, then there is no process q from b to a. Assume that both exist. Then there is a process r from a to a. In general, r = poq with o a temporal joining operation. But then r is a point event. This is a contradiction since an event spanning over two processes is itself a process. That is, a reversal of time requires that certain processes are point events. This is impossible by the definition of what a process is which is in turn justified by the observation that every natural process has distinct initial and terminal parts. The situation with rods is somewhat different. If p is a rod from a to b$\neq$a, then there is clearly a rod q from b to a. The rod q is simply p rotated to point in the opposite direction. This would not be possible if space were unidimensional like time. That is, having distinct endpoints only provides a unique direction within a unidimensional manifold. Notice, however, that there is nevertheless no such thing as a join r of p to q at b for rods. For processes, there is such a join if the inversion q of p exists.

There is therefore no need to search for very special non-reversible processes such as the expansion of circular waves in order to confirm that time has a unique direction. The reversing back and forth swing of a pendulum suffices since even this process has distinct initial and terminal parts.

Another important consequence of that no process is a point event is that, if p and p$'$ $\in$PF, In p = In p$'$, and Tm p = Tm p$'$, then pCp$'$. That is, processes with the same bounding events take equally long. For, if not, then the longer process, say p$'$, has an o-part qCp. Hence, In q = In p and Tm q = Tm p. But then p$'$ has a projecting o-part r such that In r = Tm r.

4. *Temporal order and simultaneity*

What does it mean to say that one event e is earlier than or simultaneous with another e$'$? Intuitively, it means that the date of e is smaller than that of e$'$ or identical with that of e$'$ respectively. However, dates can only be properly assigned to events if temporal order has already been established. Hence, this procedure is circular. Some physicists have tried to define the earlier than relation in terms of a causal connection between e and e$'$. However, unconnectible events are then simultaneous and simultaneity is then not an equivalence relation although it seems that it should be. Even worse, the earlier than relation is then not a strict ordering since such a relation has as a characteristic that the set of unrelated pairs is an equivalence relation. Finally, there are clearly causally unconnectible events (such as my writing this page today and a explosion on a star millions of light years away yesterday) which nevertheless stand in the earlier than relation. Hence, the definition of temporal order in terms of the causal connectibility of the concerned events is untenable. Those who adopt it have apparently jumped from the fact that e is earlier than e′ if e causes e′ to the unwarranted conclusion that the converse also holds.

One adequate procedure for defining temporal order is indirect and depends on a strict order P of processes according to their durations and a positive joining operation o from P. In such a case, r = <Po> can be called an additive process system. Such an r is dichotomously closed just when the following conditions hold.

1) For any distinct events e and e′ ∈ the r-events = the union of PF and {i:there is a p∈PF such that i = In p or i = Tm p}, there are processes p and p′ ∈PF such that In p = In p′, Tm p = In e, and Tm p′ = In e′.
2) For any p and p′ ∈PF such that In p′ = Tm p, pop′ ∈PF.

That is, both e and e′ branch out along two processes with a common initial part. This part could be the terminal part of a signalling process or of a common cause for both the events. Also, processes which can be joined end to beginning are so joined.

A system of temporal ordering is a dichotomously closed additive process system. Given such a system r, if e and e′ are r-events, e is r-prior to or r-earlier than e′ just when there are processes p and p′ ∈PF such that In p = In p′, Tm p = In e, Tm p′ = In e′, and pPp′. That is, the branch to e is shorter than the one to e′. Notice that e≠e′ since not pCp′. If C is the equivalence relation determined by P and e and e′ are r-events, e is r-simultaneous with e′ just when either e=e′ or there are processes p and p′ ∈PF such that In p = In p′, Tm p = In e, Tm p′ = In e′, and pCp′. In this case, the branches are

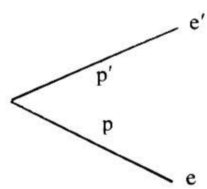

Figure 1. Defining temporal order by means of branching processes p and p′ to distinct events e and e′. The events are simultaneous just when p and p′ take equally long. If this is not so, then the earlier event is the one on the shorter branch. Of course, any event is also simultaneous with itself without any use of branches.

equally long. In what follows, E_r = {ee′: e and e′ are r-events such that e is r-earlier than e′} and S_r = {ee′: e and e′ are r-events such that e is r-simultaneous with e′}.

It can be shown that, if p,p′,q, and q′ ∈PF, e and e′ are r-events, In p = In p′, In q = In q′, Tm p = Tm q = In e, Tm p′ = Tm q′ = In e′, m and m′ ∈PF, In m = In m′, Tm m = In p = In p′, and Tm m′ = In q = In q′, then pPp′ just when qPq′. Consequently, since C = the equivalence relation determined by P, pCp′ just when qCq′. This important fact can be called the uniqueness lemma for branches. Assume the antecedent of the lemma. Then mop C m′oq and mop′C m′oq′ since these joins ∈PF and have common initial and terminal parts in both the first and second pairs. If pPp′, mop P mop′ since o-joins with the same first arguments preserve order. Consequently, m′oqP m′oq′. But then, by the cancellation property of o, qPq′. The proof of pPp′ from qPq′ is analogous. That pCp′ just when qCq′ then follows immediately from that C is the equivalence relation determined by the strict ordering P. However, it could also be proven on its own like the first part of the consequent.

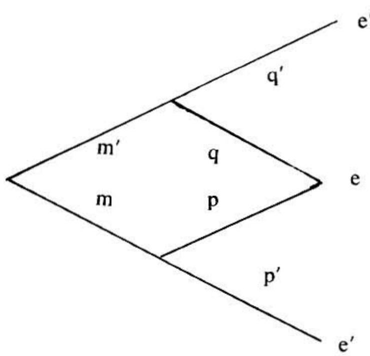

Figure 2. The setup of the uniqueness lemma for branches. The letters to the right denote events and the others processes.

Another important fact is the transitivity lemma for branches. This is the assertion that, if p,p′,q, and q′ ∈PF, e, e′, and e″ are r-events, In p = In p′, In q = In q′, Tm p = In e, Tm p′ = In e′ = Tm q, Tm q′ = In e″, m and m′ ∈PF, In m = In m′, Tm m = In p = In p′, Tm m′ = In q = In q′, pPp′, and qPq′, then mop P m′oq′. Also, pCp′ and qCq′ only if mop C m′oq′.

For the proof, assume the antecedent. Since the joins have the same endpoints, mop C m′oq. Also, mop P mop′ and m′oq P m′oq′ from the uniqueness lemma since pPp′ and qPq′. Consequently, mop P m′oq′ since P carries over C-equivalent arguments. The argument for C is analogous, but depends on the transitivity of C rather than that of P over C.

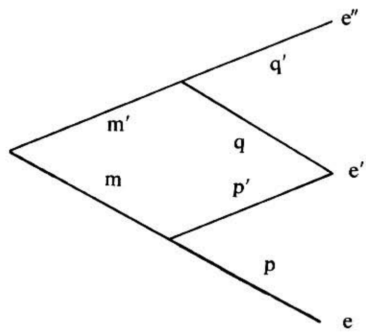

Figure 3. The setup of the transitivity lemma for branches. The letters to the right denote events and the others processes.

These two lemmas are needed to prove what can be called the fundamental theorem of temporal ordering. This is the apparently simple assertion that, if r is an additive process system, then E_r is a strict ordering and S_r = the equivalence relation determined by E_r. By the uniqueness lemma and the asymmetry of P, E_r is asymmetric. Also, by the symmetry of C, S_r is symmetric. The transitivity of both E_r and S_r follow from the transitivity lemma. If eS_re', not eE_re' and not $e'E_re$ by the uniqueness lemma and the mutual exclusiveness of C and P. On the other hand, if e and e′ are r-events and neither eE_re' nor $e'E_re$, then eS_re' follows from the mutual exhaustiveness of P and C. This proves the theorem.

It should be noted that, if P and P′ are process strict orderings, PF=P′F is a set of processes, C is the equivalence determined by P, and C′ is the equivalence relation determined by P′, then pPp′ just when pP′p′ and pCp′ just when pCp′ for any p and p′. That is, P and P′ as well as C and C′ are absolute in that no alternative orderings order their fields in a different way. This is because of the way both P and P′ and so C and C′ are determined by part to part correspondences between the constituents of processes. Also, if o and o′ are positive joining operations from P and P′, then pop′ = po′p′ if either pop′ $\in$PF or po′p′ $\in$P′F since o and o′ join the same processes in the same way initial part to terminal part. Consequently, if r = <Po> and r′ = <P′o′> are systems of temporal ordering and the r-events = the r′-events, then $eE_{r'}e'$ just when eE_re' and $eS_{r'}e'$ just when eS_re'. Consequently, E_r and S_r are also absolute. If PF = the set of all natural processes, then E_r and S_r are the unique absolute earlier than and simultaneity relations for all natural events = the set of all r-events.

The use dichotomously closed additive process systems for the above constructions may at first appear a bit strong. However, the additive closure is automatic for natural processes since they come joined to any processes starting at their terminal parts. Also, if the universe has evolved from a single primeval explosion or "big bang" as astronomers now hold to be very likely, then there is a single cause b for all subsequent natural events. It should be assumed that b is a point event. If it is also assumed that there is a pre-explosion process p such that Tm p = b, then the set of all natural events separate from p with b branches back to In p and can be temporally ordered by the procedure given above. Thus, this procedure seems to fit nature well. Notice that temporal priority and simultaneity are thereby only given definite correct meanings. It should not be expected that they always allow us to decide whether priority or simultaneity holds between two natural events. If all events are traced back to a single ancient event such as the primeval explosion, then there are clearly very early events whose temporal order we cannot determine since we do not even know of their existences. Thus, just like an explicit definition of truth does not provide the decidability of the truth or falsity of any statement, explicit definitions of temporal priority and simultaneity do not allow us to settle all questions of the temporal ordering of natural events.

5. *Axiomatizing temporal order*

Perhaps due to a lack of logical background, scientists have heretofore not noticed that concepts of temporal order need not be defined, but only axiomatized. That is, the special and weak kind of axiomatization involved in definition can be dropped in favor of a more general and powerful kind. One such axiomatization can be based on the main results obtained above by means of definitions. That is, it can be assumed that there are relations P, C, E, and S satisfying the following conditions.

1) The fields of P and C are the set of all natural processes, P is a strict ordering, and C is the equivalence relation determined by P.
2) The fields of E and S are the set of all natural events, E is a strict ordering, and S is the equivalence relation determined by E. Of course, it is assumed that In p and Tm p are natural events if p is a natural process.
3) For any natural processes p and p′, pPp′ just when every part of p corresponds to one of p′ and not vice-versa under assumed concurrence.
4) For any natural events e and e′ and natural processes p and p′ such that In p = In p′, Tm p = In e, and Tm p′ = In e′, eEe′ just when pPp′ and so eSe′ just when pCp′.

Notice that E and S are also absolute. That is, if r is a system of temporal ordering and every r-event is natural, then, for any r-events e and e′, eE_re' just when eEe′ and eS_re' just when eSe′. As before, the proof depends on the fact that P and C are defined in the usual standard manner.

These axiomatic notions of temporal order are superior to the defined ones in that they do not require dichotomous closure or additivity under a natural join although they lead to the same results as those obtained by means of these conditions when they are satisfied.

6. *Absolute simultaneity and the theory of relativity*

According to relativity theory, there is no such thing as absolute simultaneity. Are then the explications absolute simultaneity given above erroneous? The answer is no. It is in fact the relativistic view which is erroneous.

The relativistic objection to absolute simultaneity is based on variants of a single argument which is logically unsound. The argument in its standard form is that a vehicle with a so-called observer inside of it passes with constant rectilinear motion another such observer outside the vehicle. A light is flashed in the middle of the vehicle. According to be observer in the vehicle, the flash reaches the front and back walls of the vehicle simultaneously since the velocity of light is the same in both directions and has to cover equal distances. Notice that this claim presupposes a constant velocity of light. On the other hand, according to the outside observer, the flash reaches the back wall of the vehicle first since it is moving towards the position in his space at which the flash started while the front of the vehicle is moving away from this position. The remarkable conclusion drawn from this paradox in relativity is that both observers are right and so that simultaneity is relative to a frame of reference. However, from a logical point of view, at least one observer is wrong since they are making mutually contradictory claims about a single phenomenon. This can best be seen from the fact that simultaneity is by formal necessity an equivalence relation among natural events determined by the earlier than strict ordering among natural events independently of at least all moving coordinate systems. Although apparently unnoticed by relativitists for decades, this relation exists even in relativity as simultaneity in all frames at rest with respect to the fixed stars. Hence, if a is earlier than or simultaneous with b, it is so in all coordinate systems irrespective of what times are determined for them by transported or disturbed clocks. Consequently, if a is earlier thab b according to the outside observer, a is simultaneous with b according to the inside observer, and both are right, then a is earlier than a via the attributes of simultaneity. This is a contradiction. The situation seems to be that, if the outside observer is at rest with respect to the fixed stars, then he is right and the inside observer is wrong on the basis of the classical wave theory of light. That is, synchronous clocks isolated from disturbing influences provide simultaneity by identity of readings only in such systems. Moving observers must convert their clock readings to such reliable ones by means of calculations or special gadgetry before they can use them to determine temporal order.

Of course, the Lorentz transormations imply that the outside observer has different flash arrival clock readings while the inside observer has the same ones. However, the clocks of the inside observer are here unreliable. In particular, the assumption of a constant value c for the velocity of light in all inertial systems implies that clocks run haywire for no reason at all at different positions in a moving inertial frame. This casts no doubt whatever on absolute simultaneity, but rather on the constancy assumption with its ad hoc consequences. Indeed, it was shown in [5] that the assumption is a contradiction. Thus, although the assumptions used above to refute the relativity of simultaneity are unavoidable, those used to establish it in relativity are untenable. (4)

7. *Dates*

To assign a date to an event e, some instant part of e must be selected as the event to be dated. In general, this seems to be the initial part In e of e. Some well-recorded event a connected by processes with directly or indirectly measurable durations with the majority of the events to be dated must also be selected as a base event which is to have a base date. It is just the coordination of a number with a base event which binds a date scale to a sequence of events. The base event is in general a past one since few present or future events have the needed properties. Those which do are of necessity connected in some definite way with one or more past ones that are employed like base events in actual date assignment.

Now let s = <Poukd> be a system of fundamental measurement of duration. Assume that every e ∈PF is a natural process, that r=<P o>, and that a is an r-event. If p and q ∈PF, p is an r-subprocess of q just when p=q, or there is a u ∈PF such that q=pou or q=uop, or there are u and v ∈PF such that q=uopov. Also, e is r-connected with a

(4) For an introduction to relativity, the reader is referred to [1]. A more extensive and detailed critique of relativity is given in [5]. It is perhaps of interest that [5] was once the present section, but became too extensive and important to remain so. The section is mostly a part of the beginning of that paper.

by p just when e is an r-event, p ∈PF, In p S In e and Tm p = In a if eEa, and In p = In a and Tm p S In e if aEe. That is, p goes from an event simultaneous with e to a if e is earlier than the base event a and in the opposite direction if a and e are in the opposite temporal order. Assume finally that, for any r-events e and e′ such that not eSa and not e′Sa, there are p and p′ such that e and e′ are r-connected with a by p and p′ respectively and pop′ ∈PF if Tm p=In p′. Also, if eEe′Ea or aEeEe′, then p′ and p are r-subprocesses of p and p′ respectively. This means that the processes connecting e and e′ with a overlap if both e and e′ are before or after a. It can be called the linearity property of r. In such a case, the system d=<san> with n a real number can be called a dating system. Let the d-events = the r-events. Also, let l be the fundamental measure of duration determined by s. Certain d-events e can be assigned dates by the dating function t determined by d as follows.

1) For any d-event e such that eSa, t(e)=n.
2) Assume that e is a d-event such that not eSa and that e is r-connected with a by p. Then t(e) = n− l(p) if l(p) exists and eEa. On the other hand, t(e)=n+ l(p) if l(p) exists and aEe. Of course, if l(p) does not exist, t is not defined for e. When every d-event ∈tD, t is fully d-defined.

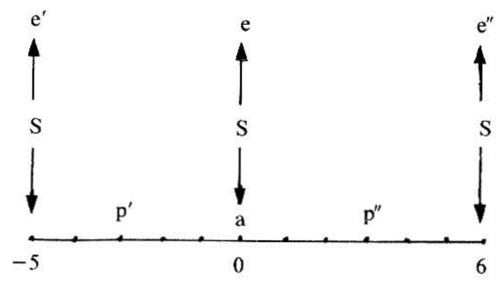

Figure 4. The dating setup. The events e′ and e″ are connected by the processes p′ and p″ with the 0-event a. Also, e is simultaneous with a while e′ and e″ are simultaneous with the initial and teminal parts of p′ and p″ respectively. Then a and e have the date 0. Also, since the durations of p′ and p″ are 5 and 6 respectively, the dates of e′ and e″ are 0−5 = −5 and 0+6 = 6 respectively.

The adequacy of this method of dating is expressed in what can be called the fundamental theorem of dating. This is the assertion that, if d is a dating system, t is the dating function determined by d, and t is fully d-defined, then, for any d-events e and e′, eEe′ just when $t(e) < t(e')$ and eSe′ just when $t(e) = t(e')$. The proof from the temporal E to the numerical $<$ is by analyzing the cases of eEe′Ea, eEe′Sa, eEaEe′, aSeEe′, and aEeEe′ via the transitivity of E and S, the linearity property of the concerned $r = <Po>$, and the fact that p and q P poq for p and $q \in PF$ such that $poq \in PF$. The proof from S to $=$ is also by analyzing cases via the transitivity of S. The converses then follow from the proved part of the theorem and eEe′ or eSe′ or e′E e via the asymmetry of $<$ and the incompatibility of $<$ and $=$.

For a dating system $d = \ll Poukl \rhd\ a\ n >$ like the one now being used on most of the earth, u is one revolution of the earth about the sun, $k=1$, $l=365$, a an event assumed to be simultaneous with the birth of Jesus, and $n=0$. In this way, a date of year and day is assigned to the covered natural events with positive dates corresponding to A.D. dates and negative dates to B.C. dates by the dating function determined by d. Notice that a day here does not coincide exactly with a single rotation of the earth since the extra leap year days are omitted.

It is of interest that it is not impossible that there is a system $d' = \ll Pou'kl \rhd\ a\ n >$ whose only difference from d is that the unit u′ is one period of a process of exactly the same kind as u which is thought to be of the same duration as u, but is in fact slightly longer or shorter. The dates obtained by means of d′ will then be smaller and greater than those obtained by means of d for events which are common to both systems respectively.

On the other hand, a process p dilated or contracted in the same sort of way as u′ will have its normal duration. So, for example, a free-floating astronaut should age more quickly than his twin brother on earth and his watch should have contracted hours, minutes, and seconds because of the absence of the time-dilating effects of gravitation. Thus, it is not at all incompatible with time dilation that there is an absolute temporal order. The mistaken viewpoint of incompatibility seems to be widespread among physicists.

8. *Past, present, and future*

An aspect of time which is ignored in physics and poorly understood in philosophy is that of the transition from future to present and of present to past. A moment m can be thought of as a simultaneity class of natural events. That is, m = {e:e is a natural event and e is simultaneous with e′} for some natural event e′. Since e′ is simultaneous with itself, it follows that e′ ∈m. If m and m′ are moments, m is earlier than m′ just when there are e ∈m and e′ ∈m′ such that e is earlier than e′. It follows that m is earlier than m′ just when e is earlier than e′ for any e ∈m and e′ ∈m′. Similarly, m is simultaneous with m′ just when there are e ∈m and e′ ∈m′ such that e is simultaneous with e′, and so just when e is simultaneous with e′ for any e ∈m and e′ ∈m′. Hence, m is simultaneous with m′ just when m=m′. Also, the earlier than relation among moments is a strict ordering whose equivalence relation is the identity relation among moments.

Moments are useful for the analysis of temporal concepts. Time itself can of course be thought of as the set of all moments. The present or now can be thought of as a sequential object involving moments in whose stages alone events exert causal influences and existence is actual. Future events have not yet come to be and past events have passed away from actuality. The shifting of events into the past can be understood to be the evolution of the now into the future. Speaking pictorally, the now can be likened to a flame. It is a flame of causal action and actual existence which reaches out through space and gaplessly consumes its constituents into its future configurations. The main point of this analogy can be reformulated in an analytic manner which indicates the structure of the now. If m is a moment, the past of m = {m′:m′ is a moment earlier than m}. A past P is the past of m for some moment m and the present of P = the unique moment m such that P is the past of m. The now N can then be defined as the function defined on the set of all pasts which assigns to each P in its domain the present of P. In this way, N(P) is a moment which N has transformed the section of time up to that moment into and N strides through time in that time is the range of N. The future of a moment, a future, and the present of a future can be defined in the same sort of way as the corresponding notions for the past.

Presumably, the latest stage of the now is the transformational

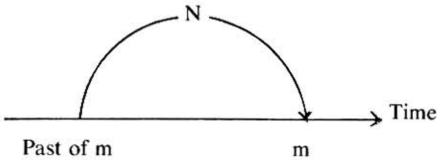

Figure 5. The now N is a function which irrevocably transforms the past of a moment m into m.

product of a primordial explosion which was perhaps the result of previous stages of contraction. That moments, actuality, and causality should be transformed or consumed stagewise in this manner is puzzling. However, it is perhaps no easier to explain this than why there is mass or gravitation.

Notice that, if the now attains future moments by stagewise converting the earlier ones into them, then both travel to the past and rapid travel into the future are impossible. This is also expressed rather strongly by the irrevesibility of temporal direction and causal action together with the impossibility of reaching an effect without going through a process which realizes it over a temporal interval. Expressing the matter in another way, we cannot cause past events involving ourselves since causes have only future effects. Also, we can only cause such future events by planning for them in the ordinary manner and over the same processes that they require to come to be if they come at all. That is, we must wait for the future moment to come to be by itself with its events.

The whole idea of time travel seems to arise from the possibility of having a linear diagram or model of time in which indices or images of all moments are at least in principle laid out. Since the representations of all the moments are copresent, it is possible to move freely into the past or future in the diagram from a moment image. It is then apparently imagined that the same kind of motions can be made among the represented moments although such motion is in fact out of the question for the reasons given above.

The shiftiness or relativity to moments which the notions of past, present and future display has caused some philosophers to claim that they, and particularly the now, are purely subjective or at least unreal. Since every moment is at least a now value, the additional claim that all of time is unreal has also been made. However, all of these views are erroneous. The shiftiness is not subjective, but objective and relational in the same sort of way that the shiftiness of operations over their values are. For example, although the father of John $\neq$ the father of George, this shiftiness of the father operation does not make it subjective or unreal, but only relational or argument dependent. That is, there is no person the father, but only different fathers for different persons. Nevertheless, although it is not a person, the father operation also exists. The same holds for the concepts of past, present, and future.

There are perhaps other notions of the unreality of time. However, durations are real in that processes really have relative lengths. Temporal orderings are real in that they express durations from common initial events. Although the numerals used to index moments in dates denote numbers and so are in a sense unreal, dates are real in that they express how many times a basic process goes into another whose initial and terminal parts are simultaneous with the initial parts of selected events. Even time dilated dates are real in this way just as measures of length obtained by means of a hot meter rod are real although shorter than those obtained with such a rod at a lower temperature. It can therefore be concluded that the only kinds of unreality to which temporal concepts can be subject are those due to various kinds of abstractions or errors which can be made by those who determine durations, temporal orders and dates. Of course, such things can also be made in the determinations of other quantities such as those of weight or length.

9. *Completing time*

Time as the set of moments characterized above is in fact not intuitively complete since the moments concerned must be determined via the initial parts of particular natural events. For example, suppose that the history of the universe consists of just two point events a and b with a earlier than b. Then it is intuitively necessary that time was running from a to b even if nothing happened in between. Thus, the time hitherto dealt with is not intuitively complete in that it may only contain the moments at which natural events do start and not the moments at which they could start. Such a time is like a space which may contain only the positions occupied by natural objects and not all possible positions. However, if all possible moments are to be in time along with the occupied ones, then time must be unbounded, dense, and continuous in the mathematical senses. This is perhaps surprising to some intuitions, but formally necessary.

To obtain the result that time has these features, it is sufficient to axiomatically strengthen E and S in the expected mathematical manner. Since the moments which result also consist of natural events, there are natural events between events like our two above which have a dubious reality. However, this reality is just like that of unoccupied spatial positions. Perhaps it only seems dubious because it is less familiar than that of unoccupied spatial positions. However, since spatial positions are occupied at some times and not at others, each of them must exist at any time. Consequently, the dubiousness can be removed by understanding the events in question to be those of the existences of p for all spatial positions p at the moments to be determined.

Let H = the history of the universe = the set of all natural events = the fields of both E and S. That time is unbounded above and below is expressed by the following assertion.

5) For any $e \in H$, there are d and f such that dEeEf. That is, for any natural event, there are an earlier and a later natural event.

The density of time is expressed by

6) For any d and f such that dEf, there is an e such that dEeEf. If a natural event is earlier than another, then there a third such event in between them.

For continuity, the usual mathematical apparatus can be used. Let E′ = the relation of not being earlier than among natural events = $\{ef: e \in H, f \in H, \text{and not } eEf\}$. A $b \in H$ is an upper or lower bound of a set $M \subseteq H$ just when eE′b or bE′e respectively for any $e \in M$. Also, b is a least or greatest upper or lower bound of M just when b is an upper or lower bound of M and there is no c such that cEb or bEc and c is an upper or lower bound of M respectively. The assertion of the continuity of time is then the following one.

7) For any non-empty $M \subseteq H$ such that there is a b which is an upper or lower bound of M, there is a c which is a least upper bound or greatest lower bound of M respectively.

To complete time is simply to add 5) through 7) to the previously given four axioms for temporal order.

From these axioms, it follows that, even if there were no matter or energy in the universe, the now would nevertheless evolve in empty space with each moment looking like every other one. This rather dull universe of spacetime without matter or energy seems to be what some philosophers have inappropriately called the "nothing". If matter or energy can suddenly appear in the universe and then somehow disappear again, then the more interesting phases of its history could be interspersed with such dull phases. Such a phased universe seems to be similar to the famous Tad Ekam or "That" of the Ṛg-Veda. (5)

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(5) For the moving Nāsadīya hymn about "That", see pp. 14-15 of [6].