IDENTITY, INDISCERNIBILITY AND GEACH

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P. T. Geach in his recent article «Identity» (1) argues for the thesis that identity is relative. He accuses proponents of absolute identity such as Frege and Quine (2), of not being in the position to offer an adequate account of the notion of absolute identity. In this article, I want to take up one main argument which Geach presents for this view (3) and to show that it is not sound. In this connection, I shall also discuss some questions concerning the relationship between identity and indiscernibility.

I.

According to Quine, we can obtain an adequate theory of identity by adding to the vocabulary of standard quantification theory the two-place predicate '=' (4) as a primitive, and to its set of axiom-schemata the axiom

(A1) $$x = x$$

and the axiom-schema

(A2) $$x = y \,.\, Fx \supset Fy$$ (5)

For convenience, we shall denote from now on the conjunction of (A1) and (A2), by (W). The so obtained theory of identity, called by Geach, «the classical theory of identity» (6), is, as is well known, consistent and complete. (7)

Geach however, isn't satisfied with this theory and in spite of its notable features, he contends that it cannot serve as an adequate account of the notion of absolute identity. The route by which he reaches this conclusion is this. Geach basically assumes that a satisfactory theory of absolute identity must provide a necessary and sufficient condition for a predicate to express absolute identity. (8) Consequently, the question that arises for him is whether or not the classical theory can meet this requirement. Geach considers two ways in which the classical theory of identity can be said to do so. One possibility is to say that

(P1) A two-place predicate $\ulcorner\alpha\urcorner$ expresses absolute identity in a given theory T iff it is an I-predicate in T. (9)

By an «I-predicate in T», Geach understands a predicate whose substitution in (W) for '=', yields a schema that comes out true under any interpretation which assigns to 'F' a predicate in T, and to 'x' and 'y' referring expressions in T. (10)

However Geach, doesn't accept (P1). He argues that a predicate $\ulcorner\alpha\urcorner$ that functions as an I-predicate in a given theory T, need not express in T absolute identity. The open sentence $\ulcorner\alpha\, x\, y\urcorner$, need, according to him, mean no more than that x an y are indiscernible by the predicates that form the descriptive resources of T. (11). That is, an I-predicate in a given theory, rather than expressing absolute identity, need capture only indiscernibility in that theory. (12)

Geach stresses this point by the following consideration. He draws our attention to the fact that individuals which are indiscernible in a specific theory T, may be perfectly discernible in another theory T^1, that differs from T in including a predicate that enables us to distinguish between any two objects indiscernible in T. As a result, the possibility may arise of T including a predicate $\ulcorner\beta\urcorner$ that is an I-predicate in T, but which is not an I-predicate in T^1. Geach claims now that someone who would wish to consider (P1) as a necessary and sufficient condition for absolute identity, would be in this case committed to the absurd consequence of saying that the same predicate $\ulcorner\beta\urcorner$ that expresses absolute identity in T, doesn't do so in T^1, in spite of the fact, that its meaning didn't change when passing from T to T^1. (13) As a result of that, Geach concludes that one can't take an I-predicate to express absolute identity, and suggests therefore to reject (P1).

The second attempt, considered by Geach, to provide a necessary and sufficient condition for absolute identity on the basis of the classical theory of identity, reads as follows.

(P2) x = y iff *whatever* is true of x is true of y, and conversely. (14)
In other words, a predicate $\ulcorner\alpha\urcorner$ expresses absolute identity iff it satisfies schema (W) for *all* substitutions for 'F'.

The idea behind this suggestion is clear. (P2) tries to avoid the shortcomings of (P1), by placing no restrictions on the predicates that fall under the scope of (W).

Geach however, isn't willing to accept (P2) either. He argues that the unrestricted use of 'whatever is true of' in (P2) leads directly to the well known semantical paradoxes. ([15]) As a result, Geach concludes that the classical theory of identity cannot serve as a satisfactory account of the notion of absolute identity. For Geach however, this conclusion is crucial, since in his opinion no alternative account of absolute identity, which could replace the one provided by the classical theory, has thus far been suggested. ([16])

II.

We shall turn now to the examination of Geach's argument. On the basis of what has been said till now it goes as follows:

(1) Any satisfactory account of absolute identity must give us a necessary and sufficient condition for a predicate to express absolute identity.

(2) The classical theory of identity does not meet this requirement.

(3) But no other account of the notion of absolute identity, which could replace the one provided by the classical theory of identity has thus far been suggested.

(4) Hence, the proponent of absolute identity is in no position to provide a satisfactory account of the notion of identity to which he adheres.

Our problem is now to find out whether this argument is a sound one or not.

Let us begin with the examination of premise (3) of Geach's argument. In «Identity» Geach does not provide any justification for its plausibility. Also, he does not discuss such alternative definitions of absolute identity as

(D1) $x = y$ iff every set which contains x also contains y and conversely.

and

(D2) $x = y$ iff every property of x is also a property of y and conversely.

An absolutist may therefore object that as long as Geach hasn't demonstrated the inadequacy of (D1) and (D2), premise (3) of his argument is unfounded.

It is not difficult, however, to fill in the gap in Geach's argument. As to (D1), Geach could reply that it is simply circular. For (D1) is stated in terms of sets. However the principle of individuation for sets of individuals, clearly presupposes a prior understanding of the notion of identity for individuals. ([17])

A closely related reply could be given by Geach with respect to (D2). Using Quine's well known ontological principle: «No entity without identity», Geach could claim that since the absolutist doesn't have a satisfactory criterion for the identity of properties, he is not allowed to use (D2) as an adequate explication of absolute identity. After all, of what worth is an account of absolute identity, if it is based on entities whose ontological status is obscure.

Let us turn now to the examination of the second premise of Geach's argument, namely, that the classical theory of identity does not provide a necessary and sufficient condition for a predicate to express absolute identity.

We saw in the last section that Geach justifies this premise in two steps. 1) He assumes that all proposals for a necessary and sufficient condition for absolute identity based on the classical theory of identity, can be reduced either to (P1) or to (P2), and 2) he demonstrates the inadequacy of each of these proposals. We shall begin with his objections to (P1) and (P2).

As to (P1), Geach seems to be perfectly right that it doesn't provide a necessary and sufficient condition for absolute identity. (P1) says that

(P1) A two-place predicate $\ulcorner\alpha\urcorner$ expresses absolute identity in a given theory T iff it is an I-predicate in T.

But as Geach correctly points out, the fact that a predicate $\ulcorner\alpha\urcorner$ is an I-predicate in a given theory assures us only that it captures indiscernibility within that theory, and that of course can't always be taken to be coextensive with absolute identity. ([18])

In face of this criticism, the absolutist may attempt to seek within the language of first-order logic with identity a stronger requirement for absolute identity than that provided by schema (W). However, Geach can undermine this move by proving that such a requirement simply does not exist within that language. The proof for this claim might be the following.

(a) According to Gödel's completeness theorem for first-order logic with identity, each valid formula in that language can be deduced from (A1) $x = x$ and (A2) $Fx \,.\, x = y \supset Fy$, by using the standard axiom-schemata and rules of inference of first-order logic. ([19])

(b) Since (W) forms the conjunction of (A1) and (A2), the completeness theorem equally holds for (W), that is, each valid formula of first-order logic with identity is also deducible from (W) and the standard machinery of first-order logic.

(c) Suppose now that there exists within the language of first-order logic with identity an axiom-schema for absolute identity that is stronger than (W). As such, it obviously must be valid. But then, as already established in (b), it is in principle deducible from (W) and standard first-order logic.

(d) Hence, there doesn't exist within the language of first-order logic with identity a stronger requirement for absolute identity than (W).

Granting the correctness of Geach's criticism of (P1), let us turn now to the examination of his objection to (P2).

As remembered, Geach's objection was that

(P2) $x = y$ iff whatever is true of x is true of y and conversely,

leads to semantical paradoxes, as a result of involving an unrestricted use of 'whatever is true of'.

It seems that we must admit that Geach is right on this point. For if indeed, we use 'whatever is true of' unrestrictively, we easily run into the danger of including under its scope such semantical predicates as '——' is true; '——' is true of; '——' names; etc., predicates whose appearance in the object-language is, as we know, at the bottom of the semantical paradoxes. ([20])

But the proponent of absolute identity may easily avoid these shortcomings of (P2). What he may do is simply to propose a slightly modified version of (P2), say (P2′), in which 'whatever is true of' does not range over semantical predicates. ([21]) In «Identity», Geach doesn't discuss this proposal, but it would appear that his attitude towards it would be a negative one. The question of course is how he could justify that attitude.

It seems, that one possible course for him to take would be to claim that indiscernibility within a certain theory or language can *never* serve as a necessary and sufficient condition for absolute identity. ([22]) And in (P2′) it is clearly such a notion of indiscernibility in terms of which identity is defined.

However, such a claim on Geach's part couldn't be correct. A situation in which absolute identity can be defined in terms of indiscernibility within a certain theory, is one in which the predicates of a particular theory T enable us to distinguish between any two distinct individuals of T. In such a theory, since no *two* of its individuals could be indiscernible relative to its predicates, the condition that for any x and y in T, x = y iff x and y are indiscernible in T, would clearly hold.

But there is still another line of argument, which Geach might choose in order to refute (P2′). Geach might claim that though the absolutist may have succeeded in showing that there are theories for which a definition of absolute identity in terms of indiscernibility is available, no such definition can be provided for theories ranging over individuals with more than a denumerable number of properties. The idea behind this claim is that in such theories, in order to capture absolute identity in terms of indiscernibility, we would need to have the means for expressing a non-denumerable number of properties. But since (P2′) is formulated in terms of predicates and there is at most a denumerable number of these, (P2′) won't be strong enough to define identity for objects having a non-denumerable number of properties.

Convincing as this argument may sound, it nevertheless rests on a mistake. The root of the mistake is the tacit assumption that a definition of absolute identity for objects with a non-denumerable number of properties is possible only by making reference to all their properties. But such an assumption is obviously false. There are theories ranging over individuals having a non-denumerable number of properties, in which identity may be defined in terms of indiscernibility without having to quantify over properties. One example is the theory of real numbers containing the two-place relation symbol '<' as its sole predicate. Now in such a theory we may clearly have that

$$x = y \text{ iff } (z)(z < x \equiv z < y)$$

and this in spite of the fact that real numbers have more than a denumerable number of properties. [23] We see, therefore, that what really matters to the possibility of defining the identity relation for the objects of a given theory in terms of indiscernibility, is not the question of how many properties they have, but whether we are able to mark the difference between any two of them, by the predicates of the theory concerned.

Keeping this result in mind, we may consider one more objection that Geach might raise with respect to (P2′). Geach might claim that there is after all at least one situation where (P2′) won't be strong enough to capture absolute identity. This might happen if we had in our ontology objects distinguishable *only* with reference to a *nondenumerable* number of properties. [24] Now, in order for (P2′) to cope with such a case, 'whatever is true of' in (P2′) would have to range over a non-denumerable number of predicates. But as already mentioned, only a denumerable number of them is available.

Now whether or not this last objection is a sound one, seems to me to be an open question. The reason for this is that it rests upon the rather theoretical assumption that there might be such objects whose identity conditions can be formulated only with reference to a non-denumerable number of properties. The intellegibility of this assumption however won't be discussed here. But nevertheless, one has to admit that at least from a theoretical point of view the foregoing objection may be a sound one, and that in this sense one would have to drop a definition of absolute identity as (P2′).

Having established the first half of Geach's defence of the second premise of his argument, let us take a look at its second half, namely, that (P1) and (P2) appear to be the only proposals for a definition of absolute identity that can be provided by the classical theory of identity. It seems that in spite of the difficulties Geach has raised, there still is a way of introducing absolute identity which appears to be adequate. In order to understand this way, let us, parallel to Geach, introduce again the notion of an I-predicate for a given theory T. We shall say that

(E) A two-place predicate $\ulcorner\alpha\urcorner$ is an I-predicate in a given theory T iff its substitution in (W) for '=', yields a schema that comes out true under any interpretation which assigns to 'F' a predicate in T and to 'x' and 'y' individuals in the universe of discourse of T. ([25])

Now for the proponent of absolute identity there are obviously two predicates which in a given theory T can function as I-predicates. 1) the absolute identity predicate '=', and 2) the predicate expressing indiscernibility in T, and the question arises whether he can effectively distinguish between the two. The following example will illuminate the answer to this question.

Consider for a moment a theory T that ranges over a domain of persons, such that it's impossible to distinguish in terms of its predicates between any *two* persons having the same nationality. It is clear that in such a theory the predicates '=' and 'has the same nationality as', can both function as I-predicates. Now, however, let us see what happens to the two predicates in case we enrich the ideology of T with a new predicate that enables us to distinguish between two persons having the same nationality. We see that 'has the same natinality as', which previously expressed only indiscernibility in T, ceases now to function as an I-predicate, as opposed to the absolute identity predicate, which fulfills this function also in the new theory. The difference between these two kinds of I-predicates appears to be therefore the following. A predicate that expresses only indiscernibility within a given theory, might very well lose its property of being an I-predicate when passing from the original theory to a new one. However, a predicate expressing absolute identity, retains its status as an I-predicate, no matter in which theory it might appear. Having got this result we can suggest now the following explication of absolute identity:

(F) A two-place predicate $\ulcorner \alpha \urcorner$ expresses absolute identity iff in any theory, it can function as an I-predicate. ([26])

Once having vitiated the second premise of Geach's argument against absolute identity, let us finally consider its first one. This premise says, as remembered, that in order for the notion of absolute identity to make sense, one must be able to provide for it a necessary and sufficient condition. Unfortunately, Geach doesn't provide any justification for this premise in his article «Identity», and the impression one gets is that it is simply unfounded. This impression can be also strengthened by the following consideration.

Suppose for a moment that there is a proof, according to which, one in principle couldn't provide such conditions for absolute identity. Would this imply, as Geach says, that 'is the same as' is «just a vague expression of halfed formed thought»? Why can't we, for example, treat it as a primitive notion and regard schema (W), or any of its equivalent versions, as a partial characterization of it? But even if we suppose that Geach is right that the notion of absolute identity is unclear, there still remains the question of what are his criteria for the various relative identity predicates? At one place([27]), Geach tried to meet this objection by saying that expressions of the form 'is the same so-and-so as', play an essential role in our language and thereby stand in no need of justification. But then one might ask why the proponent of absolute identity can't say the same thing about 'is identical with'. The quite remarkable difference between the two positions, in case neither notion of identity can be defined, would be that the proponent of relative identity is committed to an infinite number of undefinable identity predicates, while the absolutist is committed to only one. ([28])

III.

Concluding now our discussion of Geach's argument, we can say that it rests on the following problematic assumptions: (1) that a satisfactory account for absolute identity must provide for us a necessary and sufficient condition for a predicate to express absolute identity and (2) that the classical identity-theory doesn't meet this requirement. As to Geach's second assumption, we have shown that contrary to Geach's contention, one can provide such conditions for absolute identity within the framework of the classical identity-theory. And as far as his first assumption is concerned, we have seen that it is either groundless, or it raises for Geach at least the same difficulties, as it does for the proponent of absolute identity. As a result of that, I would like to conclude that this argument by Geach doesn't provide sufficient grounds for abandoning the notion of absolute identity. ([29])

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NOTES

(1) P.T. GEACH, «Identity», *Review of Metaphysics*, 21 (1976-68), 3-12.

(2) G. FREGE, *Grundgesetze der Arithmetik* (Jena, 1903), p. 254. W.V. QUINE, «Review of Geach's *Reference and Generality*», *Philosophical Review*, 63 (1964), p. 101.

(3) Geach has in «Identity» also another argument against absolute identity (starting on p. 6, 3-rd paragraph). However this argument, won't be discussed here.

(4) The predicate '=' is usually taken to mean 'is identical with' or 'is the same as'.

(5) See W.V. QUINE, *Set Theory and its Logic* (Cambridge, 1963), p. 12. Since for Quine identity is absolute, I will from now on refer to the notion of identity designated by '=', by 'absolute identity'.

(6) The theory of identity in the text differs somewhat from the one cited by Geach in «Identity». There he uses (p. 3) instead of (A1) and (A2) the single axiom-schema $\ulcorner Fy \equiv (\exists x)(Fx.x = y)\urcorner$. Both theories however are, as Geach also notes, equivalent. For the proof of their equivalence, see again Quine's *Set Theory and its Logic*, p. 13.

(7) For a detailed proof of the consistency and completeness of first-order logic with identity, see G. HUNTER, *Metalogic*, (London, 1971), pp. 197-198.

(8) Geach, *op. cit.*, p. 6. (See especially the second paragraph on p. 6).

(9) *Ibid.*, p. 5.

(10) *Ibid.*, p. 4.

(11) *Ibid.*, p. 5.

(12) By 'indiscernibility in that theory', I understand indiscernibility by the predicates of that theory.

(13) *Ibid.*, p. 6.

(14) *Ibid.*, p. 5. The term 'whatever is true of', will be understood as a quantifier ranging over extensional English predicates. The term 'is true of' will be accordingly relativised to 'true of in English'. By 'English', I mean a fragment of English which is in canonical notation or can be paraphrased into canonical notation, i.e., a language Quine has in mind in *Word and Object* (Cambridge, 1960), Chapter 5.

(15) Geach, *op. cit.*, p. 5.

(16) *Ibid.*, p. 6. (See especially the second paragraph).

(17) This is so, since the criterion of identity for sets says that sets are identical iff their members are identical.

(18) It's very easy to construct a theory in which indiscernibility in that theory and absolute identity in it won't be coextensive. Consider thus a theory T, whose individual variables range over a domain of persons and whose predicates don't enable us to distinguish between two persons having the same nationality. If T has among its predicates 'has the same nationality as', then this predicate expresses indiscernibility in T, but it doesn't express absolute identity in T. For two persons may have the same nationality, without being identical.

(19) K. GODEL, «Die Vollständigkeit der Axiome des logischen Funktionen-kalküls», *Monatshefte für Mathematik und Physik*, 37 (1930), 349-360. See also note 7.

(20) See A. TARSKI, «The semantic conception of truth and the foundations of semantics», in L. LINSKY (ed.), *Semantics and the philosophy of Language* (Urbana, 1952).

(21) In general, I don't think that the elimination of semantical predicates is a good strategy for avoiding the semantical paradoxes. Thus I consider it here only as a possible way out for saving a definition of absolute identity as (P2).

(22) Up to now, Geach has only shown that indiscernibility in a theory need not *always* be coextensive with absolute identity. (See e.g., note 22). He has thereby left open the possibility that in a rich enough theory, e.g., the one underlying (P2′), the two notions may still coincide. The claim in the text is intended to rule out this possibility.

(23) See W. V. QUINE, *Set Theory and its Logic*, p. 114.

(24) I have in mind a case, where for any denumerable number of predicates, there will always be two different objects that are indiscernible by these predicates.

(25) Personally, I like this definition of an I-predicate better than the one suggested by Geach, which is purely syntactical. (See p. 2 of this article). However, the difference between these two definitions is immaterial to the issue involved here.

(26) By theory, I mean here a fully interpreted first-order theory or a theory in Quine's sense of the term. (See W. V. QUINE, *Ontological Relativity and Other Essays* (New York, 1969), p. 51).

(27) See P. T. GEACH, «A Reply», *Review of Metaphysics*, 22 (1969), p. 559.

(28) This point has been also made by F. FELDMAN, «Geach on Relative Identity», *Review of Metaphysics*, 22 (1969), p. 555.

(29) I am grateful to Alex Blum and Dale Gottlieb for helpful comments on an earlier draft of this paper.