ON THE IMPOSSIBILITY OF AN ORTHODOX SOCIAL THEORY AND OF AN ORTHODOX SOLUTION TO ENVIRONMENTAL PROBLEMS

R. Routley

Let us begin on what may be castigated as a foolish enterprise:

> Only those who fail to appreciate [the present imperfect] condition [of economics] are likely to attempt the construction of universal systems (von Neumann & Morgenstern, [1], p. 2).

Von Neumann and Morgenstern appear to have missed the importance of general (and mostly inapplicable) models in establishing negative results, for example, that such-and-such is impossible, that the so-and-so problem is unsolvable. The design of universal systems is important in the program of showing that certain general social problems are unsolvable, for example that *an orthodox economic solution of environmental problems is impossible,* that a general assessment procedure in education, or sociology, is impossible.

A social or economic system, like an ethical system, is first of all a system. A system in the general sense, may be represented by a relational structure [1]. That is to say, a (*general*) *system* S is represented by a relational structure or model $S = (K, \mathbf{R})$ where K is a set of elements (of alternatives or states or worlds, in the cases to be studied) and $\mathbf{R} = \{R_i\}$ is an indexed class of relations where each relation R_i with index i is a relation on K, i.e. a general structure is effectively a set with a batch of relations on it (cf. Bertalanffy's 'sets of elements standing in interaction', [8], p. 38). Thus algebras, which are sets

[1] The exposition of general systems theory, general optimisation theory, and the synthesis of decision theory methods sketches that elaborated in [7], Appendix 6.

with certain two-place relations defined on them, and geometries, which are sets with three-place relations such as betweenness on them, are systems. The expressive power of systems is rich, e.g. whatever can be said in quantificational logic, and much more, can be expressed. Many familiar notions can be defined for systems, as work in model theory reveals. In particular $S' = \langle K', R' \rangle$ is a subsystem of $S = \langle K, R \rangle$ iff K' is a subset of K and $\mathbf{R}'$ consists of the restrictions of the relations of $\mathbf{R}$ to K'. Thus every subset of K generates a subsystem of $\langle K, \mathbf{R} \rangle$.

General systems theory, as envisaged by Bertalanffy, can begin at this point, and its success in unifying work in biological sciences is already substantial enough to deal a heavy blow to the von Neumann-Morgenstern contention as to the point of constructing universal systems. Some of the main distinctions Bertalanffy is concerned to emphasize can be developed almost at once. For example, an *unorganised system* is a system such that the (extensional) properties of the system can be derived from the properties of (proper) components of the system (given just intersystemic relations such as spatio-temporal ones and general laws). An *organised* system is one that is not thus unorganised, i.e. where the whole is more than the lawlike consequences of the distributed components. It is immediate that an organised system cannot be deductively reduced to features of its components. If, as appears certain, organised systems actually occur, then deductive reductivism (the main form of mechanism) is bound to fail.

The systems commonly considered in system theory, e.g. input-output models, automata, feedback models, formal systems, equational models, are all special sorts of systems obtained by considering certain specific relations, e.g. input and output functions, derivability relations, etc. Decision theory and optimisation theory, branches of general systems theory, also usually begin with a much more specifc structure. And *some* structural elaboration is essential in order to introduce optimisation notions. A *General optimisation system* OS is represented by a structure $\langle K, \mathbf{R}, f \rangle$ where $\langle K, \mathbf{R} \rangle$ is a system and f is an n-place function (the *objective function* of OS) defined on a subset of K. The *objective* in OS is to maximize values of f — subject to constraints imposed, in effect, by relations **R**. In decision theory the problem is usually more circumscribed (in ways indicated, e.g., in [7]).

Economic systems — there is much evidence for claiming — are special sorts of optimisation systems. For though the systems studied invariably concern optimisation, or the determination of equilibrium states, the factors optimised are specialised; they vary from some mix of profits, efficiency, states, etc., to some mix of GDP and welfare and amenity factors for example. Furthermore the systems will be expected to include characteristically economic elements or relations, such as producers, consumers, budgetary and commodity restraints, or such like.

Now the conditions imposed on the objective function through the systematic relations in economic systems may render optimisation impossible; indeed the conditions may render an overall consistent ranking, on which optimisation depends, impossible. This is the genesis of impossibility results. Moreover the conditions imposed on the objective function or overall ranking may be realistic or reasonable ones — which can thus be reconstituted as requirements of rationality — in which case the impossibility results can hurt. Arrow's result (of [3]), for instance, has done considerable damage — the extent has still not been fully assessed — to welfare economics.

The main impossibility result to be discussed — only *the first of several limitative results* that merit investigation — is obtained by an elementary relocation of Arrow's impossibility theorem to a more general setting. For Arrow's theorem applies to much more than the determination of a social welfare or collective choice recipe. Freed from its conventional setting it is not just a problem for welfare economics, it is a problem for social and economic assessment generally. The transposition — to a general impossibility result in the social sciences — is achieved simply by replacing the social welfare or collective choice function to be determined from individual preference rankings, by an overall ranking to be constrained by factor rankings, and observing that the conditions imposed on a social welfare ranking can be replaced by appealing conditions on the overall ranking rule. In short, individuals can be convincingly replaced by factors, and individual reduction assumptions removed. The working examples should clarify the scope and importance of ranking problems — all of which are subject to the general impossibility result.

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§ 1. *Ranking Problems.* (1) *Environmental decision problems with a forestry case as working example.* Consider a 3-alternative 3-factor case. The future management of a piece of wet sclerophyll eucalypt forest over the next 100 years is to be determined. Three alternatives are considered in this real life example.

a: The forest is conservatively managed as a eucalypt forest and retained more or less in its natural state.
b: The forest is intensively managed as a eucalypt forest, with species and understorey reduction, planting, etc., and also with special recreational facilities.
c: The forest is converted to a pine plantation.

Suppose, for simplicity, the following three factors only are considered.

i: timber (value)
j: recreation
k: wildlife. Almost any other environmental factor could supplant wildlife in the example, e.g. soil, flora. Alternatively, and better for subsequent purposes, k can be considered as an amalgam of these environmental factors.

The rankings of the alternatives with respect to each factor can be represented in the following table in which the columns show the placement (first, second or third) of each alternative with respect to the factor which controls the column.

Ranking table 1

alternative\factor	i	j	k
a	3	2	1
b	2	1	2
c	1	3	3

Where the strict ranking with respect to factor f is represented by P_f, and xP_fyP_fz abbreviates xP_fy and yP_fz, the table may be expressed linearly: cP_ibP_ia, bP_jaP_jc, aP_kbP_kc.

The problem for forestry decision-making should be to determine an overall ranking P of the alternative set $\{a, b, c\}$ with one factor ranked above the other two, so that a rational choice of management alternatives can be made. For, as elsewhere (e.g. [2], p. 24), a necessary condition for a rational choice is that the most highly ranked (preferred) alternative be chosen.

The problem is thus exactly parallel to the social choice problem where a social preference ordering P is to be determined in terms of individual rankings P_f for each individual f.

Any cost-benefit analysis can be recast in a similar form, once rankings or placements give way to net benefit (benefit—cost) figures. In the forestry example the table would be supplanted by a matrix of the following form, with matrix values giving net benefits, e.g. in dollar terms:

Alternative\Factor

$$\begin{bmatrix} a_i & a_j & a_k \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix}$$

(Thus, for instance, the forestry cost-benefit analysis tabulated in [7], p. 84a, is readily transcribed to this form, on taking net benefits and interchanging rows and columns.) Given that ties are allowed for, by having more than one alternative having the same place (e.g. third equal) with respect to a given factor, every cost-benefit analysis will yield a ranking table with rankings representable in terms of a ranking relation P. Thus cost-benefit analysis — such as those for supersonic transport, dams, nuclear installations — yield further examples, and these examples will be subject to the general impossibility theorem to be adduced.

(2) *Educational assessment problems.* It should suffice to indicate the range of examples by specifying kinds of alternatives and factors.

E1.	Career alternatives:	executive, manual worker, academic,... .
	Factors:	salary, hours, job satisfaction,... .
	Objective:	determination of career or overall ranking of career opportunities.
E2.	Course combination alternatives:	arts, science, law,....
	Factors:	chance of passing, ease, interest,
	Objective:	course determination.
E3.	Alternatives:	members of class, e.g. Avril, Anne, Isobel,... .
	Factors:	subjects, e.g. mathematics, music, Maori, conduct,...
	Objective:	overall class ranking, and class prizes.

It is amusing that voting paradoxes extend to such examples if a majority decision method is used, e.g. to determine overall class ranking in case E3. For consider the following placement table.

Ranking table 2.

	Music	Mathematics	Maori
Avril	1	3	2
Anne	2	1	3
Isobel	3	2	1

Then under the majority test for class prize: Avril P Anne P Isobel P Avril P... i.e. transitivity of P fails and it is impossible to determine best choice.

With these cases at hand it is easy to multiply decision problem examples and extend the range into other areas, e.g. work value, and ethics (2), where conspicuous difficulties emerge for utilitarians — since utilitarians have somehow to devise overall happiness or net pleasure rankings on the basis of similar rankings for each member of the base class, e.g. persons, sentient creatures, or whatever (and for further difficulties, see [15]).

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§ 2. *Impossibility Results.* In order to show that the results apply to a wide range of ranking problems we abstract the relevant formal structure that the examples given all display.

A *decision (model) structure* (d.s.) is a system $\mathbf{C} = \langle K, F \rangle$ where K is a non-null set of alternatives (or worlds) and F a non-null set of factors (or subjects). Thus **C** is a table, or matrix, structure. A d.s. is *usual* iff K has more than 2 elements (3). The basic framework adopted thus not only generalises on standard expositions of Arrow's theorem by removing the interpretational restriction to social welfare contexts, but also removes standard limitations, e.g. to finite sets of (exclusive) alternatives and to finite sets of subjects.

A *full ranking* **p** *on* a d.s. **C** is a function which assigns, for each factor i in F, a 2-place relation $P_i = \mathbf{p}(i)$, called a *factor ranking,* defined on KxK, and also an *overall ranking* $P = P_{\Phi} = \mathbf{p}(\Phi)$ on KxK, where Φ is an arbitrary item not in $F \cup K$. Thus **p** is defined on $F^* = F \cup \{\Phi\}$ with values in K^2. **P** is subject to the following (ranking) requirements:

(2) One interesting set of factors for ethical cases is provided by the virtues, in terms of which one may try to ethically rank, as better or worse persons, members of a community.

(3) The results that follow also hold where K has exactly 1 element, but *not* where K has 2 elements: see [3], p. 48.

B. For each factor j in F, P_j is a strict partial ordering, i.e. it is transitive and asymmetric, on K; that is,

if xP_jy and yP_jz then xP_jz, for x,y,z in K (transitivity); and if xP_jy then it is not the case that yP_jx, for x,y in K (asymmetry).

A set of factor orderings is *admissible* (consistent) if it conforms to requirement B.

D. Overall ranking P is strongly transitive and asymmetric on K, i.e.

if xPy and $\sim$zPy then xPz, for x,y,z in K (strong transitivity).

Strong transitivity and asymmetry guarantee transitivity, thus: — Suppose otherwise, for x,y,z in K, xPy and yPz but $\sim$xPz. Then by asymmetry, $\sim$yPx, but, by strong transitivity, yPx which is impossible, by asymmetry again.

Strong transitivity, not required for factor rankings, incorporates a connectedness, requirement, and so enables the recovery of the usual postulates for relation R (of «preference or indifference»; see, e.g., [3] pp. 12-13, [4], pp. 2-4); namely

R-connectedness: For x,y in K, either xRy or yRx; and
R-transitivity: For x,y,z in K, xRy and yRz strictly imply xRz.

For, upon defining xRy as $\sim$yPx, R-connectedness follows from asymmetry and R-transitivity follows from strong transitivity using the principle of Antilogism. Relation R will not be used in what follows. Nor is it required that factor rankings meet such implausible requirements as that xP_iy & $yP_iz \supset .xP_iz$ or that xP_iy & $yI_iz \supset .\ xP_iz$ (where 'I' represents indifference), thereby eliminating one familiar source of criticism of Arrow-style theorems.

It is sometimes more illuminating to separate the rankings into two classes, factor rankings, P_i, for each factor i in F, and an overall ranking, P, which is supposed to be in some measure determined in terms of the factor rankings — commonly a function of these.

A *decision table* $\mathbf{M} = \langle K,F,\{P_i{:}i{\in}F\}, P\rangle$ on $\mathbf{C} = \langle K,F\rangle$ is a d.s. **C** with factor and overall rankings on it. Alternatively it may be represented by the structure $\langle K,F,\mathbf{p}\rangle$ where **p** is a ranking. Thus decision tables formalise the ranking tables, with overall rankings, given in the examples above. Note that **M** can be expressed in the initial systemic form by recasting as the structure $\langle K,\{P_j\}\rangle$ where j is in the index set F*. It is the general character of the relations admitted that makes the structure of economic interest — though its interest is by no means confined to economics.

A decision table should satisfy conditions which — unlike conditions B and D — relate the overall ranking to factor rankings. The most straightforward, and plausible, of the conditions to be imposed are these: —

T. (*Pareto,* or *Factor Unanimity*). For any x,y in K, if xP_iy for every i in F, then xPy. That is, if every factor ranks alternative x above y then so does the overall ranking.

M. (*Multiple Value,* or *Non-Dominance*). There is no i in F such that, for every full ranking **p** and every x,y in K, if xP_iy then xPy. Factor j is *dominant* iff for every x,y in K if xP_jy then xPy. According to M no factor is dominant, i.e. there is no factor which determines overall rankings in every case contrary to multiple use principles. For multiple use implies taking due account of all relevant factors in decision making, which in turn means there will be some cases at least where a single factor does not dominate. Multiple use, which is legislatively required of forestry (and land) management practices in United States, and which can be explicated, in a way the legislation suggests, in terms of an optimisation modelling (cf. [7], p. 225 ff), thus requires as a minimal condition non-dominance. Of course in a single factor decision structure one factor will dominate, but American legislation on, and optimisation modellings of, multiple use both prescribe several factors that are to be taken into account. M is thus no ad hoc condition, but fundamental in environmental, and multifactor, decision making.

An underlying assumption, to be expected from analyses of social welfare and collective choice, is that P is a function of factor rankings, that is

E. P is a function of elements of $\{P_j : j \in F\}$, i.e. in finite factor cases $P = f(P_1, \ldots, P_n)$ for some function f.

This assumption — made in effect in conventional expositions of Arrow's theorem (e.g. [4], p. 41) — is however *nowhere* required in the generalised theorem. Nor is it always desirable since overall rankings may not (e.g. in holistic cases) be simply a matter of, reducible to, or analysable in terms of, factor rankings; all that is required is that P and $\{P_j : j \in F\}$ stand in some relationship, just restricted by conditions such as T and M. Part of the role intended for E is absorbed in the following *weak reduction* requirement:

I. (*Relevant Factor Completeness, or Independence of Irrelevant Alternatives*). Let **p** be a ranking (of a given sort) on d.s. $\mathbf{C} = \langle K, F \rangle$ and let **p′** be any other ranking (of the same sort) on **C**, and let P_i and P_i' be the corresponding defined relations for $i \in F^*$; and finally let S be any subset of K. Then if, for every x,y in S and every j in F, xP_jy iff $xP_j'y$, then, for every x,y in S, xPy iff xP′y;

or in classical logical formalism:

$$(x,y \in S)(j \in F)(xP_jy \equiv xP_j'y) \supset . (x,y \in S)(xPy \equiv xP'y) \ (^4)$$

I follows from the principle

I+. (*Extensionality*).

$$(j \in F)(xP_jy \equiv xP_j'y) \supset . xPy \equiv xP'y, \qquad \text{for } x,y \in K.$$

This given the classical content of the usual statement that xPy *depends only on* the factor rankings xP_jy for $j \in F$, which

(4) This is the principle IP of [11].

implies the familiar statement (e.g. [2], p. 79) that the ordering of a set S of alternatives should depend only on the ordering of the factors over S regardless of orderings over K-S.

The critical — and disputable — condition I has a key role in specifying how overall rankings are determined through factor rankings. The formal statements of I and M already involve — since they consider alternative or variant rankings to a given ranking — sets of rankings over d.s. And what the remaining familiar condition (labelled U) of universality or unrestricted domain really requires is that all sets of rankings, conforming to the given conditions, be considered — in short, consideration of the class of all decision tables conforming to T, I and M.

A *general ranking method* (GRM) on a d.s. **C** is a procedure (relation or rule) which relates an overall ranking to each admissible set of factor orderings on **C**; hence it determines a full ranking **p** on **C** given a set of factor rankings on **C**. Thus too a GRM on **C** satisfying conditions TIM should define some subject — *an orthodox decision (or assessment) subset* (under GRM) *on* **C** — of the class of all decision (or assessment) tables which contain all admissible factor rankings on **C**. In the important special case discussed in choice literature a GRM is a function (device or machine) which given *any* admissible set of factor rankings delivers an overall ranking. A *general* ranking method should, like a decision procedure, work for every admissible factor ranking, not just for special cases. Moreover a general social or economic theory should — so it is not unreasonable to require — work for every case, not just for specially selected cases. But, according to the main impossibility result to be adduced, no such general procedure can succeed.

Impossibility Theorem. There is no general ranking method satisfying conditions T, I and M on any usual decision structure. In other words, there is no orthodox decision (or assessment) set on a usual d.s.

The proof of the theorem is but a (somewhat tortuous) rectification and extension — designed to make the role of condition I and other conditions transparent — of a proof given by Arrow (Arrow's proof is set out in [3], p. 97ff.; the corrected extended version appears in [11]).

Since the conditions TIM are orthodox requirements on ranking or assessment procedures in the social sciences (whether empiricist or Marxist in orientation), the upshot of the theorem is that *there is no orthodox solution to ranking and assessment problems in the social sciences.*

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§ 3. *Speculative Corollaries.* The prototype for all these corollaries is the fairly pervasive economic assumption (5) that Arrow's theorem somehow renders welfare economic theory impossible — or, in weaker more metaphorical form, creates an (impenetrable) barrier to welfare aggregation and group decision making.

1. *An orthodox economic theory is impossible.* The essence of the argument is simply this: an economic theory would imply a general ranking method satisfying — when orthodox — TIM on decision structures, which is impossible. The speculative details concern the character of an orthodox economic theory, and how it is thereby required that there be such a general ranking method.

Economic theory involves the determination of the best choices between alternatives in each economic system. It is a commonplace of economic textbooks (whether true or not need not concern us) that economic theory arises out of scarcity (or more generally constraints). Scarcity forces decisions. So economic theory emerges as a general theory of choice, making superior choices among competing alternatives (thus, e.g., Boulding, Samelson, Bergson) (6). As such the theory must involve the equivalent of an overall ranking of alternatives (or an objective function on which to maximize) on

(5) The assumption is not of course universal; see, e.g., Little [13] and Arrow's remarks in [3].

(6) According to Bergson [12], *the* problem of economics is «the optimum allocation of resources».

economic systems. This first point may be argued for in a number of ways. One familiar way is this: Economic theory implies the determination of best choice in each economic system, of maximisation on the objective function — choice as determined through maximisation constitutes the foundation of economic theory (thus, e.g., Heilbroner [10], and sources cited therein: accordingly economic theory is but a branch of optimisation theory). Since subsystems of economic systems are economic systems, economic theory implies merit rankings in each 2 alternative system. Hence, since each system can be decomposed into a string of 2 alternative subsystems (as argued in Arrow [3]), economic theory implies an overall ranking, in terms of merit, of alternatives in each system, that is, it implies a general overall ranking method.

Another familiar way of arguing the first point (Arrow's succinct way, [3], p. 22, p. 104) is this: — The aim, of economic theory, is to maximize on what is desired (e.g. net value, utility, social welfare) subject to relevant constraints (e.g. resource, technological, ethical, etc.), that is again, the aim is optimisation (over rankings) in economic systems. But a necessary condition for such optimisation is, in each case, an overall ranking satisfying ordering conditions D.

Secondly, an overall ranking of alternatives is not independent of, but related to, and characteristically derivative from, rankings of the factors taken into account in the ranking assessment (from which factors the overall ranking is typically a composite). The ranking and assessment problems of § 1 illustrate the point. And at least in orthodox economic cases — in all *reasonable* cases most would want to claim — the overall ranking and factor rankings conform to conditions T, I and M. Indeed the central economic example of a competitive market does, according to Arrow ([3], p. 110): where such a market allocation system fails is as regards generality (7).

(7) The question is not whether the market procedure — any more than a Pareto procedure — provides satisfactory rankings where it does, but whether it provides a general procedure conforming to TIM. That it does not offer a satisfactory procedure where it does work is all too evident, especially from environmental cases.

Hence, thirdly, an economic theory characteristically involves, at bottom, full rankings on economic systems constrained by orthodox (or Arrow-rationality) conditions. And so an economic theory, which has to work for every case, implies a general ranking method on economic systems. But the impossibility theorem renders this impossible; that is an orthodox economic theory is impossible.

Though part of the argument resembles the initial part of Arrow's argument ([3], p. 22 ff; also p. 104) that a welfare theory typically involves rational determination, and optimisation, of a social welfare function on the basis of individual rankings subject to technological and resource constraints, it is worth remarking that the argument does *not* require the further, and dubious, moves Arrow makes in defence of his position, notably,

(i) the strong *reduction* assumption, that social rankings reduce to, or depend only on, the rankings of the individuals that make up the society, and, connected with this,
(ii) the assumption of *methodological individualism*, that social wholes are always analysable in terms of their individual components,

a thesis defended by implicit appeal to

(iii) the verification principle and other rightly questioned tenets of positivism.

For example, the argument given does not exclude organic or holistic features of overall rankings — it is simply required that such rankings are appropriately related to *factor* rankings (and perhaps not just to individual rankings even in the social case since there may be other factors to take account of) And while the independence condition I comes close in Arrow's social welfare context to a reduction assumption (compare, e.g. the statement and defence of the reduction assumption, in the form 'The overall relative ranking of a pair of alternatives is dependent only on the ranking of the pair for each individual'

p. 22 ff, with that of I, p. 26 ff.), I and the strong reduction assumption diverge in the larger context where factors other than individual preferences may make a difference to the overall (social) ranking. So though a strong reduction assumption of this type 'serves as a justification of both political democracy and laissez-faire economics' ([3], p. 23), it is *not* incorporated in the arguments offered. Nor should it be adopted as a basic assumption. In view of the growing opposition to methodological individualism in the social sciences such an assumption can no longer be expected to gain such widespread acceptance as in the heydays of positivism. Moreover there are straightforward environmental reasons for believing that the strong reduction assumption is mistaken, in particular (where the social rankings are tied in familiar ways with what the society should do, see e.g. [15]).

What is being canvassed is the impossibility of an orthodox (and general) economic theory, not the impossibility of economics (8). Needless to say, descriptive economics and many parts of theoretical economics are unaffected. But though descriptive economics, which is concerned merely with the description of choices and market decisions actually made and such-like, is not impeded by the impossibility result, economics, were it to reduce to the descriptive, would really vanish into a branch of sociology. Nor is it being denied that economic theory could work in limited areas (its current lack of success in many areas is again evident), or even that a good theory may sometimes accurately predict future economic decisions and behaviour. Here only the generality of such an economic theory (as a general theory of choice and economic behaviour) is being denied — a denial that has a special bearing on economic issues conspicuously beyond the reach of current economic theory, such as environmental economics, where new factors, that are notoriously hard to quantify or to compare in terms of the current framework, figure prominently, and where reduction condition I appears to fail.

(8) It is partly no doubt for this reason that Heilbroner [10] — which has a different argument and thesis — is entitled 'Is economic theory possible ?'.

2. An orthodox economic solution of environmental problems is impossible. For the argument applying in the case of orthodox economic theory certainly applies in the special case of environmental problems, where questions of interfactor comparison become crucial. That is to say, the earlier argument is rerun and it is observed that the incontrovertibility of the premisses is not diminished in the environmental case — as the environmental ranking problems of § 1 are intended to show.
3. An orthodox optimisation theory is impossible. For if it were, so would an orthodox theory of economics be possible ([9]).
4. There is no orthodox engineering or technological solution to environmental problems. For there is no orthodox theoretical solution — which these presuppose — by 2 and 3. Thus the familiar *unqualified* technological optimism, according to which there are no insoluble developmental or environmental problems, is unjustified, and can be demolished logically ([16]). Similar results apply to more specific areas such as land use planning.
5. An orthodox social theory is impossible. More specifically, there is no orthodox solution to ranking and assessment problems in such disciplines as psychology, education and sociology. The corollary puts another major obstacle in the way of typical unified science programs since such programs characteristically presuppose an orthodox social theory.
6. There is no general method of factor analysis. For suppose one starts with an overall ranking or assessment, e.g. of group or individual intelligence, and aims to uncover the factor rankings on which the overall ranking is based. Because the respective rankings will commonly stand in relations satisfying Arrow-rationality conditions, and so will in general include all cases required for the application of the impossibility theorem,

([9]) It is anticipated that interesting and more specific results as to the unsolvability of classes of optimisation problems will emerge from recursion theory. And these will provide the genesis of further impossibility results in economics.

([10]) The extraordinary claim sometimes made on behalf of technological optimism, that there are no insoluble problems is refuted by theorems from recursion theory. Optimism of this kind is, accordingly, irrational.

the general problem of determining factors succumbs to the impossibility theorem. For otherwise there would be a full Arrow-rational ranking on every usual decision structure, contradicting the theorem.

This corollary emphasizes that there really are *two classes of limitations* emerging from the general impossibility result — a limit on amalgamation, as in the original Arrow result, and a limitation on analysis. There is a way up — composition, synthesis or aggregation from factor rankings to overall rankings — and a way down — decomposition or analysis from an overall ranking to factor rankings. Social choice theory illustrates the serious limitations there are on the first, factor analysis on the second.

7. The general problem of assigning cardinals to all ranked alternatives, and so of absolute interfactor comparisons, is unsolvable. Hence, firstly there is no general pricing mechanism which will cover all factors (assigning cardinals). For if there where such a mechanism there would be an absolute interfactor method, contrary to 2 and violating 1. A second outcome is that cost-benefit analyses are not of general applicability.

A critical issue of course — on which much of environmental economics hinges — is the matter of interfactor comparisons. The matter of interpersonal comparison has had a protracted discussion in the literature; but many of the more favoured alternatives for delivering cardinal comparisons in the interpersonal case, e.g. methods using minimal or just noticeable dicriminable differences — none of which work in the intended context — are either inapplicable or do not extend plausibly to interfactor comparisons where persons may not be (at all directly) involved. Of course if, in defiance of Arrow rationality conditions, a method of interfactor comparisons which delivered cardinal assigments could be devised, then the unsolvable problems cited would no longer be unsolvable on the score argued: but several theoretical and empirical considerations (the latter set out in Robbins [14]) come together to rule out this possibility.

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§ 4. *Enforcing orthodox requirements?* The speculative corollaries come to nought if orthodox (rationality) conditions such as B, D, T, M, and especially I, on which the underlying impossibility theorem is premissed, cannot be made to stick appropriately.

Without B there are no factor rankings to relate to the overall rankings and decision and assessment procedures characteristic of the social sciences cannot even begin — rational assessment procedures are virtually dissolved, since overall rankings, when challenged are almost invariably defended, and often can only be defended, by appeal back to the factors or considerations on which they are based. The asymmetry and transitivity requirements of B on rankings may of course be questioned, but if a factor *ranking* is given an asymmetric ranking can always be defined (by excluding cases of equality) and its transitive closure taken. In short, B can be guaranteed.

Requirement D is a necessary condition for optimisation and for best or better solutions to problems. Thus in the case of economics the very characterisation of economic theory, in terms of optimum choice in constraint-imposing economic systems, has a consequence requirement D. For the dimension-limiting conditions for an optimum to be even defined ensure an appropriately connected strict partial ordering. In any case strong transitivity could be again enforced by taking the strongly transitive closure of any putative overall ranking relation that is given (and the subsequent method of factor enrichment need not clash with the initial procedure of taking transitive closures of rankings).

But it is not really B and D, or for that matter T and M, that are at issue as rationality requirements. It is I that has rightly been under challenge. There are however ways of enforcing conditions I, T and M on a ranking **p** on d.s. **C**, so that a general solution to ranking problems involves a general solution to ranking problems satisfying I, T and M. A common, but quite unsatisfactory, way of trying to enforce orthodox requirements — especially I and such supporting doctrines as that there is no (empirical) method for assigning interpersonally viable cardinal preference rankings — is a hard-line way, e.g. appeal to the verification principle and other methodological assumptions of positivism (along lines commonplace in the social sciences, especially economics, and well exhibited in Robbins [14], p. 139 ff). But a less assumption-loaded, and more direct, way of trying to enforce orthodox requirements I, T and M, as thoroughly reasonable in the enlarged context, is *the method of extra factors*. The method is simply to go on inserting additional factors until conditions TIM are satisfied (11). The intuitive considerations at work are these: —

ad T. Suppose for some ranking $\mathbf{p}$ on some d.s. $\mathbf{C} = \langle K, F \rangle$, for some x, y in K, xP_iy for every i in F, but x is not ranked above y, i.e. $\sim xPy$. In this case there must be some factor h, not in F, which *makes the difference,* and for which x is not ranked above y, i.e. $\sim xP_hy$; that is there must be such a factor which accounts for the discrepancy. Thus it should be possible to extend F to a class F^+ of factors, with $F \subset F^+$, such that for rankings on $C^+ = \langle K, F^+ \rangle$, condition T is satisfied.
ad I. Suppose for some environment $S \subseteq K$, P_i' coincides with P_i for every factor i (i.e. for x, y $\in$ S $xP_i'y$ iff xP_iy), but P' diverges from P, say xP'y but $\sim xPy$ for some x, y $\in$ S. Then, insofar as P' and P are controlled by the respective factor rankings, there must be some factor rankings, e.g. that for $h \notin F$ such that $uP_h'v$ and $\sim uP_hv$ (or vice versa) for some u and v in S. But in this case the addition of further relevant factors to F, which ensure completeness, should guarantee condition I. In short, I fails only because of the incompleteness of factors taken into account in the assessment.

There is presumably then, a way of enlarging d.s. — not generally available in the social welfare case, where the class of individuals is fixed — by adding factors so as to ensure that

(11) The method of extra factors, at the same time, blocks any presumed solution of social ranking problems by assignment of cardinals to each factor ranking thereby providing a method of interfactor comparison and simple arithmetical methods for obtaining overall rankings. For overall rankings are not invariant in the way they should be, when extra relevant factors are introduced, under presumed cardinal solutions, since the extra factors may alter overall orderings.

conditions T and I hold [12]. In fact T can be guaranteed in a rather trivial way, but at the cost of violating M. For simply define (for x, y $\in$ K and h $\in$ F $\cup$ K)P_h thus: xP_hy iff xPy, and set $F^+ = F \cup \{h\}$. Then T is automatically satisfied, since if $\sim$xPy then $\sim xP_hy$. However the definition makes h dominant, violating M. This defect can be avoided by defining P_h in a zig-zag or diagonal fashion, but first let us guarantee I. A simple way to do this is to replace F by F*. For then

$$(j \in F^*)(xP_j'y \equiv xP_jy) \supset . \; xP_\Phi'y \in xP_\Phi y$$
$$\supset . \; xP'y \equiv xPy$$

T is guaranteed as well, but, since factor Φ is dominant, M is violated. It appears however that M can be ensured along with T and I by having the further factors dominant only over proper subsets of K. In particular, factor j is *locally dominant wrt* x and y iff if xP_jy then xPy. It suffices then to introduce into F^+ a set of factors Φ(x,y), for each x and y in K, which are locally dominant. Set $xP_{\Phi(x,y)}y \equiv xP_\Phi y$, and $uP_{\Phi(x,y)}v \equiv \sim uPv$, for {u,v} distinct from {x,y}. Then Φ is represented in F^+ for each pair of K^2 but not uniformly for every pair. Such considerations motivate the

Extra Factor Thesis. For any ranking **p** on usual d.s. **C** = ⟨K,F⟩ which satisfies M, there is a set of factors F^+ which includes F and a ranking p^+ on ⟨K,F^+⟩ which satisfies TIM. i.e.. there is an orthodox ranking on K,F^+.

Call ⟨K, F'⟩ an *extension* of ⟨K,F⟩ where F $\subseteq$ F'. The import of the extra factor thesis is that even where there is a general ranking method satisfying M on a usual d.s. **C** there are extensions of **C** for which there are no general ranking methods. So the damaging effects of the impossibility theorem, e.g. for environmental economics, are apparently not removed simply by qualifying or abandoning I or T: given the thesis, the effects

(12) The method resembles the method of further worlds in extending the scope of extensional-style semantical analysis.

will reappear in a larger context, thereby destroying, so it is suggested, prospects for a general theory.

There are similar strategies, of wide applicability, which appear to weaken the force of objections to other quite rightly questioned assumptions used in the underlying impossibility theorem. If, for instance, the overall ranking fails to conform to strong transitivity, then it will be possible to define a new overall ranking which does conform (e.g., by taking closure) and then to increase the class of factors in order to reinstate conditions that are thrown out by strengthening transitivity.

If the orthodox requirements can be so imposed, in one way or another, there is, it thus appears, no easy escape from the impossibility theorem or its corollaries for the social and environmental sciences. But appearances are here deceptive. Even if ranking methods on an extension of a usual d.s. **C** succumb to the impossibility theorem, methods on **C** itself, if unorthodox, may well not. To argue to impossibility on this basis is scarcely better than arguing to undecidability of a system on the basis that it has undecidable extensions. The requirements of orthodoxy cannot be enforced in unorthodox procedures by the extra factor method or other such methods. The impossibility result for orthodox social and environmental theories does not extend — at least without further and different ado — to unorthodox theories, any more than Gödel's impossibility theorems for classical theories extend, without further ado, to nonclassical theories. On the contrary, the prospects for unorthodox social and environmental theories which make due allowance for interdependence and interrelations of factors, and accordingly abandon requirement I, are bright.

Research School of Social Sciences R. ROUTLEY (13)
Australian National University.

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