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# EPISTEMIC PLURALISM

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Abstract

The present paper wants to promote epistemic pluralism as an alternative view of non-classical logics. For this purpose, a bilateralist logic of acceptance and rejection is developed in order to make an important difference between several concepts of epistemology, including information and justification. Moreover, the notion of disagreement corresponds to a set of epistemic oppositions between agents. The result is a non-standard theory of opposition for many-valued logics, rendering total and partial disagreement in terms of epistemic negation and semi-negations.

## Introduction

It seems obvious how to use "yes" and "no" in normal communication: by saying either of these, not both. Indeed, "well, yes and no" appears to be nothing but a harmless expression of partial agreement and disagreement that does not seriously challenge Aristotle's Principle of Non-Contradiction. And yet, agents may be faced to doubt when they deal with opposite arguments of equal weight: should they then remain silent, thereby meaning "neither yes nor no" or "none"? The way to deal with yes- and no-answers directly relates to the bounds of rationality. These bounds also relate to the existence of epistemic norms any agent ought to follow to reason correctly.

What does "correctness" mean? There exists a literature devoted to this issue, especially with respect to the logic of direct answers (e.g. Matilal 2000, Rumfitt 2000, Dummett 2002). In Dummett (2002, 290), the basic role of judgment occurs in a logic of yes- and no-answers:

> In affirming a sentence, what does a speaker say? He expresses a judgment that an assertion made by uttering it would be correct.

A tricky case arises when a no-answer is given by an agent, thereby opposing two views of negation: *unilateralism*, where negation is basically a sentential operator used to express the assertion of a negative sentence; *bilateralism*, according to which denial is both independent from assertion and prior to sentential negation. Dummett (2002, 291) makes use of the special symbolism of Rumfitt (2000) to make his point about this controversy.

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What, in denying a sentence, does a speaker say? This is where Rumfitt parts company from me. (...) The denial is indeed, for him, equivalent to asserting its negation: from '−A' it will be possible to infer '+(¬A)' and conversely.

In the following, we want to follow Dummett's position in order to bring alternative models of formal epistemology. The general objective is clearly summarized by Dummett (2002, 291), again:

> We must lay down the condition for [a sentence] to be correctly asserted, but also the condition for it to be correctly denied.

The paper wants to address such an issue, by means of a pluralist epistemology that makes an essential difference between the concepts of information and justification.

## 1. Information and justification

Let us introduce the two corresponding logical systems for information and justification: $\mathbf{AR}_4$ and $\mathbf{AR}_{4\blacksquare}$, respectively. The two main speech-acts of affirmation (including assertion as a special case) and denial play a central role in these, leading to opposite attitudes of acceptance and rejection by agents.

### 1.1. *Information*

Any normal agent is assumed to take decision on the basis of available information: empirical evidence, proofs, or whatever can be used as a argument for or against the truth of a given sentence. For example, seeing a table in front of me is an information that argues *for* the truth of the sentence $\varphi$, "There is a table in front of me". Such a perceptual experience is also an information *against* the opposed view expressed by the negative sentence $\varphi$: "There is no table in front of me".

The two attitudes of acceptance and rejection may be expressed by speechacts in a question-answer game, as depicted by Frege (1919) for scientific investigation. If I am asked: "Is there a table in front of you?", I answer "Yes!" by doing a speech-act of affirmation if my related attitude is belief. Therefore, a normal agent accepts $\varphi$ by having evidence *for* $\varphi$ and accepts $\neg\varphi$ by having evidence *against* $\varphi$. Conversely, I answer "No!" by doing a speech-act of denial if my related attitude is disbelief. Thus, an agent rejects $\neg\varphi$ by having no evidence *for* $\varphi$ and rejects $\varphi$ by having no evidence *against* $\varphi$.

Now an important controversy arises about the relation between accepting $\neg\varphi$ and rejecting $\varphi$. Are they equivalent with each other? Not according to bilateralism, which assumes that affirming $\varphi$ (or $\neg\varphi$) includes asserting $\varphi$ (or $\neg\varphi$) as a special case of exclusive affirmation. A way to express their difference consists in weakening the meaning of no-answers as "No, I have no evidence for $\varphi$", rather than "Yes, I have evidence for $\varphi$". The latter has the sense of a "strong" rejection and relates to the speech-act of negative assertion.

Is there a proper logic for epistemic attitudes? A famous case is modal epistemic logic, introduced in Hintikka (1962) and assuming consistent agents throughout a set of possible words. In the following, an important distinction is made between evidence and attitudes.

The first logic $\mathbf{AR}_4 = \langle \text{INF}, \neg, \wedge, \vee, \rightarrow, \mathbf{4}, \mathcal{D} \rangle$ deals with information, independently from any attitudes of agents towards it. It consists of a set of formulas INF including a set of logical constants: negation $\neg$, conjunction $\wedge$, disjunction $\vee$, and implication $\rightarrow$, following the Backus-Naur form:

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi$$

**4** is the domain of logical values assigned to sentences of INF. Importantly, these logical values are not single properties of propositions but ordered answers to corresponding questions about the truth-value of sentences. Thus, for every truth-value $\mathbf{A}(\varphi)$ there is a set of four ordered answers $\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \mathbf{a}_2(\varphi) \rangle$ in which every single answer $\mathbf{a}_i(\varphi)$ maps onto $\{1,0\}$: 1 for "yes" or acceptance, 0 for "no" or rejection (in the weak, broader sense of the word). An explanation of these constructive values runs as follows, matching with the twist-structure representation of e.g. Rivieccio (2014) but without the underlying lattice theoretical background:

$\mathbf{A}(\varphi) = 11$ iff:
there is evidence for $\varphi$
there is evidence against $\varphi$

$\mathbf{A}(\varphi) = 10$ iff:
there is evidence for $\varphi$
there is no evidence against $\varphi$

$\mathbf{A}(\varphi) = 01$ iff:
there is no evidence for $\varphi$
there is evidence against $\varphi$

$\mathbf{A}(\varphi) = 00$ iff:
there is no evidence for $\varphi$
there is no evidence against $\varphi$

From our antirealist perspective, "evidence" is whatever counts either for or against the truth of a given sentence $\varphi$: empirical observation, authoritative testimony, or logical deduction. There is no conclusive evidence *per se*, and no truth- or falsity-claim can be made without having some minimal reason to argue for or against $\varphi$. Accordingly, a sentence $\varphi$ is said "true for a given agent" in the sense that $\varphi$ is made true in the light of the agent's epistemic norms.

$\mathcal{D} = \{11, 10\}$, it is the subset of designated sentences in **4** such that the agent accepts their truth: $\mathbf{A}(\varphi) \in \mathcal{D}$ iff $\mathbf{a}_1(\varphi) = 1$. Logical consequence is defined in the usual sense of truth-preservation:

$$\varphi \vDash_{\mathbf{AR}_4} \psi \text{ whenever } \mathbf{a}_1(\varphi) = 1 \text{ entails } \mathbf{a}_1(\psi) = 1$$

The logical constants of $\mathbf{AR}_4$ can be defined bitwise by means of the lattice-theoretical operators of meet $\sqcap$ and join $\sqcup$. Let 1 and 0 be the yes- and no-answers of single items $\mathbf{a}_i(\varphi)$, where $1>0$. Then, for any such items $a, b$:

$a \sqcap b = \min(a,b)$ and $a \sqcup b = \max(a,b)$, logical constants are defined according to their truth- and falsity-conditions and expressed by speech-acts of acceptance and strong rejection.

**Negation**
The formula $\neg\varphi \in$ INF is:
accepted iff there is evidence against $\varphi$
strongly rejected iff there is evidence for $\varphi$
$\mathbf{A}(\neg\varphi) = \langle \mathbf{a}_2(\varphi), \mathbf{a}_1(\varphi)\rangle$

**Conjunction**
The formula $\varphi \wedge \psi \in$ INF is:
accepted iff there is evidence for $\varphi$ and for $\psi$
strongly rejected iff there is evidence against $\varphi$ or against $\psi$
$\mathbf{A}(\varphi \wedge \psi) = \langle \mathbf{a}_1(\varphi) \sqcap \mathbf{a}_1(\psi), \mathbf{a}_2(\varphi) \sqcup \mathbf{a}_2(\psi)\rangle$

**Disjunction**
The disjunction $\varphi \vee \psi \in$ INF is:
accepted iff there is evidence for $\varphi$ or for $\psi$
strongly rejected iff there is evidence against $\varphi$ and against $\psi$
$\mathbf{A}(\varphi \vee \psi) = \langle \mathbf{a}_1(\varphi) \sqcup \mathbf{a}_1(\psi), \mathbf{a}_2(\varphi) \sqcap \mathbf{a}_2(\psi)\rangle$

**Implication**
The formula $\varphi \longrightarrow \psi$ is:
accepted iff there is evidence for $\varphi$ and for $\psi$
strongly rejected iff there is evidence for $\varphi$ and against $\psi$
$\mathbf{A}(\varphi \rightarrow \psi) = \langle \mathbf{a}_1(\varphi) \sqcap \mathbf{a}_1(\psi), \mathbf{a}_1(\varphi) \sqcap \mathbf{a}_2(\psi)\rangle$

The whole system includes a number of theorems matching with some other systems like Belnap (1977)'s FDE, or Nelson (1949)'s logic $\mathbf{N}_4$ with strong negation. Some standard or classical theorems that fail in $\mathbf{AR}_4$ are also described below.

| | |
|---|---|
| $\varphi \vDash \neg\neg\varphi$ | $\psi \nvDash \varphi \vee \neg\varphi$ |
| $\neg\neg\varphi \vDash \varphi$ | $\varphi \wedge \neg\varphi \nvDash \psi$ |
| $\varphi \wedge \psi \vDash \varphi$ | $\varphi \vee \psi, \neg\varphi \nvDash \psi$ |
| $\varphi \wedge \psi \vDash \psi$ | $\varphi \rightarrow \psi \nvDash \neg\psi \rightarrow \neg\phi$ |
| $\varphi \vDash \varphi \vee \psi$ | $\varphi \nvDash \psi \rightarrow \varphi$ |
| $\psi \vDash \varphi \vee \psi$ | |
| $\varphi \rightarrow \psi, \varphi \vDash \psi$ | |
| $\varphi \rightarrow \psi, \psi \rightarrow \beta \vDash \varphi \rightarrow \beta$ | |
| $\varphi \wedge (\psi \vee \beta) \vDash (\varphi \wedge \psi) \vee (\varphi \wedge \beta)$ | |
| $\varphi \vee (\psi \wedge \beta) \vDash (\varphi \vee \psi) \wedge (\varphi \vee \beta)$ | |

$\mathbf{AR}_4$ is both paracomplete and paraconsistent, as well as a relevant system through the stronger definition of implication (see Schang 201X). Again, the above features only concern the primary logic of information where sentences are connected to each other in terms of available evidence.

A note is to be made about logical values. Realist-minded logicians may be reluctant at the proper use of the preceding values 11 and 00: both mean that agents are undecided, because of overabundant or lacking sources that make their investigation incomplete and require more to achieve a definite assessment. A Fregean logician or philosopher would reply that every scientific investigation tends to unanimity about the value of sentences, unlike provisory states of mind that create a dangerous confusion between truth-values and propositional attitudes (Dubois 2008). However, the present logic of information is no logic of beliefs: yes- and no-answers do not refer to beliefs in $\mathbf{AR}_4$, they are just answers concerning available evidence for or against arbitrary sentences (Wansing & Belnap 2010). The notion of justification has to be explained, accordingly.

### 1.2. *Justification*

The usual assumption of logical independence between truth, justification and belief will be expelled from our following pluralist logic of beliefs, in which one and the same set of available evidence may lead to different epistemic attitudes. In a nutshell, we endorse about the concept of truth the position Prawitz (1998, 23) called "relativist" or "subjective constructivism", according to which no distinction can be made between, on the one hand, a statement being true and, on the other hand, it being taken to be true or treated as true by us.

This also leads to a "social" view of epistemology, in the sense that different criteria of justification correspond to different groups of agents in a whole community.

For this purpose, one needs a precise definition of justification. A typical one is to be found in the entry "Theory of Justification" of Wikipedia:

> Justification is the reason why someone *properly* holds a belief, the explanation as to why the belief is a true one, or an account of how one knows that one knows.

Now our antirealist account requires some substantial change in this realist-minded reading of justification, especially about what is to be called as a "proper" or "true" belief. Hence this minimal definition of justification, with no reference to truth:

*A sentence $\varphi$ is justified iff the agent is entitled to believe $\varphi$.*

What does "entitle" an agent to believe a given sentence? Is there any common ground for justification? A quick overview of our antirealist epistemology is required, in order to motivate such a definition of justification.

First and foremost, an antirealist approach to truth needs a deep revision of the Platonician definition of knowledge. According to the latter, knowledge is justified true belief and each of the three definitional components are *logically independent* from each other. So if we assume the definition $K\varphi =_{df} B\varphi \wedge T\varphi \wedge J\varphi$, logical independence means that $B\varphi$ may hold without $T\varphi$, $B\varphi$ without $J\varphi$, and $T\varphi$ without $J\varphi$.

Such an explanation holds for mainstream epistemic logics, indeed; but it does not in our antirealist epistemology. Firstly, $J\varphi$ and $T\varphi$ are not logically independent from each other: not any justification is *sufficient* for a sentence $\varphi$ to be true; but at the same time, any justification is *necessary* for a sentence $\varphi$ to be entitled to be true. Second, $T\varphi$ is not the consequent but, rather, the antecedent of $K\varphi$: an agent knows $\varphi$ once $\varphi$ is acknowledged to be true, and an antirealist should argue that the *utterance* of $T\varphi$ and $K\varphi$ are on a par for want of a transcendental status of truth. For this reason, there is no logical independence between $T\varphi$ and $J\varphi$, insofar as these mean the same in the epistemological background of $\mathbf{AR}_4$. Thirdly, the revised relation between the definitional concepts of knowledge yields the following: $T\varphi = B\varphi\ J\varphi$, whilst $K\varphi$ is equated with $T\varphi$ because the latter is not transcendent over the agent's epistemic capacities. Fourthly, what counts as a truth or knowledge may not be the same for different agents; so "justification" means the same as "sufficient justification" in the following logic $\mathbf{AR}_{4\blacksquare}$.

Our account of justification consists of two sorts of condition, thereby arguing for epistemic pluralism. On the one hand, such an entitlement may depend on the *sentential content* itself, depending upon whether sentences are of formal or empirical kind. For example, a skeptic agent should not be entitled to accept the truth of a sentence whenever it is not based on conclusive evidence. On the other hand, the acceptance of sentential content also relies upon *agents'* epistemic norms, i.e., the strength of evidence required by them to believe it. The borderline between available evidence and sufficient evidence (for belief) is the borderline between the logics of information and justification. In $\mathbf{AR}_4$, logical values stand for the mere occurrence of evidence for or against sentences. In $\mathbf{AR}_{4\blacksquare}$, they stand for the attitude of a given agent towards evidence.

Any evidence can be assessed in various ways by agents, according to the nature of evidence (empirical, or not), their degree of plausibility, and the epistemic norms adopted by agents. An evidence can be said merely available, but also adequate, proper, conclusive, plausible, sufficient, and the like. For example, the availability of evidence for a sentence $\varphi$ need not be a sufficient reason to accept it. The process achieved in order to complete an investigation and give final assessment stands outside logic and belongs to epistemology, relating to what e.g. a "case" may signify (Beall & Restall 2006). According to epistemic pluralism, there is not only one sort of assessment in the general process of rational argumentation. A sample of these can be taken from religious epistemology, opposing atheists to agnostics (see Schang 2011); another area is legal epistemology, where the burden of proof is divided into presumptions of guilt or innocence and opposes defense to prosecution (Kapsner 2016); in philosophy of science, constructive epistemology equally makes a distinction between two ways of accepting a sentence, viz. verificationism versus falsificationism (Kapsner 2014).

How to classify the different criteria of justification? Recent works have been done in this respect with the so-called logics of justification in Artemov (2008), da Costa (1999), or Carrara & Chiffi & De Florio (2016). Modal logic is taken as a common pattern in all these logics, as witnessed by a common distinction between strong and weak justification. In the first case, strong justification $J$ is viewed as an assertion and refers to the occurrence of conclusive evidence for a given sentence. In the second case, weak justification $J'$ is on a par with hypothesis and corresponds to the occurrence of provisory or defeasible evidence.

In sum, the above operators $J$, $J'$ follow the modal pattern by matching with the operators $\Box, \Diamond$. Possible-world semantics is then considered to be an adequate model for these variants of epistemic and doxastic logics, defining justification in terms of totally or partially ordered relations in a set of worlds. However, the following logic of justification departs from this current trend by favoring a many-valued model of yes- and no-answers. Despite the limited translatability of modal logics into characteristic many-valued matrices (Dugundji 1940), we opt for the latter in order to give a richer typology of epistemic agents.

Thus, the second logic of justification: $\mathbf{AR}_{4\blacksquare} = \langle JUS, \mathbf{4}, \neg, \sim, \wedge, \vee, \rightarrow, \blacksquare \rangle$ is an extension of $\mathbf{AR}_4$ by augmenting it with a set of belief operators $\blacksquare$ and a second kind of negation $\sim$. This new system includes an extended set of statements JUS expressing belief attitudes, following the Backus-Naur Form

$$\text{JUS} \quad \varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid \blacksquare\varphi \mid \sim\blacksquare\varphi$$

Acceptance and rejection are now answers to what agents believe or not, rather than what they merely count as available evidence or not. For every logical value $\mathbf{A}(\blacksquare\varphi)$, the ordered pair of answers $\mathbf{A}(\blacksquare\varphi) = \langle \mathbf{a}_1(\blacksquare\varphi), \mathbf{a}_2(\blacksquare\varphi) \rangle$ is afforded by a function $\blacksquare$ mapping from $\mathbf{4}$ onto $\mathbf{4}$. This formal system brings formal contribution to two perspectives of epistemology (Kusch 2015): in the first, local perspective, justification is centered on a single criterion and concerns only one operator in $\blacksquare$; in the second, global perspective, justification relies upon criteria holding for every kind of operator in $\blacksquare$. By doing so, $\mathbf{AR}_4$ also helps to answer to the question asked by Prawitz (1998, 24):

> Is there a stable notion of truth that is both constructive and absolute, or does the constructive approach face a dilemma: either it drifts into relativism or, if steps are taken to avoid that, it becomes indistinguishable from a realist notion of truth?

We are going to show that *absolute* constructivism is expressed by the general properties of $\blacksquare$, whilst *relative* constructivism concerns the special properties of single operators in $\blacksquare$.

Any logical system including $m = 4$ logical values gives rise to a maximal set of $(m^m)^n = (4^4)^1 = 256$ unary operators. Such a huge number of operators may appear reluctant to epistemologists, whereas a modal approach to epistemology favors possible-world semantics with bivalent accessibility relations. However, the possibility of partitioning yes- and no-answers is of intrinsic interest for the issue of justification, thereby motivating an investigation of the relevant attitudes inside these 256 operators.

There is a way to characterize the most relevant operators, as proper candidates for the role of belief operators: first, by specifying four sorts of constraint on truth- and falsity-conditions, which are made independent from each other in our bilateralist perspective; secondly, by assuming *rationality* in epistemic attitudes.

*An agent is* rational *by trusting the least available evidence about $\varphi$, for or against $\varphi$*:

$$\mathbf{a}_i(\blacksquare\varphi) \neq 0 \text{ if } \mathbf{a}_i(\varphi) = 1 \text{ and } \mathbf{a}_j(\varphi) = 0 \text{ (where } i, j = 1 \text{ or } 2).$$

In other words, a rational agent is someone who does not believe against available evidence.

It results from the first precondition a set $\blacksquare$ of $16 \times 4 = 64$ single belief operators, following this general pattern:

$$\mathbf{A}(\blacksquare\varphi) = \langle \mathbf{a}_1(\blacksquare\varphi), \mathbf{a}_2(\blacksquare\varphi)\rangle$$

$$\mathbf{a}_1(\blacksquare\varphi) = 1 \text{ iff } \mathbf{a}_1(\varphi) = y \sqcap\backslash\sqcup\, \mathbf{a}_2(\varphi) = y$$

$$\mathbf{a}_2(\blacksquare\varphi) = 1 \text{ iff } \mathbf{a}_1(\varphi) = y \sqcap\backslash\sqcup\, \mathbf{a}_2(\varphi) = y \text{ (with } y = 1 \text{ or } 0)$$

Only 4 among the 64 operators happen to be serious candidates for the role of epistemic attitudes in social epistemology. These are rational behaviors that distinguish from each other by various criteria of justification.

The first pair of these is a symmetrical opposition between those agents who favor some evidence over other ones, whether for or against a given sentence. This can be exemplified with Pascal's Wager and the Precautionary Principle (PP).

In the first case, take the sentence $\varphi$, "God exists". Pascal claimed that any argument for the truth of $\varphi$ is to be favored over its negation $\neg\varphi$, "God does not exist". As to atheists, their rationale is contrary to Pascal's one in the sense that any evidence for $\neg\varphi$ is to be preferred to those for $\varphi$. Due to its affinity for positive arguments, let us take Pascal's Wager as an instance of "positivist" agents.

Positivists: $\boxbar \in \blacksquare$
If an agent $a$ is a *positivist*, then
$a$ accepts $\boxbar\varphi$ by having evidence for $\varphi$ or no evidence against $\varphi$
$a$ strongly rejects $\boxbar\varphi$ by having no evidence for $\varphi$ and evidence against $\varphi$
$\mathbf{A}(\boxbar\varphi) = \langle \mathbf{a}_1(\varphi) \sqcup \overline{\mathbf{a}_2(\varphi)}, \overline{\mathbf{a}_1(\varphi)} \sqcap \mathbf{a}_2(\varphi) \rangle$

In the second case, take the sentence $\psi$, "GMOs are helpful to humans". According to those who endorse PP, any evidence against $\psi$ (i.e., any evidence for $\neg\psi$) is to be favored over evidence for $\psi$. As to lobbyists of, e.g., the Monsanto company, any evidence for $\psi$ is to be favored over evidence against $\psi$. Due to its affinity with negative arguments, let us take PP as an instance of "negativist" agents.

Negativists: $\boxminus \in \blacksquare$
If an agent $a$ is a *negativist*, then
$a$ accepts $\boxminus\varphi$ by having evidence for $\varphi$ and no evidence against $\varphi$
$a$ strongly rejects $\boxminus\varphi$ by having no evidence for $\varphi$ or evidence against $\varphi$
$\boxminus\varphi = \langle \mathbf{a}_1(\varphi) \sqcap \overline{\mathbf{a}_2(\varphi)}, \overline{\mathbf{a}_1(\varphi)} \sqcup \mathbf{a}_2(\varphi) \rangle$

The above two examples show that agents favor one sort of argument (for or against a sentence) over its contradictory (against or for a sentence), thus differing in how they view the "burden of proof".

A comparison can be also made with the area of legal epistemology (Kapsner 2016), where positivists play the role of prosecution advocates who favor guilt over innocence. At the opposite side, negativists play the role of defense advocates who favor innocence over guilt.

Both kinds of agents may be blamed for justifying their beliefs with arbitrary biases for or against truth. Nevertheless, these can be viewed as decision-makers in ambiguous situations: they need to make a choice either for or against a given sentence, for want of any conclusive evidence at hand. At any rate, their behavior is perfectly consistent by restoring bivalence through their binary decisions: a sentence is believed whenever its sentential negation is disbelieved, and conversely.

At the same time, another category of agents matches with standard norms of rationality while breaking with bivalence: it is the case in which agents do not believe a sentence without sufficient evidence for it. The notion of sufficiency being unclear, it is rendered in $\mathbf{AR}_{4\blacksquare}$ by an exclusive or one-sided occurrence of available evidence. Let us call "skepticism" this third category of agents, insofar as its stricter truth-conditions are reminiscent of the skeptic search for conclusive arguments.

Skeptics: $\boxtimes \in \blacksquare$
If an agent $a$ is a *skeptic*, then
$a$ accepts $\boxtimes\varphi$ by having evidence for $\varphi$ and no evidence against $\varphi$
$a$ strongly rejects $\boxtimes$ by having no evidence for $\varphi$ and evidence against $\varphi$
$\boxtimes\varphi = \langle \mathbf{a}_1(\varphi) \sqcap \overline{\mathbf{a}_2(\varphi)}, \overline{\mathbf{a}_1(\varphi)} \sqcap \mathbf{a}_2(\varphi) \rangle$

Just as negativists are dually opposed to positivists, skeptics are similarly opposed to another category of agents: these believe a sentence unless there is sufficient evidence against it. Insofar as such agents tend to accept evidence from opposed theories, let us call them *eclecticists*.

Eclecticists: $\boxplus \in \blacksquare$
If an agent $a$ is an *eclecticist*, then
$a$ accepts $\boxplus\varphi$ by having evidence for $\varphi$ or no evidence against $\varphi$
$a$ strongly rejects $\boxplus\varphi$ by having no evidence for $\varphi$ or evidence against $\varphi$
$\boxplus\varphi = \langle \mathbf{a}_1(\varphi) \sqcup \overline{\mathbf{a}_2(\varphi)}, \overline{\mathbf{a}_1(\varphi)} \sqcup \mathbf{a}_2(\varphi) \rangle$

A common point between positivists and negativists is that they assume a *burden of proof* in their attitude, unlike skeptics and eclecticists. The difference between each of these rational agents lies in their own epistemic norms with respect to truth and falsity, and the usual theorems of modal logic are variously validated or invalidated by them (see Schang & Costa-Leite 2016). At the same time, some general properties hold or fail for every agent whilst restoring some logical theorems usually claimed by the adversaries of four-valuedness. These will be listed later on, after introducing epistemic negation and its import for the other main notion of disagreement.

Let us consider now how all of this leads to disagreements between agents.

## 2. Epistemic disagreement

Disagreement is an important concept of epistemology: it expresses a situation in which agents do not share the same beliefs. From our bilateralist perspective of truth-values as independent speech-acts, disagreements are made more complex than the standard view of disagreement between truth- and falsity-claims. Let us consider their main features in three steps: as a relation between either sentences, or beliefs; as different occurrences of negation; as a form of opposition between epistemic agents.

### 2.1. *Disagreements*

A basic distinction holds between two kinds of disagreement, echoing some previous results of Zaitsev & Shramko (2013). The latter may come from an incompatibility between the truth- or falsity-conditions of any two sentences; or it may come from an incompatibility between the epistemic norms of agents with respect to the same sentence. The first kind of disagreement is *ontic*, because it relies upon the sentential content.

*Any two sentences $\varphi,\psi$* disagree *with each other iff there is disagreement in their truth- or falsity-conditions.*

This primary form of disagreement is that currently mentioned in the theory of oppositions, especially in its application to oppositions between binary sentences (Blanché 1957). For example, there is a disagreement between the sentences $\varphi = p \wedge q$ and $\psi = p \wedge \neg q$ with respect to their truth-conditions: there cannot be evidence for both sentences at once; but there is no such disagreement between their falsity-conditions, in the sense that there can be evidence against them at once.

The second version of disagreement is more natural: it designates a relation between agents, rather than sentences. Again, bilateralism makes a substantial difference between the logical relations of sentences in $\mathbf{AR}_4$ and the logical behavior of beliefs in $\mathbf{AR}_{4\blacksquare}$: any two agents may disagree about the same sentence not because they disagree about their truth-values, but because they do not share the same epistemic norms with respect to the same valuation.

*Any two agents* disagree *with each other about $\varphi$ iff they disagree about its truth-conditions, or they disagree about its falsity-conditions.*

A second difference with the standard valuation is about the *degree* of a disagreement. In a unilateralist view of truth-values, agreement and disagreement are as absolute as the difference between truth and falsity. In our bilateralist view, disagreement may be either total or partial between either sentences or agents.

*A disagreement is* total *iff the disagreement is both about truth-conditions and falsity-conditions.*

*A disagreement is* partial *iff the disagreement is about either truth-conditions or falsity-conditions, but not both.*

Thus, agents may disagree partly about specific valuations. For example, positivists and eclecticists agree about the truth-conditions of a given sentence while disagreeing about its falsity-conditions. Given this partition of epistemic attitudes, a second negation is to be introduced into $\mathbf{AR}_{4\blacksquare}$.

### 2.2. *Negations*

Negation is a candidate for the expression of disagreement. Disagreement between sentences is expressed by *sentential* negation $\neg$, which has already been defined in the logic of information $\mathbf{AR}_4$. Disagreement between agents' beliefs is expressed by *epistemic* negation and applies to belief operators $\blacksquare$ only.

$\sim\blacksquare\varphi$ *is the* epistemic negation *of* $\blacksquare\varphi$ *by disagreeing both about truth-conditions and about falsity-conditions.*

$$\mathbf{A}(\sim\blacksquare\varphi) = \langle \overline{\mathbf{a}_1(\blacksquare\varphi)}, \overline{\mathbf{a}_2(\blacksquare\varphi)} \rangle$$

A corresponding distinction is to be made between ontic and epistemic negations, according to their use in $\mathbf{AR}_4$ and $\mathbf{AR}_{4\blacksquare}$. Let us symbolize by $\mathrm{N} \in \{\neg, \sim\blacksquare\}$ a generic operation of negation that applies to any formulas, whether ontically or epistemically. Then any such negation is *Morganian* if and only it satisfies the formulas (i)-(iv):

(i) $\mathrm{N}(\varphi \wedge \psi) \vDash \mathrm{N}\varphi \vee \mathrm{N}\psi$

(ii) $\mathrm{N}\varphi \vee \mathrm{N}\psi \vDash \mathrm{N}(\varphi \wedge \psi)$

(iii) $\mathrm{N}(\varphi \vee \psi) \vDash \mathrm{N}\varphi \wedge \mathrm{N}\psi$

(iv) $\mathrm{N}\varphi \wedge \mathrm{N}\psi \vDash \mathrm{N}(\varphi \vee \psi)$

Also, negation is *Boolean* if and only if it satisfies the formulas (v)-(vi) about double negation:

(v) $\varphi \vDash \mathrm{NN}\varphi$

(vi) $\mathrm{NN}\varphi \vDash \varphi$

Then we have the following result about negations:

*Ontic negation is non-Morganian and Boolean. Epistemic negation is Morganian and Boolean.*

The logical form of epistemic negation is $\mathbf{a}_i(\sim\blacksquare\varphi) = \overline{\mathbf{a}_i(\blacksquare\varphi)}$, turning affirmation into denial and conversely. Echoing Prawitz's search for absolute constructivism, the following properties of $\blacksquare$ hold or fail for every agent and help to restore some classical theorems that were lost by the confusion between an information $\varphi$ and its justification $\blacksquare\varphi$.

| | |
|---|---|
| $\vDash \blacksquare\varphi \vee \sim\blacksquare\varphi$ | $\nvDash \blacksquare\varphi \vee \blacksquare\neg\varphi$ |
| $\vDash \sim(\blacksquare\varphi \wedge \sim\blacksquare\varphi)$ | $\nvDash \sim(\blacksquare\varphi \wedge \sim\blacksquare\varphi)$ |
| $\blacksquare(\varphi \rightarrow \psi), \blacksquare\varphi \vDash \blacksquare\psi$ | $\blacksquare(\varphi \rightarrow \psi) \nvDash (\blacksquare\varphi \rightarrow \blacksquare\psi)$ |
| $\blacksquare\neg(\varphi \vee \psi) \vDash \blacksquare\neg\varphi \wedge \blacksquare\neg\psi$ | $\blacksquare(\varphi \rightarrow \psi) \nvDash \blacksquare\sim\psi \rightarrow \sim\blacksquare\varphi$ |
| $\blacksquare\neg(\varphi \wedge \psi) \vDash \blacksquare\neg\varphi \vee \blacksquare\neg\psi$ | $\blacksquare\varphi \nvDash \sim\blacksquare\neg\varphi$ |
| $\blacksquare\varphi, \sim\blacksquare\varphi \vDash \blacksquare\psi$ | $\blacksquare\varphi, \blacksquare\neg\varphi \nvDash \blacksquare\psi$ |
| $\blacksquare(\varphi \vee \psi), \sim\blacksquare\varphi \vDash \blacksquare\psi$ | $\blacksquare(\varphi \vee \psi), \blacksquare\neg\varphi \nvDash \blacksquare\psi$ |
| | $\blacksquare(\varphi \vee \psi) \nvDash (\blacksquare\varphi \vee \blacksquare\psi)$ |
| | $\blacksquare(\varphi \wedge \psi) \nvDash (\blacksquare\varphi \wedge \blacksquare\psi)$ |

The difference made here above between ontic and epistemic negation is a difference between a classical and paraclassical (paraconsistent or paracomplete) behavior of agents.

Now agents may disagree about only one truth-value, whether truth or falsity. For example, positivists agree with eclecticists and disagree with skeptics with respect to the truth-conditions; at the same time, positivists disagree with eclecticists and agree with skeptics with respect to falsity-conditions. The notion of relative disagreement can be formulated in terms of *semi-negations*, borrowing from the terminology of Zaitsev & Shramko (2013) whilst departing from their valuation including both realist- and antirealist- minded perspectives.

$\sim_j\blacksquare\varphi$ is an epistemic *semi-negation* of $\varphi$ by disagreeing:

either about truth-conditions ($j = t$)

$$\mathbf{A}(\sim_t\blacksquare\varphi) = \langle\overline{\mathbf{a}_1(\blacksquare\varphi)}, \mathbf{a}_2(\blacksquare\varphi)\rangle$$

or about falsity-conditions ($j = f$)

$$\mathbf{A}(\sim_f\blacksquare\varphi) = \langle(\mathbf{a}_1(\blacksquare\varphi), \overline{\mathbf{a}_2(\blacksquare\varphi)}\rangle$$

Epistemic semi-negations and negation can be also combined to create double negation, or make transitions between different sorts of agents. These operations resort to properties of group theory and can be described as follows, for every $i,j$ in $\{t,f\}$:

$$\mathbf{A}(\sim_i\sim_j\blacksquare\varphi) = \mathbf{A}(\sim_j\sim_i\blacksquare\varphi) = \mathbf{A}(\sim\blacksquare\varphi)$$
$$\mathbf{A}(\sim_i\sim_i\blacksquare\varphi) = \mathbf{A}(\sim_j\sim_j\blacksquare\varphi) = \mathbf{A}(\blacksquare\varphi)$$
$$\mathbf{A}(\sim\sim_i\blacksquare\varphi) = \mathbf{A}(\sim_j\blacksquare\varphi)$$
$$\mathbf{A}(\sim\sim_j\blacksquare\varphi) = \mathbf{A}(\sim_i\blacksquare\varphi)$$

A final step consists in revisiting the logical concept of opposition in the same way as the above relation of disagreement, namely: from an ontic and an epistemic point of view. The latter may give rise to a new non-standard theory of epistemic oppositions.

### 2.3. *Oppositions*

Disagreement and epistemic opposition are on a par and cannot be separated from each other. Nevertheless, the traditional view of opposition is related to logical relations between sentences, rather than belief attitudes. A reviewed definition of epistemic opposition runs as follows:

*Any two agents c,d are* opposed *to each other about a given sentence $\varphi$ iff they disagree about its truth- or falsity-conditions.*

Let Op be the relation of opposition (see Schang 2013). It means that any two agents are opposed whenever their attitude is different from each other.

$\mathrm{Op}(\blacksquare^c\varphi, \blacksquare^d\varphi) =_{df}$ there is some $\mathbf{a}_i \in \mathbf{A}$, such that $\mathbf{a}_i(\blacksquare^c\varphi) \neq \mathbf{a}_i(\blacksquare^d\varphi)$ (with $i = 1$ or 2)

Now there can be different ways of disagreeing, in accordance to the variety of oppositions. Even if two agents have globally compatible beliefs, a broader sense of opposition can be applied to them by saying that they partly disagree with respect to falsity-conditions while fully agreeing about truth-conditions. The following definitions of epistemic oppositions deal with agreement and disagreement irrespective of the truth-value.

The attitudes of any two agents $c,d$ stand in a relation of $\varphi$:

*contrariety* iff $c$ and $d$ cannot agree at once about $\varphi$:
$\mathbf{a}_i(\blacksquare^c\varphi) = 1 \Rightarrow \mathbf{a}_i(\blacksquare^d\varphi) = 0$

*contradictoriness* $c$ and $d$ cannot agree and disagree at once about $\varphi$
$\mathbf{a}_i(\blacksquare^c\varphi) = 1 \Leftrightarrow \mathbf{a}_i(\blacksquare^d\varphi) = 0$

*subcontrariety* iff $c$ and $d$ cannot disagree at once about $\varphi$
$\mathbf{a}_i(\blacksquare^c\varphi) = 0 \Rightarrow \mathbf{a}_i(\blacksquare^d\varphi) = 1$

*subalternation* iff $d$ always agrees with what $c$ agrees about $\varphi$
$\mathbf{a}_i(\blacksquare^c\varphi) = 1 \Rightarrow \mathbf{a}_i(\blacksquare^d\varphi) = 1$

A relevant feature of epistemic oppositions is their non-standard behavior in $\mathbf{AR}_{4\blacksquare}$: there may be more than one sort of epistemic opposition between any two agents, according to their attitudes towards truth- and falsity-conditions. In other words, partial disagreements entail partial oppositions: any two agents $c,d$ may have *different* relations about a given sentence $\varphi$, with respect to its truth-conditions or its falsity-conditions.

This gives rise to a more complex formalization of oppositions, which appears now as an ordered pair of two relations between agents $c,d$:

$$\mathrm{Op}(\mathbf{A}(\blacksquare^c\varphi), \mathbf{A}(\blacksquare^d\varphi)) = \langle \mathrm{Op}(\mathbf{a}_1(\blacksquare^c\varphi), \mathbf{a}_1(\blacksquare^d\varphi)), \langle \mathrm{Op}(\mathbf{a}_2(\blacksquare^c\varphi), \mathbf{a}_2(\blacksquare^d\varphi))\rangle$$

How to determine the relation between different epistemic agents, consequently? Given the previous conceptual and formal device we introduced thus far, it is possible to develop a calculus of oppositions on the basis of the above definitions. For this purpose, let us borrow from the methods introduced in, e.g., Schang (2012), Schang (2013), and Smessaert & Demey (2014).

For one thing, each belief attitude is depicted by its characteristic matrix. By using the latter as functions onto **4**, we take oppositions as epistemic disagreements in the following way.

Let $\mathfrak{m}(\blacksquare\varphi) = \text{(i)(ii)(iii)(iv)}$ be the characteristic matrix function of $\blacksquare\varphi$, where (i) $= \mathbf{A}(\blacksquare 11)$, (ii) $= \mathbf{A}(\blacksquare 10)$, (iii) $= \mathbf{A}(\blacksquare 01)$, (iv) $= \mathbf{A}(\blacksquare 00)$

The epistemic relation $\mathrm{Op}(c,d)$ between agents $c, d$ can be determined by means of $\mathrm{Op}(\mathfrak{m}(\blacksquare^c\varphi), \mathfrak{m}(\blacksquare^d\varphi))$.

A bilateralist approach results in three different sorts of oppositions: total, partial, and mixed. *Total* oppositions are oppositions which express the same sort of disagreement with respect to both truth- and falsity-conditions, as with bivalent unilateralist sentences. *Partial* oppositions express a disagreement in either of truth- and falsity-conditions, whereas the other one stands for full agreement between agents. Finally, *mixed* oppositions express different sorts of oppositions with respect to truth- and falsity-conditions.

Here is a square of total oppositions, each vertex being related to other ones by strict relations of contradiction, contrariety, subcontrariety, or subalternation between their separate truth- and falsity-conditions. The vertical valuations of the right side correspond to the top-bottom ordering of outputs values in a matrix from (i) to (iv).

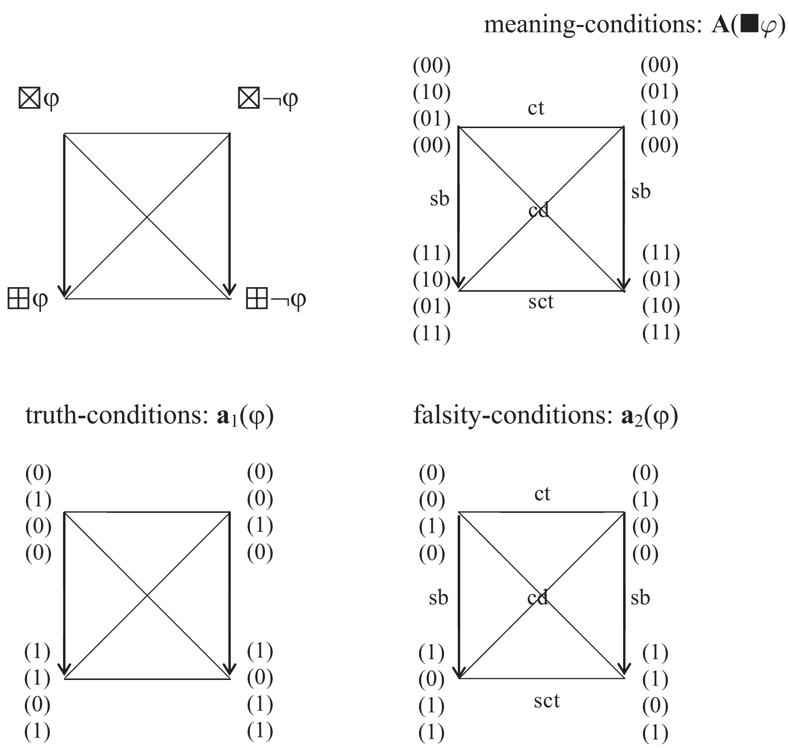


**Figure 1.** Squares of total oppositions

The following logical relations correspond to mixed oppositions between negativists and positivists in $\mathbf{AR}_{4\blacksquare}$, including both ontic and epistemic negations in their vertices.

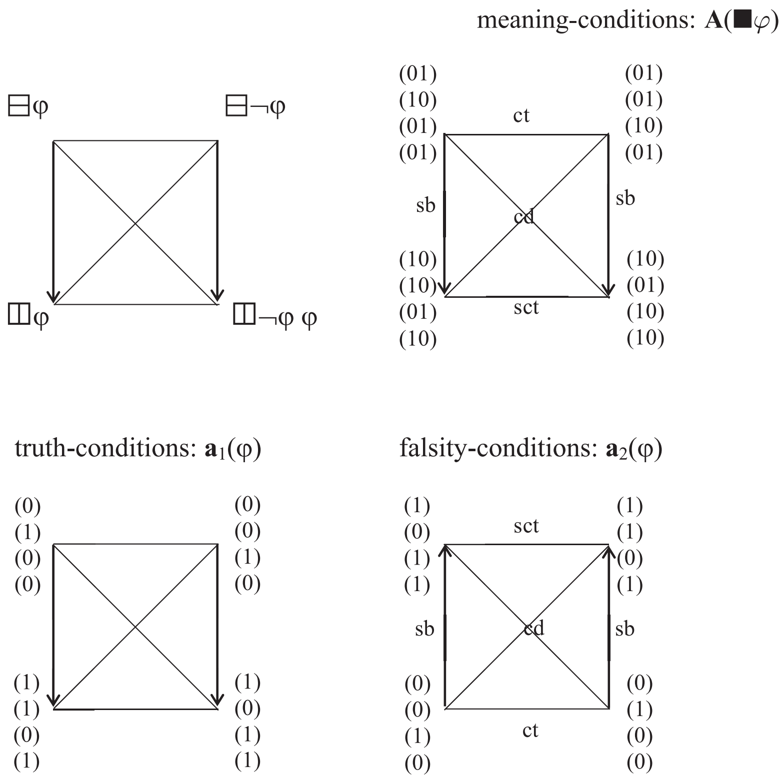


**Figure 2.** Squares of mixed oppositions

The above squares of mixed relations show that any two agents can have reversed relations with respect to truth-falsity-conditions: contrary or sub-contrary, subaltern or superaltern. More generally, all these squares also help to equate the behavior of different agents, by means of a combination of ontic and epistemic negations. Then:

$$\boxbar\varphi = {\sim}\boxminus\neg\varphi$$
$$\boxtimes\varphi = {\sim}\boxplus\neg\varphi$$

Moreover, the behavior of ontic and epistemic negations is also reminiscent of the four operations depicted by Piaget (1972) in his INRC group: I for identity, N for negational, R for reciprocity, and C for correlation. These also correspond to Klein's four-group, which is an Abelian abstract group $\mathbb{Z}_2 \times \mathbb{Z}_2$ including four elements. By applying these operations to matrix functions

$\mathfrak{m}(\blacksquare\varphi) = \text{(i)(ii)(iii)(iv)}$, they can also be combined with each other to yield a restricted set of opposition-forming operators. Thus for any formula $\blacksquare\varphi$:

$$\mathrm{N}(\blacksquare\varphi) = \mathrm{cd}(\blacksquare\varphi)$$

$$\mathrm{R}(\blacksquare\varphi) = \{\mathrm{ct}(\blacksquare\varphi), \mathrm{sct}(\blacksquare\varphi)\}$$

$$\mathrm{C}(\blacksquare\varphi) = \{\mathrm{sb}(\blacksquare\varphi), \mathrm{sp}(\blacksquare\varphi)\}$$

R and C turn their operands into contrary or subcontrary formulas, depending upon the initial valuation of $\mathfrak{m}(\blacksquare\varphi)$. The same variable result also holds with C, given that C = NR = RN and due to the functional definition of subalternation as a "contradictory of contrary" in Béziau (2003).

## Conclusion

Our plea for epistemic pluralism has been based on two fundamental assumptions of logical dependence: between truth and justification, according to an antirealist-minded theory of truth as justified assertibility; between negation and denial, according to a bilateralist theory of speech-acts. Although such assumptions deserve a more comprehensive debate on its own right, we take these to lead to interesting issues in social epistemology. The first is a distinction between logics of information and justification; the second is the rise of new non-standard oppositions between epistemic agents.

Further works are intended to confirm and develop the results of the present paper, namely: a generalization from $\mathbf{AR}_2^2 = \mathbf{AR}_4$ to $\mathbf{AR}_m^n$, $m$ and $n$ being arbitrary numbers of ordered answers and related questions; a further comparative analysis between belief attitudes and modal operators, including multimodalities adapted to formulas like, e.g., $\boxminus\boxtimes\varphi$; a translation of mainstream logics (classical, paracomplete, paraconsistent, and the like) into $\mathbf{AR}_m^n$.

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