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PRESUPPOSITIONS AND TWO KINDS OF NEGATION

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Abstract

In this paper I deal with sentences that come with a presupposition that is entailed by the positive as well as negated form of a given sentence. However, there are two kinds of negation, namely narrow-scope and wide-scope negation. I am going to prove that while the former is presupposition-preserving, the latter is presupposition-denying. Thus the main contribution of this paper is the proof that these two kinds of negation are not equivalent. This issue has much in common with the difference between topic and focus articulation within a sentence. Whereas articulating the topic of a sentence activates a presupposition, articulating the focus frequently yields merely an entailment. My background theory is Transparent Intensional Logic (TIL). TIL is an expressive logic apt for the analysis of sentences with presuppositions, because in TIL we work with partial functions, in particular with propositions with truth-value gaps. Moreover, procedural semantics of TIL makes it possible to define a general analytic schema of sentences associated with presuppositions, which is another novel contribution of this paper.

Keywords: presupposition; wide-scope vs. narrow-scope negation; definite descriptions; topic-focus articulation; Transparent Intensional Logic; if-then-else-fail function

Introduction

Sentences often come attached with a presupposition that is entailed by the positive as well as negated form of a given sentence. Thus if the presupposition of a sentence *S* is not true, the sentence *S* can be neither true nor false. I follow Frege and Strawson in treating survival under negation as the most important test for presupposition. However, there are two kinds of negation, namely Strawsonian *narrow-scope* and Russellian *wide-scope negation*. While the former is presupposition-preserving, the latter is presupposition-denying.

This issue has much in common with the difference between topic and focus articulation within a sentence. I find that whereas articulating the topic of a sentence activates a presupposition, articulating its focus frequently yields merely an entailment. The point of departure is that sentences of the form "The *F* is a *G*" are ambiguous. Their ambiguity stems from different topic-focus articulations of such sentences. The issue is this. If 'the *F*' is the topic phrase then this description occurs extensionally, i.e. with *de re*

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supposition, and the Strawsonian analysis appears to be what is wanted. On this reading the sentence presupposes the existence of the descriptum of ‘the *F*’, because the property *G* is ascribed to the object, if any, referred to by ‘the *F*’. The other option is ‘*G*’ occurring as topic and ‘the *F*’ as focus. This reading corresponds to Donnellan’s attributive use of ‘the *F*’ and the description occurs intensionally with *de dicto* supposition. On this reading the Russellian analysis gets the truth-conditions of the sentence right. The existence of a unique *F* is merely entailed.

From a logical point of view, the two readings differ also in the way their respective negated form is obtained. Whereas the Strawsonian narrow-scope negated form is “The *F* is not a *G*”, the Russellian wide-scope negated form is “It is not true that the *F* is a *G*”. Thus in the former case the property of not being a *G* is ascribed to the object, if any, that is referred to by the topic phrase ‘the *F*’. On the other hand, in the Russellian case the property of not being true is ascribed to the whole proposition that the *F* is a *G*. I am going to prove that these two readings are not equivalent, because they denote different propositions (truth-conditions individuated up to logical equivalence). While “The *F* is not a *G*” lacks a truth-value at those states of affairs where the *F* does not exist, the wide-scope negation “It is not true that the *F* is a *G*” is true at such states of affairs where there is no *F*.

To capture this difference, a logic of partial functions is needed. My background theory is Transparent Intensional Logic (TIL).[1] TIL is an expressive logic apt for the analysis of sentences with presuppositions, because in TIL we work with partial functions, in particular with propositions with truth-value gaps.

The rest of this paper is organised as follows. The relevant foundations of TIL are introduced in Section 1. In Section 2 the difference between narrow-scope and wide-scope negation is explained. Using the difference between these two kinds of negation, I also differentiate presupposition from mere entailment. In Section 3 I deal with ambiguities stemming from topic-focus articulation of a sentence. Finally, in Section 4 I generalise these results and propose a general analytic schema for sentences that come attached with a presupposition.

1. Foundations of TIL

The terms of the TIL language denote abstract procedures (roughly, Church’s functions-in-intension) that produce set-theoretical mappings (functions-in-extension).[2] These procedures are rigorously defined as TIL *constructions*.

[1] See Tichý (1988), Tichý (2004), Duží, Jespersen and Materna (2010).

[2] As an extreme case the produced function/mapping can be a nullary function, that is, an atomic object such as an individual, number, or a truth-value.

Being procedural objects, constructions are designed to be executed in order to operate on input objects (of a lower-order type) and produce the object (if any) they are typed to produce, while non-procedural objects, i.e. non-constructions, cannot be executed. Thus non-procedural objects cannot be constituents of constructions, and there are two simple constructions that present objects to be operated on. They are *Trivialization* and *Variables.*

The operational sense of Trivialization is similar to that of constants in formal languages. It presents an object X without the mediation of any other procedures. Using the terminology of programming languages, the Trivialization of X, in symbols '${}^0X$', is just a pointer to X.

Variables produce objects dependently on valuations: they are said to v-construct. We adopt an objectual variant of the Tarskian conception of variables. For each type (see Definition 2 below), there are countably many variables assigned that range over this type. Objects of each type can be arranged into infinitely many sequences. A valuation v selects such a sequence of objects of the respective type, and the first variable v-constructs the first object of the sequence, the second variable v-constructs the second object of the sequence, and so on. Thus the execution of a Trivialization or of a variable never fails to produce an object.

The execution of some other, compound, constructions can fail to present an object they are typed to produce. In such a case we say that they are *v-improper*. There are two kinds of improperness. Either a construction is compounded in a type-theoretically incoherent way, or it is an application of a function to an argument at which the function is not defined (i.e. it lacks a value at this argument). Here is the definition of *construction*.

Definition 1 (*construction*)

(i) *Variables* $x, y, \ldots$ are *constructions* that construct objects (elements of their respective ranges) dependently on a valuation v; they v-construct.

(ii) Where X is an object whatsoever (even a *construction*), ${}^0X$ is the *construction Trivialization* that constructs X without the mediation of any other constructions.

(iii) Let $X, Y_1, \ldots, Y_n$ be arbitrary constructions. Then the *Composition* $[X\, Y_1 \ldots Y_n]$ is the following *construction*. For any v, the Composition $[X\, Y_1 \ldots Y_n]$ is *v-improper* if one or more of the constructions $X, Y_1, \ldots, Y_n$ are v-improper, or if X v-constructs a function that is not defined at the n-tuple of objects v-constructed by $Y_1, \ldots, Y_n$. If X v-constructs a function that is defined at the n-tuple of objects v-constructed by $Y_1, \ldots, Y_n$ then the Composition $[X\, Y_1 \ldots Y_n]$ v-constructs the value of this function at the n-tuple.

(iv) The (λ-)*Closure* $[\lambda x_1 \ldots x_m\ Y]$ is the following *construction*. Let x_1, $x_2, \ldots, x_m$ be pair-wise distinct variables and Y a construction. Then $[\lambda x_1 \ldots x_m\, Y]$ *v-constructs* the function f that takes any members $B_1, \ldots, B_m$

of the respective ranges of the variables $x_1, \ldots, x_m$ into the object (if any) that is $v(B_1/x_1, \ldots, B_m/x_m)$-constructed by Y, where $v(B_1/x_1, \ldots, B_m/x_m)$ is like v except for assigning B_1 to $x_1, \ldots, B_1$ to x_m.

(v) Where X is any object whatsoever, ^1X is the *construction Execution* that v-constructs what X v-constructs. Thus if X is a v-improper construction or not a construction at all, then ^1X is *v-improper*.

(vi) Where X is any object whatsoever, ^2X is the *construction Double Execution*. It v-constructs what is v-constructed by the construction v-constructed by X. Thus if X is not itself a construction, or if X does not v-construct a construction, or if X v-constructs a v-improper construction, then ^2X is *v-improper*.

(vii) Nothing is a *construction*, unless it so follows from (i) through (vi).*

Note that Closure $[\lambda x_1 \ldots x_m\ Y]$ is never v-improper for any valuation v, as it always v-constructs a function. Even if the constituent Y is v-improper for all valuations v, the Closure is not v-improper. Yet in such a case the thus constructed function is a bizarre object; it is a degenerate function that lacks a value at any argument.

With constructions of constructions, constructions of functions, functions, and functional values in our stratified ontology, we need to keep track of the traffic between multiple logical strata. The ramified type hierarchy does just that. The type of first-order objects includes all objects that are not constructions. Therefore, it includes not only the standard objects of individuals, truth-values, sets, etc., but also functions including functions defined on possible worlds (i.e., the intensions germane to possible-world semantics). The type of second-order objects includes constructions of first-order objects, and functions that have such constructions at their domain or range. The type of third-order objects includes constructions of first- and second-order objects, and functions that have such constructions at their domain or range. And so on, ad infinitum.

Definition 2 (*ramified hierarchy of types*). Let B be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

$\mathbf{T_1}$ *(types of order 1)*.

i) Every member of B is an elementary *type of order 1 over B*.

ii) Let $\alpha, \beta_1, \ldots, \beta_m$ $(m > 0)$ be types of order 1 over B. Then the collection $(\alpha\ \beta_1 \ldots \beta_m)$ of all m-ary partial mappings from $\beta_1 \times \ldots \times \beta_m$ into α is a functional *type of* order 1 *over* the base B.

iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

$\mathbf{C_n}$ *(constructions of order n)*

i) Let x be a variable ranging over a type of order n. Then x is a *construction of order n over B*.

ii) Let X be a member of a type of order n. Then ${}^0X$, ${}^1X$, ${}^2X$ are *constructions of order n over B*.

iii) Let $X, X_1, ..., X_m$ ($m > 0$) be constructions of order n over B. Then $[X X_1 ... X_m]$ is a *construction of order n over B*.

iv) Let $x_1, ...x_m, X$ ($m > 0$) be constructions of order n over B. Then $[\lambda x_1...x_m X]$ is a *construction of order n over B*.

v) Nothing is a *construction of order n over B* unless it so follows from $\mathbf{C}_n$ (i)-(iv).

Tn+1 (types of order *n* + 1)

Let $*_n$ be the collection of all constructions of order n over B. Then

i) $*_n$ and every type of order n are *types of order* $n + 1$.

ii) If $m > 0$ and $\alpha, \beta_1, ..., \beta_m$ are types of order $n + 1$ over B, then $(\alpha\ \beta_1\ ...\ \beta_m)$ (see T_1 ii)) is a *type of order* $n + 1$ *over B*.

iii) Nothing is a *type of order* $n + 1$ *over B* unless it so follows from (i) and (ii).*

We model sets and relations by their characteristic functions. Thus, for instance, $(o\iota)$ is the type of a set of individuals, while $(o\iota\iota)$ is the type of a relation-in-extension between individuals. For the purposes of natural-language analysis, we are assuming the following base of *ground types*:

o: the set of truth-values $\{\mathbf{T}, \mathbf{F}\}$;

ι: the set of individuals (the universe of discourse);

τ: the set of real numbers (doubling as discrete times);

ω: the set of logically possible worlds (the logical space).

Empirical expressions denote *empirical conditions* that may or may not be satisfied at some world/time pair of evaluation. We model these empirical conditions as possible-world-semantic *(PWS) intensions.* PWS intensions are entities of type $(\beta\omega)$: mappings from possible worlds to an arbitrary type β. The type β is frequently the type of a *chronology* of α-objects, i.e., a mapping of type $(\alpha\tau)$. Thus α-intensions are frequently functions of type $((\alpha\tau)\omega)$, abbreviated as '$\alpha_{\tau\omega}$'. *Extensional entities* are entities of a type α where $\alpha \neq (\beta\omega)$ for any type β. Where w ranges over ω and t over τ, the following logical form essentially characterizes the logical syntax of empirical language: $\lambda w \lambda t\ [\ldots w \ldots . t \ldots]$.

Examples of frequently used PWS intensions are: *propositions* of type $o_{\tau\omega}$, *properties* of individuals of type $(o\iota)_{\tau\omega}$, binary *relations*-in-intension between individuals of type $(o\iota\iota)_{\tau\omega}$, individual *offices* (or roles) of type $\iota_{\tau\omega}$, *magnitudes* of type $\tau_{\tau\omega}$.

Logical objects like *truth-functions* and *quantifiers* are extensional: $\wedge$ (conjunction), $\vee$ (disjunction) and $\supset$ (implication) are of type (ooo), and

$\neg$ (negation) of type (οο). *Quantifiers* $\forall^{\alpha}$, $\exists^{\alpha}$ are type-theoretically polymorphous, total functions of type (ο(οα)), for an arbitrary type α, defined as follows. The *universal quantifier* $\forall^{\alpha}$ is a function that associates a class A of α-elements with **T** if A contains all elements of the type α, otherwise with **F**. The *existential quantifier* $\exists^{\alpha}$ is a function that associates a class A of α-elements with **T** if A is a non-empty class, otherwise with **F**.

Notational conventions. Below all type indications will be provided outside the formulae in order not to clutter the notation. The outermost brackets of a Closure will be omitted whenever no confusion arises. We often use infix notation without Trivialization for the application of truth-functions and identities to make the formulae easier to read. Furthermore, 'X/α' means that an object X is (a member) of type α. '$X \rightarrow_v \alpha$' means that X is typed to v-construct an object of type α, if any. We write '$X \rightarrow \alpha$' if what is v-constructed does not depend on a valuation v. Throughout, it holds that the variables $w \rightarrow_v \omega$ and $t \rightarrow_v \tau$. If $C \rightarrow_v \alpha_{\tau\omega}$ then the frequently used Composition $[[C\ w]\ t]$, which is the intensional descent (a.k.a. extensionalization) of the α-intension v-constructed by C, will be encoded as 'C_{wt}'.

We invariably furnish expressions with procedurally structured meanings, which are explicated as TIL constructions. Thus TIL constructions are assigned to expressions as their context-invariant meanings, and the analysis of an unambiguous expression consists in discovering the logical construction encoded by a given sentence. To this end we have developed the *TIL method of analysis* that consists of three steps:

1) *Type-theoretical analysis*, i.e., assigning types to the objects that receive mention in the analysed expression.
2) *Type-theoretical synthesis*, i.e., combining the constructions of the objects *ad* (1) in order to construct the object (if any) of the respective type denoted by the whole expression.
3) *Type-theoretical checking*, i.e. checking whether the proposed analysis is type-theoretically coherent.

To illustrate the method, we analyse the stock example "The King of France is bald" *à la* Strawson. *First*, type-theoretical analysis. The sentence mentions the following objects. $King\_of/(\iota\iota)_{\tau\omega}$ is an empirical function that dependently on $\langle w,t \rangle$-pairs assigns to one individual (a country) another individual (its king) or else nothing, depending on whether the country is a monarchy and the monarch is a king rather than a queen; $France/\iota$; $King\_of\_France/\iota_{\tau\omega}$; $Bald/(o\iota)_{\tau\omega}$; the whole sentence denotes a proposition, that is, an object of type $o_{\tau\omega}$.

Second, synthesis. Now we are to combine the constructions of the objects *King_of* and *France* in order to produce the office *King_of_France* and then ascribe *Baldness* to the holder of the office. Since we intend to arrive at the *literal* analysis of the sentence, the objects denoted by semantically simple expressions are constructed by their Trivializations: $^0King\_of$, ^0France, ^0Bald. In order to construct the office *King_of_France*, we have to combine $^0King\_of$ and ^0France. The function *King_of* must be *extensionalized* first *via* the Composition $^0King\_of_{wt} \rightarrow_v (\iota\iota)$, and the result is then applied to *France*; thus we get $[^0King\_of_{wt}\ ^0France] \rightarrow_v \iota$. Abstracting over the values of w and t we obtain the Closure that constructs the royal office:

$$\lambda w \lambda t\ [^0King\_of_{wt}\ ^0France] \rightarrow \iota_{\tau\omega}.$$

But the property of being bald cannot be ascribed to an individual office. Instead it is ascribed to the individual (if any) occupying the office. Thus the office has to be extensionalized first: $\lambda w \lambda t\ [^0King\_of_{wt}\ ^0France]_{wt} \rightarrow_v \iota$. The property itself has to be extensionalized as well: $^0Bald_{wt}$. By composing these two Compositions,

$$[^0Bald_{wt}\ \lambda w \lambda t\ [^0King\_of_{wt}\ ^0France]_{wt}] \rightarrow_v o$$

we obtain either a truth-value (**T** or **F**) or nothing, according as the King of France is bald, or does not exist, respectively. Finally, by abstracting over the values of the variables w and t, we construct the proposition:

$$\lambda w \lambda t\ [^0Bald_{wt}\ \lambda w \lambda t\ [^0King\_of_{wt}\ ^0France]_{wt}] \rightarrow_v o_{\tau\omega}$$

Third, type checking. To this end we usually draw the derivation tree as illustrated by Fig. 1.[3]

$$\lambda w \lambda t\ [^0Bald_{wt}\ \lambda w \lambda t\ [^0King\_of_{wt}\ ^0France]_{wt}]$$

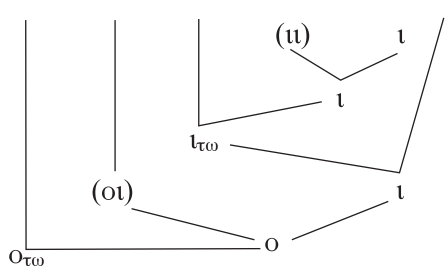

Figure 1. Derivation tree

So much for the basic notions of TIL and its method of analysis.

[3] To simplify the tree, I apply these rules: if $C \rightarrow_v \alpha_{\tau\omega}$ then $C_{wt} \rightarrow_v \alpha$, and if $D \rightarrow_v \alpha$ then $\lambda w \lambda t\ D \rightarrow_v \alpha_{\tau\omega}$. Indeed, unpacking the abbreviations '$\alpha_{\tau\omega}$' and 'C_{wt}', we have: $C \rightarrow_v ((\alpha\tau)\omega)$, $[Cw] \rightarrow_v (\alpha\tau)$, $[[Cw]t] \rightarrow_v \alpha$. Similarly the second rule: $D \rightarrow_v \alpha$, $\lambda t\ D \rightarrow_v (\alpha\tau)$, $\lambda w \lambda t\ D \rightarrow_v ((\alpha\tau)\omega)$.

2. Presuppositions and the two kinds of negation

As stated at the outset, sentences often come with a presupposition that is entailed both by the sentence and its negation. The entailment relation is defined as usual. A proposition P is analytically entailed by a proposition S, $S \vDash P$, if P takes the truth-value **T** at all $\langle w, t\rangle$-pairs at which S takes the value **T**.[4]

To define analytic entailment formally, we need the propositional property $True/(\mathrm{oo}_{\tau\omega})_{\tau\omega}$ which is defined as follows. Let P be a propositional construction ($P/*_n \to \mathrm{o}_{\tau\omega}$). Then

$$[{}^0True_{wt}\, P] \text{ } v\text{-constructs } \mathbf{T} \text{ iff } P_{wt} \text{ } v\text{-constructs } \mathbf{T}\text{, otherwise } \mathbf{F}.$$

For completeness, there are two other properties of the same type, namely *False* and *Undefined*, defined as follows:

$$[{}^0False_{wt}\, P] \text{ } v\text{-constructs } \mathbf{T} \text{ iff } P_{wt} \text{ } v\text{-constructs } \mathbf{F}\text{, otherwise } \mathbf{F}$$

$$[{}^0Undefined_{wt}\, P] \text{ } v\text{-constructs } \mathbf{T} \text{ iff } P_{wt} \text{ is } v\text{-improper, otherwise } \mathbf{F}$$

Note that $[{}^0True_{wt}\, P]$ v-constructs **F** in two cases, namely if P_{wt} v-constructs **F** or if P_{wt} is v-improper. Hence, for instance, if a proposition v-constructed by P is not true at a given $\langle w, t\rangle$-pair, it does not have to be false, because there is the third possibility of being undefined. Formally, we have these relations ($=/(\mathrm{ooo})$):

$$[{}^0True_{wt}\, P] = \neg[{}^0False_{wt}\, P] \wedge \neg[{}^0Undefined_{wt}\, P]$$

$$[{}^0False_{wt}\, P] = \neg[{}^0True_{wt}\, P] \wedge \neg[{}^0Undefined_{wt}\, P]$$

$$[{}^0Undefined_{wt}\, P] = \neg[{}^0True_{wt}\, P] \wedge \neg[{}^0False_{wt}\, P]$$

Analytical entailment is defined as follows (P, $S/*_n \to \mathrm{o}_{\tau\omega}$, $\vDash /(\mathrm{oo}_{\tau\omega}\mathrm{o}_{\tau\omega})$).[5]

$$(S \vDash P) \text{ iff } \forall w\, \forall t\, [[{}^0True_{wt}\, S] \supset [{}^0True_{wt}\, P]]$$

The logical difference between a presupposition and mere entailment is this:

$$P \text{ is a } \textit{presupposition} \text{ of } S \text{ iff } (S \vDash P) \text{ and } (\textit{non-S} \vDash P)$$

Thus if P is not true at a given $\langle w, t\rangle$-pair, then *neither S nor non-S* is true. Hence, S has no truth-value at such a $\langle w, t\rangle$-pair at which its presupposition is not true.

On the other hand:

$$P \text{ is } \textit{merely entailed} \text{ by } S \text{ if } (S \vDash P) \text{ and neither } (\textit{non-S} \vDash P) \text{ nor } (\textit{non-S} \vDash \textit{non-P})$$

[4] For the slight difference between *analytical* and *logical* entailment see Duží (2010).

[5] Again, I use the infix notation '($S \vDash P$)' instead of the proper TIL notation '$[{}^0 \vDash S\, P]$' to make the formulae easier to read.

Hence if *S* is not true we cannot deduce anything about the truth-value, or lack thereof, of *P*.

However, in order to decide whether there is a presupposition of *S*, we have to take into account two ways in which the negated form *non-S* can be obtained. To illustrate the situation, consider again the sentence "The King of France is bald". If the royal office is occupied and we want to say that its holder is *not* bald, we would simply use the form "The King of France is *not* bald". This is Strawsonian *narrow-scope negation*. The property of not being bald is ascribed to the holder of the royal office.[6] Thus the analyses of the Strawsonian reading of the sentence and of its negation amount to these constructions:

(S) $\lambda w\lambda t\ [{}^0Bald_{wt}\ \lambda w\lambda t\ [{}^0King\_of_{wt}\ {}^0France]_{wt}]$

($non\text{-}S$) $\lambda w\lambda t\ \neg[{}^0Bald_{wt}\ \lambda w\lambda t\ [{}^0King\_of_{wt}\ {}^0France]_{wt}]$

However, if the royal office is not occupied, one would protest, saying, "No, it is not true that the King of France is bald, for the King of France does not exist". This is Russellian *wide-scope negation.* The property of not being true is ascribed to the whole proposition that the King of France is bald.[7]

In order to analyse Russellian wide-scope negation, we apply the above defined propositional property $True/(oo_{\tau\omega})_{\tau\omega}$:

($non\text{-}R$) $\lambda w\lambda t\ \neg[{}^0True_{wt}\ \lambda w\lambda t\ [{}^0Bald_{wt}\ \lambda w\lambda t\ [{}^0King\_of_{wt}\ {}^0France]_{wt}]]$

($non\text{-}R$) is the wide-scope negation of the proposition that it is true that the King of France is bald constructed by

(R) $\lambda w\lambda t\ [{}^0True_{wt}\ \lambda w\lambda t\ [{}^0Bald_{wt}\ \lambda w\lambda t\ [{}^0King\_of_{wt}\ {}^0France]_{wt}]]$

This is not exactly Russell's analysis. The Russellian rephrasing of the sentence "The King of France is bald" is "There is a unique individual such that he is the King of France and he is bald". The analysis of this sentence comes down to

(R^*) $\lambda w\lambda t\ [{}^0\exists\lambda x\ [[x = \lambda w\lambda t\ [{}^0King\_of_{wt}\ {}^0France]_{wt}] \wedge [{}^0Bald_{wt}\ x]]]$.

Additional types. $\exists/(o(o\iota))$; $=/(o\iota\iota)$; $x/*_1 \rightarrow_v \iota$.[8]

[6] There is still a difference between being *non-bald* (property negation) and being an *x* such that *not: x is bald* (boolean negation). However, this issue is out of the scope of this paper.

[7] For the difference between narrow-scope and wide-scope negation with respect to a presupposition, see also Hajičová (2008).

[8] Note that in TIL we do not need a construction to specify the uniqueness of the King of France, because it is inherent in the meaning of 'the King of France'. The meaning of definite descriptions like 'the King of France' is a construction of an individual office of type $\iota_{\tau\omega}$ occupied in each $\langle w, t\rangle$-pair by at most one individual. See also Duží (2009).

Yet (R) gets the truth-conditions of the Russellian reading right, because (*R*) and (*R**) are equivalent in the sense of constructing the same proposition.[9] This proposition does not come with the presupposition that the King of France exists, unlike the proposition constructed by (*S*). The analysis reveals that these two readings are not equivalent. Though (*R*) and (*S*) are *co-entailing* they denote *different propositions*, which I am going to prove now.

First, the equivalence of (*R*) and (*R**).

a) Let the royal office be occupied in a given world w and time t by an individual a. Then if this individual a is bald, the proposition

$$\lambda w \lambda t\ [{}^0 Bald_{wt}\ \lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}]$$

takes the value **T** in such a world-time pair, otherwise **F**, and so does (*R*) according to the definition of the property *True.* By assumption

$$[{}^0 a = [\lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}].$$

Hence the Composition

$$[{}^0 a = [\lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}] \wedge [{}^0 Bald_{wt}\ {}^0 a]]$$

v-constructs **T** and so does

$$[\lambda x\ [x = [\lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}] \wedge [{}^0 Bald_{wt}\ x]]\ {}^0 a]$$

if a is bald. It means that the class of individuals

$$\lambda x\ [x = [\lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}] \wedge [{}^0 Bald_{wt}\ x]]$$

is non-empty according as the individual a is bald or not. Thus according to the definition of the quantifier $\exists$, the proposition constructed by (*R**) takes the value **T** if a is bald, otherwise **F**, exactly as does the proposition constructed by (R).

b) Let the royal office be vacant in a given world w and time t. Then by Def. 1, iii) the following Compositions are v-improper:

$$[\lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}]$$

$$[{}^0 Bald_{wt}\ \lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}]$$

$$[x = [\lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}]]$$

$$[[x = \lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}] \wedge [{}^0 Bald_{wt}\ x]].$$

This in turn means that the proposition constructed by

$$\lambda w \lambda t\ [{}^0 Bald_{wt}\ \lambda w \lambda t\ [{}^0 King\_of_{wt}\ {}^0 France]_{wt}]$$

9 For more details on Russell's analysis and its comparison with Strawson's analysis see Duží (2014).

is undefined and by the definition of the property *True* the proposition constructed by (*R*) takes the value **F**. But so does (*R**), because the class constructed by

$$\lambda x\, [[x = \lambda w \lambda t\, [{}^0King\_of_{wt}\, {}^0France]_{wt}] \wedge [{}^0Bald_{wt}\, x]]$$

is empty and the application of the existential quantifier $\exists$ to an empty class results in **F**.

Note that from (b) it also follows that neither (*R*) nor (*R**) comes with the existential presupposition that the King of France exists. Non-trivial existence of empirical objects is in TIL explicated as a property of intensions to be instantiated at a given $\langle w, t\rangle$-pair of evaluation.[10] Thus to say that unicorns do not exist is tantamount to saying that at the given world *w* and time *t* the property of being a unicorn has the empty class of individuals as its population. Similarly, that the King of France does not exist means that the office of the King of France is vacant at the world and time of evaluation. Hence if there were an existential presupposition, the proposition constructed by (*R*) or (*R**) would have no truth-value in case of the royal office being vacant. Yet, these propositions take the value **F**. In other words, neither (*non-R*) nor (*non-R**) entails that the royal office is occupied.

Now I am going to prove that (*S*), and thus also (*non-S*), presupposes the existence of the King of France. To this end I must prove that the following arguments are valid (though not sound):

$$\frac{\text{The King of France is (not) bald}}{\text{The King of France exists}}$$

First, the analysis of the conclusion amounts to this construction:

$$\lambda w \lambda t\, [{}^0Exist_{wt}\, [\lambda w \lambda t\, [{}^0King\_of_{wt}\, {}^0France]]]$$

where $Exist/(o\iota_{\tau\omega})_{\tau\omega}$ is the property that an office has when it is occupied. This property is defined as follows:

$${}^0Exist =_{of} \lambda w \lambda t\, \lambda c\, [{}^0\exists \lambda x\, [x = c_{wt}]].$$

Types: $\exists/(o(o\iota))$; $c \rightarrow_v \iota_{\tau\omega}$; $x \rightarrow_v \iota$; $=_{of} /(o(o\iota_{\tau\omega})_{\tau\omega}(o\iota_{\tau\omega})_{\tau\omega})$: the identity of properties of individual offices; $=/(o\iota\iota)$: the identity of individuals, $x \rightarrow_v \iota$.

Now I am ready to prove the validity of the above arguments and thus the validity of the claim that the Strawsonian reading is associated with a presupposition of the royal office being occupied.

At any $\langle w, t\rangle$-pair the following proof steps are truth-preserving:

1) $(\neg)[{}^0Bald_{wt}\, \lambda w \lambda t\, [{}^0King\_of_{wt}\, {}^0France]_{wt}]$ Ø

2) $\neg[{}^0Improper_{wt}\, {}^0[\lambda w \lambda t\, [{}^0King\_of_{wt}\, {}^0France]_{wt}]]$ 1), by Def. 1, iii)

[10] For details see Duží *et al.* (2010, § 2.3).

3)	$\neg[^0Empty\ \lambda x\ [x = [\lambda w\lambda t\ [^0King\_of_{wt}\ ^0France]]_{wt}]]$	2), by Def. 1, iv)
4)	$[^0\exists\lambda x\ [x = [\lambda w\lambda t\ [^0King\_of_{wt}\ ^0France]]_{wt}]]$	3), EG
5)	$[\lambda c\ [^0\exists\lambda x\ [x = c_{wt}]]\ \lambda w\lambda t\ [^0King\_of_{wt}\ ^0France]]$	4), λ-abstraction
6)	$^0Exist_{wt} = \lambda c\ [^0\exists\lambda x\ [x = c_{wt}]]$	Def. of *Exist*
7)	$[^0Exist_{wt}\ [\lambda w\lambda t\ [^0King\_of_{wt}\ ^0France]]]$	5), 6), substitution of identicals

Remark. At step (2) the property of being *Improper* of type $(o*_1)_{\tau\omega}$ is applied to the *construction* $[\lambda w\lambda t\ [^0King\_of_{wt}\ ^0France]_{wt}]$ of type $*_1$ that is supplied here by its Trivialisation $^0[\lambda w\lambda t\ [^0King\_of_{wt}\ ^0France]_{wt}]$ belonging to type $*_2$. On the other hand, at step (3) the property of being *Empty* of type (ο(οι)) is applied to the set of individuals $\lambda x\ [x = [\lambda w\lambda t\ [^0King\_of_{wt}\ ^0France]]_{wt}]$. These two steps are necessary in order to existentially generalize at step (4). In a logic of partial functions such as TIL we cannot carelessly generalize before having proved that the set to which the existential quantifier is applied is non-empty.

The following Table 1 illustrates the truth conditions of the propositions constructed by (*R*), (*S*), (*non-R*) and (*non-S*) with respect to the occupancy of the office of the King of France (*KF*).[11]

	KF	(*R*)	(*S*)	(*non-R*)	(*non-S*)
w_1, t_1	*a*	T	T	F	F
w_2, t_2	*a*	F	F	T	T
w_3, t_3	⊥	F	⊥	T	⊥
w_4, t_4	*b*	F	F	T	T
w_5, t_5	*b*	T	T	F	F
w_6, t_6	⊥	F	⊥	T	⊥
…					

Table 1. Russellian vs. Strawsonian analysis

Indeed, (*R*) and (*S*) are co-entailing. Whenever (*R*) is true (*S*) is true as well, and vice versa. Yet the propositions (*R*) and (*S*) are not identical, because (*non-R*) and (*non-S*) are not co-entailing. At those $\langle w, t\rangle$-pairs where the King of France does not exist, both (*S*) and (*non-S*) are undefined, the propositions having a truth-value gap, while (*R*) and (*non-R*) are false and true, respectively.

[11] I use the symbol '⊥' to mark a truth-value *gap* rather than the truth-value F.

3. Ambiguities in topic-focus articulation

The above analyses provide a solution to the almost hundred-year old dispute over Strawsonian versus Russellian definite descriptions.[12] The ambiguity of sentences of the form "The *F* is a *G*" is not rooted in a shift of *meaning* of the definite description 'the *F*'. Rather the ambiguity stems from different *topic-focus articulations* of such sentences. Whereas articulating the topic of a sentence activates a presupposition, articulating the focus frequently yields merely an entailment.[13] If 'the *F*' is the topic phrase then this description occurs extensionally, that is with *de re* supposition, and Strawson's analysis appears to be what is wanted. This reading corresponds to Donnellan's *referential use* of 'the *F*' and the sentence *presupposes* the existence of the descriptum of 'the *F*'. The other option is '*G*' occurring as topic and 'the *F*' as focus. This reading corresponds to Donnellan's *attributive* use of 'the *F*' and the description occurs intensionally that is with *de dicto* supposition. On this reading the Russellian analysis gets the truth-conditions of the sentence right. The existence of a unique *F* is merely entailed.

The received view still tends to be that there is room for at most one of the two positions, since they are deemed incompatible. But there is no incompatibility between Strawson's and Russell's positions, because they simply do not talk about one and the same meaning of the sentence "The King of France is bald". My novel *contribution* is to point out this *ambiguity* which yielded the false dilemma. Russell argued for the attributive use of 'the King of France' and Strawson for its referential use.

For illustration, consider the sentence "The Pope of the Roman Catholic Church visited the Pope of the Coptic Orthodox Church in Egypt in 2010". This sentence demonstrates multiple ambiguities and has at least four non-equivalent readings depending on topic-focus articulation. In what follows I will use 'the Catholic Pope' and 'the Coptic Pope' for short, with *topic* marked in italics.

1) *The Catholic Pope* visited the Coptic Pope in Egypt in 2010.

On this reading the sentence *presupposes* that the Catholic Pope exists now, and *merely entails* that the Coptic Pope existed in 2010 (i.e. diachronic occupation of two different offices). Hence with the additional assumption that the Catholic Pope is Francisco and the Coptic Pope in 2010 was

[12] See, for instance, Russell (1905, 1957), Strawson (1950, 1964), Donnellan (1966), von Fintel (2004), Neale (1990). A summary of this dispute can be found in Duží (2014).

[13] This assumption is based on Hajičová (2008), and supported by other linguists as well. See, for instance Gundel (1999), Gundel and Fretheim (2004) and Strawson (1952, esp. p. 173ff.).

Shenouda III, the sentence entails that Francisco visited Shenouda III in Egypt in 2010.

2) The Catholic Pope visited *the Coptic Pope* in Egypt in 2010. (Or, for clarity, "*The Coptic Pope* was visited by the Catholic Pope in Egypt in 2010")

This reading *presupposes* that the Coptic Pope exists now, and *merely entails* that the Catholic Pope existed in 2010 (i.e. diachronic occupation of two different offices). Hence with the additional assumption that the current Coptic Pope is Tawadros II and the Catholic Pope in 2010 was Benedict XVI, the sentence entails that Tawadros II was visited by Benedict XVI in Egypt in 2010.

3) The Catholic Pope visited the Coptic Pope in Egypt *in 2010.* (Or, for clarity, "*In 2010* the Catholic Pope visited the Coptic Pope in Egypt")

This reading *merely entails* that both the Catholic Pope and the Coptic Pope existed in 2010 (i.e. synchronic occupation of two different offices), because the topic is now the year *2010*. The sentence could have been uttered as an answer to the question "What happened in 2010"? Thus it does not presuppose the occupancy of either of the two offices. If one or both of them were not occupied in 2010, one would protest, for instance like this: "No, it is not true that in 2010 the Catholic Pope visited the Coptic Pope in Egypt, because the Catholic Pope did not exist in 2010". Hence, wide-scope, i.e., presupposition-denying negation, is applied. Thus with the additional assumption that the Catholic Pope in 2010 was Benedict XVI and the Coptic Pope was Shenouda III, the sentence entails that in 2010 Benedict XVI visited Shenouda III in Egypt.

4) The Catholic Pope visited the Coptic Pope in Egypt in 2010.

This is a neutral reading that comes with the *presupposition* that both the Catholic Pope and the Coptic Pope exist now (i.e. synchronic occupation of two different offices).[14] Hence if the Catholic Pope is Francisco and the Coptic Pope is Tawadros II the sentence entails that Francisco visited Tawadros II in Egypt in 2010.

It is a matter of *pragmatics*, of course, which reading is the intended one on an occasion of use. Logic cannot decide which among multiple readings happens to be the intended one. Yet, I cannot agree with Kripke (1977) on

[14] Von Fintel (2004) considers in particular such a neutral reading of sentences with definite descriptions. Thus he arrives at the conclusion that using definite descriptions is always connected with an existential presupposition.

two accounts. First, it is not *entirely* a matter of pragmatics which reading is the intended one; it is also a matter of semantics. Second, and more importantly, the Russellian account of definite descriptions cannot, by itself, account for both the referential and the attributive uses. Our fine-grained logical method of analysis as presented in this paper demonstrates that these readings are *not equivalent*, and that the Russellian reading does not take into account any presuppositions triggered by the topic of the sentence in question. Thus though logic itself cannot decide between multiple readings it can contribute to disambiguation of a sentence by making these different meanings explicit. In case the sentence is ambiguous, logic can bring out this ambiguity and, as a result, propose different meanings to be assigned to different (non-equivalent) readings. Thus choosing between them becomes also a matter of *semantics*.

4. General analytic schema for sentences with presuppositions

Up until now I have utilized the singularity of an individual office of type $\iota_{\tau\omega}$ when analysing sentences that have a presupposition. If the office denoted by 'the F' goes vacant at a given world w and time t of evaluation, the extensionalization F_{wt} is v-improper, and if the occurrence of 'the F' is referential (i.e., extensional, or *de re*) the so constructed proposition has a truth-value gap. However, the construction of a presupposition can be more complicated. In particular, the topic term does not have to be a singular one; it can be also a plural term like 'the popes of Rome and Avignon' or a general one like 'a penguin'. Thus we need a general analytic schema for sentences with presuppositions, which I am going to introduce now.

A sentence S that comes with a presupposition P encodes as its meaning this procedure:

In any $\langle w, t\rangle$-pair of evaluation,
if P_{wt} is true
then evaluate S_{wt} to produce a truth-value,
else fail to produce a truth-value.

To formulate this schema rigorously, we need to define the *if-then-else-fail* function. Here is how. The procedure encoded by "If $P(\rightarrow o)$ then $C(\rightarrow \alpha)$, else D $(\rightarrow \alpha)$" behaves as follows:

a) If P v-constructs **T** then execute C (and return the result of type α, provided C is not v-improper).

b) If P v-constructs **F** then execute D (and return the result of type α, provided D is not v-improper).

c) If P is v-improper then no result.

Hence, *if-then-else* is seen to be a function of type $(\alpha o *_n *_n)$, and its definition decomposes into two phases.[15]

First, select a construction to be executed on the basis of a specific condition P. The choice between C and D comes down to this Composition:

$$[{}^0\mathrm{I}^* \, \lambda c \, [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

Types: $P \rightarrow_v o$ *v*-constructs the condition of the choice between the execution of C or D, $C/*_n$, $D/*_n \rightarrow_v \alpha$; $c \rightarrow_v *_n$; $\mathrm{I}^*/(*_n(o*_n))$: the singularizer function that associates a singleton of constructions with the construction that is the element of this singleton, and is otherwise (i.e. if the set is empty or many-valued) undefined.

If P *v*-constructs **T** then the variable c *v*-constructs the *construction* C, and if P *v*-constructs **F** then the variable c *v*-constructs the *construction* D. In either case, the set constructed by

$$\lambda c \, [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]$$

is a singleton and the singularizer I* returns as its value either the construction C or the construction D.[16]

Second, the selected construction is executed; therefore, Double Execution must be applied:

$${}^2[{}^0\mathrm{I}^* \, \lambda c \, [[P \wedge [c = {}^0C]] \vee [\neg P \wedge [c = {}^0D]]]]$$

As a special case of P being a presupposition, *no* construction is to be selected whenever P is not satisfied. Thus the definition of the *if-then-else-fail* function of type $(\alpha o *_n)$ is this:

$${}^2[{}^0\mathrm{I}^* \, \lambda c \, [P \wedge [c = {}^0C]]]$$

Now we can apply this definition to the case of a presupposition. Thus let $P/*_n \rightarrow o_{\tau\omega}$ be a construction of a presupposition of $S/*_n \rightarrow o_{\tau\omega}$. Moreover, let $c/*_{n+1} \rightarrow_v *_n$, ${}^2c \rightarrow_v o$. Then the type of the *if-then-else-fail* function is $(oo*_n)$ and its definition is:

$$\lambda w \lambda t \, [{}^0\textit{if-then-else-fail} \; P_{wt} \; {}^0[S_{wt}]] = \lambda w \lambda t \; {}^2[{}^0\mathrm{I}^* \, \lambda c \, [P_{wt} \wedge [c = {}^0[S_{wt}]]]]$$

Gloss. In the first phase the construction S_{wt} is selected, provided P_{wt} *v*-constructs **T**. In the second phase S_{wt} is executed. In case P_{wt} does not *v*-construct **T**, no construction is selected and executed, hence ${}^2[{}^0\mathrm{I}^* \, \lambda c$

[15] The definition introduced here is a slightly adjusted version of the definition presented in Duží (2010a).

[16] Note that in this phase C and D are not constituents to be executed; rather they are merely displayed as objects to be selected by the variable c. This is to say that in TIL constructions themselves can be objects to be operated on, and without this *hyperintensional* approach we would not be able to define the *strict* function *if-then-else*.

$[P_{wt} \wedge [c = {}^0[S_{wt}]]]]$ is v-improper and the so constructed proposition has a truth-value gap, as it should have.

In what follows, instead of the above definition I will use this abbreviated notation as the *general analytic schema*:

$$\lambda w \lambda t\ [\mathit{if}\ P_{wt}\ \mathit{then}\ S_{wt}\ \mathit{else}\ \mathit{fail}].$$

For illustration, let us analyse Strawson's (1952, pp. 173ff) example

All *John's children* are asleep.

If the topic of the sentence is 'John's children' then there is a presupposition to the effect that John has children.[17] Hence the truth-conditions of this reading can be formulated like this:

If John has any children
then check whether each and every one of them is asleep
else fail to produce a truth-value.

Thus we have:

$$\lambda w \lambda t\ [\mathit{if}\ [{}^0\exists\ [{}^0\mathit{Children\_of}_{wt}\ {}^0\mathit{John}]\ \mathit{then}$$
$$[[{}^0\mathit{All}\ [{}^0\mathit{Children\_of}_{wt}\ {}^0\mathit{John}]]\ {}^0\mathit{Sleep}_{wt}]\ \mathit{else}\ \mathit{fail}]$$

Types: *Children_of* $((o\iota)\iota)_{\tau\omega}$: the empirical function (attribute) that dependently on a state of affairs associates an individual with the set of those individuals who are his or her children; *John* $/\iota$; *Sleep* $/(o\iota)_{\tau\omega}$; $\exists/(o(o\iota))$; *All* $/((o(o\iota))(o\iota))$: restricted quantifier that associates a set S of individuals with all the superset of S.

Remark. Here I use the restricted quantifier *All*, because I want to arrive at the *literal* analysis of the sentence. Such an analysis follows Frege's principle (1884, p. 60): It is simply not possible to speak about an object without somehow denoting or naming it.[18] If the unrestricted general quantifier were used the resulting construction would be:

$$\lambda w \lambda t\ [\mathrm{if}\ [{}^0\exists\ [{}^0\mathit{Children\_of}_{wt}\ {}^0\mathit{John}]\ \mathrm{then}$$

$$[{}^0\forall \lambda x\ [[[{}^0\mathit{Children\_of}_{wt}\ {}^0\mathit{John}]\ x] \supset [{}^0\mathit{Sleep}_{wt}\ x]]]\ \mathrm{else}\ \mathit{fail}]$$

[17] Hence the situation is this. We are talking about John's children, and just want to know what they are doing right now. The other option would be, for instance, the scenario of talking about those who are asleep, and the sentence would be offered as an answer, "Among those who are asleep are all of John's children". On this reading the sentence would only entail that John has children.

[18] The German original goes, "Überhaupt ist es nicht möglich von einem Gegenstand zu sprechen, ohne ihn irgendwie zu bezeichnen oder benennen."

This is an equivalent construction producing the same proposition as the above one, yet it is not the literal analysis of our sentence, because the truth-function of implication is not mentioned in the sentence.[19]

5. Conclusion

In this paper I demonstrated and proved that narrow-scope and wide-scope negation are not equivalent. If a sentence comes with a presupposition, then narrow-scope negation is the relevant one, because wide-scope negation is presupposition-denying. I also dealt with the ambiguities in natural language stemming from different topic-focus articulations within a sentence. While the topic phrase generates a presupposition, the focus phrase usually triggers merely an entailment. It is a matter of *pragmatics*, of course, which reading is the intended one on an occasion of use. Yet, our fine-grained logical method of analysis as presented in this paper demonstrates that sentences differing in point of topic-focus articulation are not equivalent, and thus choosing between particular readings becomes also a matter of *semantics*. Logic can contribute to the disambiguation of a sentence by making these hidden features explicit and logically tractable. In case there are more *non-equivalent* senses of a sentence we furnish the sentence with *different meanings*.

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