CLASSES AND SETS

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What is class ? The usual answer is that a class is any collection of entities of any sort. As a rule the words «class» and «sets» are used interchangeably. Thus having explained that a *set* is «any kind of collection of entities of any sort», Suppes (*Introduction to Logic,* D van Nostrand, 1957 p. 177) continues: «Many other words are used synonymously with 'set': for instance, 'class', 'collection', 'aggregate'». This remark of Suppes is typical of much contemporary thinking about classes. It is true that in von Neumann's Set Theory a distinction is drawn between classes and sets but the distinction is proffered not in the course of an intuitive interpretation of these concepts but rather in the attempt to find a technical way out of the set-theoretic paradoxes.

There is an appearance of circularity in the passage quoted from Suppes which is probably not due to a mere oversight. «Collection» being synonymous with «sets», Suppes' elucidation would seem to boil down to saying that a set is any *set* of entities of any sort. In justice to Suppes it should be noted that he does not claim to offer a strict definition of *set*. But the question must still remain whether, even as a conversational preliminary, such an explanation can be of any help. Be that as it may, it is something of a surprise to find that Quine apparently exults in the sort of circularity to which attention has been called. I will quote him *in extenso.*

«Set theory is the mathematics of classes. Sets are classes. The notion of class is so fundamental to thought that we cannot hope to define it in more fundamental terms. We can say that a class is any aggregate, any collection, any combination of objects of any sort; if this helps, well and good. But even this will be less help than hindrance unless we keep clearly in mind that the aggregating or collecting or combining here is to connote no actual displacement of objects, and further that the aggregation or collection or combination of say seven pairs of shoes is not to be identified with the aggregation or collection or combination of those fourteen shoes, nor with that of the twenty-eight soles and uppers. *In short, a class may be thought of as an aggregate or collection or combination of objects just so long as 'aggregate' or 'collection' or 'combination' is undrstood strictly in the sense of 'class'*». (My italics). (Quine: *Set Theory and its Logic*, Harvard, Second Revised edition, p. 1 1969).

I confess to being even more seriously hampered by Quine's elaboration. In comparison, Cantor's definition of an «aggregate» is light itself. «By an 'aggregate' (*Menge*) we are to understand any collection into a whole (*Zusammenfassung zu einem Ganzen*) M of definite and separate objects *m* of our intuition or thought. These objects are called elements of M». (Cantor: *Contributions to the Founding of the Theory of Transfinite Numbers*, Dover, 1915, p. 85) Cantor's 'aggregate' is, of course, the same as Quine's 'class'. We note also that both of them talk of 'collection'. But whereas Quine's collection is a collection which is avowedly not a collection, Cantor's collection is a mental collection. Thus if Cantor had gone on to remark that his collection «is to connote no actual displacement of objects», his meaning would have been quite straightforward. I can mentally survey all the capital cities of the African continent, *collecting* them into one whole of which Accra, Lagos, Freetown, Kampala, Nairobi... are elements. This collecting into one whole consists merely in *noting* that all these cities have *one* characteristic in common, namely *being a capital*. (Of course, this characteristic is *one* only relatively to the underlying purpose; for some other purpose one may analyse the characteristic of being *a capital* into several characteristics). Again there is, on Cantor's definition, no mystery in the fact that a class of *seven given pairs of shoes* is not identical with the class *of those fourteen shoes* or *the twenty eight soles and uppers*. For the *elements* of the relevant *collection* are to be 'definite' objects of our intuition or thought, and what secures this *definiteness* is the characteristic which forms the basis of the *collecting*. Thus a *shoe* could not possibly an element of a class of *pairs of shoes*. Change the characteristic and you change the *collection*.

Accordingly, one might attempt a pro-Cantor adaptation of Quine as follows: «In short a class may be thought of as an aggregate or collection or combination of objects just so long as 'aggregate' or 'collection' or 'combination' is understood strictly in the sense of Cantor». Cantor is often criticised on the ground that his reference to objects of our intuition and thought introduces a subjective element into the definition of an objective concept. The simple answer is that the mere reference to thought or, more generally, to a mental process or category does not imply subjectivity. It can hardly be maintained that all thinking is subjective, whatever that might exactly mean. The mental process alluded to by Cantor is simply one of specifying a *clear concept* under which entities may be subsumed.

So far the problem we have discussed in connection with the explanation of the term «class» may be phrased thus: If the collection that is said to constitute a class is not a physical collection (even when the class relates to physical objects) then what sort of collection is it ? But there are other problems, possibly more urgent. For example, we can raise questions about the scope of the collection. I will clarify this by means of an illustration. Consider the class of men. This class is presumably the 'collection' of *all* men. Does this 'collection' consist of all the men of this particular specious present or does it include past as well as future men ? There should, I think, be no hesitation in answering that the 'collection' includes *all* men, past, present, future. When we say, for example, that all men are rational (in some sense !) we cannot possibly be understood to be making an exception of our ancestors or of our posterity. Moreover, surely, the class of *men* is distinct from the class of *men existing at the present time*, for Socrates, for instance, is a member of the first class but not of the second.

This consideration leads naturally to the following alternative characterisation of classes: A class is the comprehension of a term i.e. its field of applicability. This type of definition is also sometimes given in logical texts. Thus Susanne Langer in her *Introduction to Symbolic Logic,* (Dover, Second Edition 1953), a text which devotes more space to the clarification of the concept of class than usual, states:

> A «class» may be described, [then], as a collection of all those and only those terms (1) to which a certain *concept* applies. If we collect all the individuals to which the concept «being a fox» applies, we form the class of foxes. If we would form the class of prime numbers, we must indicate all the items to which the concept «prime number» applies. We may say, then, that a class is the field of applicability of a concept. (p. 116).

There is a distinction here of which Mrs. Langer is probably unmindful. The field of *application* of a term is strictly not identical with its field of *applicability*. If we collect (mentally, no doubt) all the *individuals* to which the concept «being a fox» *applies* we obtain something short of the class of foxes; we only secure the sub-class of the class of foxes which contains the foxes existing at the present time. Obviously, to get the class of foxes, we must advert our minds not only to the individuals to which the concept «being a fox» applies but also to the individuals to which this concept *can* apply, which is what we mean by the field of applicability of the concept. The class of foxes, that is to say, comprises actual as well as possible individuals. It should be pointed out, however, that a possible individual is not a *kind* of individual. To talk of a possible individual is simply to envisage the possible application of a given concept.

There is further a certain subtle distinction between the two examples cited in the passage quoted from Mrs. Langer to which we shall recur more at length later. Suffice it to say that with regard to certain concepts, the field of application turns out to be identical in extent with the field of applicabili-

(1) Note that Mrs. Langer apparently uses «term» in the sense of «entity».

ty. This is the case with the concept of prime number, which is Mrs. Langer's second example. On the other hand, as I believe I have shown above, *application* and *applicability* are different in the case of a concept such as «being a fox» which is her first example. Possibly, Mrs. Langer's failure to draw the distinction between the application of a concept and its applicability is attributable to the fact that concepts such as «prime number» are more central to the concerns of set theorists. Before expatiating on this matter, I should like to call attention to a certain consequence which flows from the conception of a class as the field of applicability of a concept for the distinction between classes and attributes (or properties).

The concept of classes which underlies existing set theories is often said to be *extensional*. By this is meant that the concept satisfies the following principle: Two classes are equal if and only if they have the same members — the so called principle of extensionality for classes. Now, the most respectable explanation of the customary distinction between classes and attributes (or properties) is in terms of this principle of extensionality. Thus Quine states the «only intelligible difference between class and attribute» in this way:

> Classes are identical when their members are identical. This, the *law of extensionality*, is not considered to extend to attributes.

He immediately adds:

> If someone views attributes as identical always when they are attributes of the same things he should be viewed as talking rather of classes. I deplore the notion of attribute, partly because of vagueness of the circumstances under which the attributes attributed by two open sentences may be identified. (*Op cit* p. 2).

In fact, it will not do to separate classes from attributes by means of the law of extensionality. The reason is this. As already seen, the elements of a class need not be restricted to the *actually* existing entities which instantiate the given class concept. Every *possible* case of the given *sort* is to be counted among the membership of the class. Thus the law of extensionality may be interpreted as saying that two classes are identical if and only if every possible entity instantiating the class concept of the one also instantiates the class concept of the other. From which it emerges that to deny that attributes satisfy this principle is to suggest that two attributes might fail to be identical even though the one is an attribute of every possible entity of which the other too is an attribute; which is inplausible, to say the least.

To distinguish between classes and attributes by saying that classes are identical if they have the same members but attributes may be attributes of the same things without being identical since there may be *possible* entities to which one but not the other may be attributable is to betray a lack of clarity about the scope of the 'collections' that are supposed to constitute classes. With the clarification of this topic already given above, it only remains to note that the alleged distinction between classes and attributes is, standardly, illustrated with misleading examples. Adapting an illustration due to Nelson Goodman (*The Structure of Appearance,* The Bobbs-Merrill Co. Inc. New York, Second Edition, 1966, p. 4f), it might be said that given that all and only those residents of Wilmington in 1947 that weigh between 175 and 180 pounds have red hair, we are entitled to say that the class of 1947 residents of Wilmington weighing between 175 and 180 pounds is identical with the class oof red-haired 19947 resident of Wilmington, and yet the attribute of being a red-haired resident of Wilmington in 1947 is patently different from that of being a 1947 resident of Wilmington weighing between 175 and 180 pounds. But there is an oversight here. Surely, the class of 1947 residents of Wilmington weighing etc., has not been showon to be identical with the class of 1947 red-haired residents of Wilmington. It is only the class of 1947 residents of Wilmington weighing etc. *in the circumstance in which all and only those 1947 residents of Wilmington weighing etc. have red hair* that is identical with the class of 1947 red-haired residents of Wilmington. And, of course, the attributes to use here for the comparison must not be the relatively simple ones referred to above.

For parity of reasoning we must attend to the attribute of being a 1947 resident of Wilmington in the circumstance that all and only such residents are red haired, on the one hand, and on the other, the attribute of being a 1947 red-haired resident of Wilmington in the same circumstance. To say that there is any possible case in which the two attributes might fail to coincide in their applicability would imply either contradicting the assumption which defines the particular curcumstances of the illustration or proceeding on an inadequate notion of the attributes involved.

One is naturally led to suspect that there must be something wrong with the interpretation of the concept of extensionality that is associated with the concept of *class* in logic. It is freêquently said that the *extension* of a term is the actual entities to which the term applies. In this sense classes in logic will not satisfy the *corresponding* law of extensionality. Ironically, extensionality would seem to acquire more sense with respect to classes if the term is taken in its traditional acceptation. Traditionally, by the extension of a term was meant the *ranges* of entities to which the term is applicable. Thus the extension of the term «man» is constituted by red haired men, white men, black men tec. For the actual instances of a term the traditional logicians reserved the word «denotation». (See for example H.W.B. Joseph: *An Introduction to Logic,* Oxford, Second revised edition, 1916. Chap. VI esp. p. 146f.). It is an unfortunate fact that the term «extension» has no stable signification in modern logical literature. It sometimes has something similar to its traditional meaning but also frequently it appears to mean the same as *denotation* in its traditional use. [2] In this circumstance, I prefer the word «comprehension» to **«extension»** for the purpose of charaterising classes. Thus instead of saying that a class is the *extension* of a term, I prefer to say

[2] For an illustration of the first type of usage see Langer Op. cit. p. 130; «The *meaning* of a concept is called its *intension,* the range of *applicability* its *extension*». For an example of the second kind of usage, see C.I. Lewis: *Analysis of Knowledge and Valuation* p. 39 «The *denotation* or *extension* of a term is the class of all actual or existent things which the term correctly applies to or names».

that a class is the *comprehension* of a term. To consider the comprehension of a term is to consider its applicability i.e. the entities to which it does, or might, apply.

This usage of the term «comprehension» I have borrowed from C.I. Lewis. According to Lewis, the *comprehension* of a term is «the classification of all the consistently thinkable things to which the term would correctly apply — where anything is consistently thinkable if the assertion of its existence would not, explicitly or implicitly, involve a contradiction». (*An Analysis of Knowledge and Valuation,* The Open Court Publishing Company, 1946, p. 40). Note incidentally that the word «classification» in the definition is needless and may consequently be dropped. On the basis of the — if I may coin an adjective — *comprehensional* view of classes, it is a straightforward matter to explain the relation between classes and attributes: A *term* is a symbol, for example, a series of marks or sounds to which a meaning is associated. When the meaning is an abstract substantive, simple or complex, it is to be called an *attribute.* To every such term necessarily corresponds a comprehension. It is important to note that comprehension is a grammatical category. Thus the term «man» may be said to express the attribute of *being a man* or *manhood* which in turn determines the comprehension «men». Similarly the comprehension corresponding to «beautiful» which yields the attribute «being beautiful» or «beauty» is «beautiful things». And so on. It can be seen that the transition from *attribute* to *comprehension,* and, therefore, to *class* is of a purely syntactical character, using the word «syntactical» in a non-technical, very ordinary sense. It should, therefore, occasion no surprise that attributes cannot be distinguished from classes by reference to any possible difference with respect to instantiation. On this view, the attribute of being a man, for example, is not some entity designated by the concept «being a man»; the attribute *is* the concept. The difference between the attribute of being a man and the class of men is the difference between the syntactical status represented by the phrase «being a man» and that represented by the word «men». This explanation should serve to dissipate any suspicion of an untoward ontology lurking behind the concept of an attribute, at any rate, in the present usage.

One advantage of the comprehensional view of classes is that it enables us to give an intelligible explanation of how it is that a unit class can be distinguished from its sole member. The usual explanation of unit classes in the context of the «extensional» conception of classes is anything but intelligible. If, as is frequently supposed, a class is constituted by the entities which actually fall under the corresponding class concept, then the unit class must be identical with its sole member. On the other hand, since the comprehension of a term is clearly distinct from any entity that instantiates the term, there is no difficulty whatever in seeing that a unit class must be different from its sole member.

Again, on the comprehensional view of classes, it becomes possible to give a viable account of the admissibility of the null class in logic. I believe that Geach is right when he remarks: «If a class is taken as consisting of its members, then there is just no place for a null class in logic». (*Reference and Generality,* Cornell University Press, Ithaca, 1962 p. 12). First of all it is necessary to clear up a certain confusion in the conception of the null class. One often encounters statements such as this: «The null class is the class with no members». Conformably to this conception, examples such as the class of unicorns are frequently cited in illustration of the null class. (3) Curiously, when it comes to formal definition, even those who give such examples usually give the correct formulation, namely, '$\Lambda = \{x: x \neq x\}$'. On this showing, a null class must be said to be not simply a class which has no members but one which *cannot,* as a matter of logical impossibility, have any members. In other words the null class is the class which corresponds to a logically inconsistent class concept. Clearly the class of unicorns is not a null class. Incidentally similar care must be exercised in defining the unit class. This is not the class which just happens to have only one member; it is the class which *logically* can have only one member.

(3) Mrs. Langer's book (Op. cit.) is one text which avoids this confusion. See p. 128f.

Returning to the question of how there can be a null class, it might seem that even on the comprehensional view there can be no room for such a class. For, if the comprehension of a term is constituted by «all the consistently thinkable things to which the term would correctly apply», then it is difficult to see just how a logically inconsistent concept could be said to have a comprehension. This problem cannot be solved by resorting to the expedient of distinguishing between there being no comprehension at all for a term and there being only zero comprehension, and then claiming that self-contradictory concepts do have zero comprehension. This manouevre is tried by Lewis: «... a term may have zero comprehension. For example, 'round square' has zero comprehension; the classification of consistently-thinkable things so named is empty». (*Op. cit.* p. 47). But to say that a term has zero comprehension, it would seem, can only be an alternative way of saying that it has no comprehension at all.

The key to the solution of the problem lies, I believe, in the syntactical status of the concept of comprehension. It is an interesting fact that although syntactical categories are founded on very broad classificaions of types of meanings (i.e. types of materials of meaningful discourse), expressions belonging to these categories may be combined in *syntactically correct* ways without being capable of *communicating* a unified meaning. Thus a *noun* is an expression signifying a kind of entity. (It is, perhaps, prudent to state that I use the word «entity» in an ontologically neutral sense). A *verb* is an expression signifying an action or process. Yet, to borrow a famous example due to Russell (*An Inquiry into Meaning and* Truth, Allen and Unwin, London 1940 p. 166), «Quadruplicity drinks procrastination», cannot communicate any information inspite of the fact that it is an expression consisting of two nouns joined by a verb in a way which abides by the syntactical rules of sentence formation. This suggests that although, contrary to the widespread opinion, syntax is not *absolutely* independent of meaning, it nevertheless has nothing to do with issues of communicativeness.

Consider the following expression: «Of by to at and». It is definitely meaningless in a rather drastic sense. It can communicate nothing; nor is it even syntactically significant. By comparison, «Quadruplicity drinks procrastination» would seem to be afflicted with a somewhat less aggravated degree of meaninglessness. Even though it cannot communicate any message, it still makes syntactical sense. A distinction which naturally suggests itself here is this. We may distinguish between two levels of meaningfulness as follows. An expression is *syntactically* meaningful if it satisfies the relevant syntactical rules of well-formedness; it is *communicatively* — I would also say, *semantically* — meaningful if in addition to being syntactically meaningful it is *as a whole* capable of communicating something. What we have just seen is that an expression may be meaningful syntactically and yet not be meaningful communicatively.

Given that *comprehension* is a syntactical category even the brief indications given above for forming the comprehension of a term should be enough for one to recognise «round squares» as a syntactically accredited comprehension corresponding to the expression «round square». In like manner «drinkers of procrastination» corresponds as a comprehension to the open sentence «x drinks procrastination» or to the attribute «being a drinker of procrastination». The fact that for mathematical and semantical reasons (respectively) these expressions cannot as *wholes* communicate anything is obviously without prejudice to their syntactic status. Lewis' account of comprehension suffers, I think, from the weakness that it does not bring out its syntactical standing.

In the distinction between syntactic and communicative meaning we have the means of resolving the general problem of whether contradictions are meaningful or not. The answer is that they are meaningful syntactically but not communicatively. This answer is of the greatest significance for logic. Logic is a formal discipline. This implies, among other things, that the rules for the admissibility of expressions in logic must be formal. The syntactic meaningfulness of 'p & — p' follows from the usual rules for obtaining well-formed formulas in sentential logic. Similarly, the syntactic meaningfulness of '$\{x : x \neq$

x}' follows from the set-theoretical rules of well-formedness. These rules are syntactical in as much as they deal with the forms of expressions and the various ways of combining them. This is as it should be since the concern of logic is with the forms, combinations and transformations of expressions. To introduce issues of communicative meaningfulness would mean compromising the formal character of logic.

The foregoing remarks explain the otherwise unaccountable fact of the utility of contradictions in logic. The admissibility as well as the utility of the comprehensions of selfcontradictory concepts (i.e. of the null class) falls into place as a special case.

The conception of classes as the comprehensions of terms would seem from this discussion to be the one which is adequate to the requirements of class theory. It would, however, be a mistake to suppose that it is the only legitimate concept of classes. Only a little attention to language is necessary for one to see that in common discourse we frequently use the word «class» in a purely *denotational* sense. («Denotation» is intended in its traditional signification). When we speak of a teacher lecturing to his *class*, we obviously mean by «class» the actual entities which satisfy the relevant class concept. Again when a socialist revolutionary asserts that he will eliminate the capitalist and feudal classes from his fatherland, he can hardly be understood to be threatening to conjure out of existence the comprehensions of the relevant terms. It is not difficult to see that in this usage, «class» is synonymous with a certain sense of the word «set» which is also frequently met in ordinary discourse. We speak of presenting a *set* of books to, say, a library. It need hardly be pointed out that here it is the actual pack of books that we are calling a set.

In the above sense a class or set is always finite. For this reason alone, if for no others, this conception of classes or sets is inadequate to the needs of set theory. Nor will it measure up even to the purposes of the purely logical calculus of classes since, as we have already seen, within such a conception of classes there is no room for the distinction between a unit class and its sole member or for the concept of the null class. Accordingly, we must be careful not to confuse the concept of *set* appropriate to set theory with the familiar notion of *set* referred to above. The set theoretic concept of *set* corresponds to the more powerful concept of classes as the comprehensions of terms.

Nevertheless, there is an important analogy between set-theoretical sets and denotational *sets* which is probably responsible for the greater currency of the term 'set' (as compared with «class») in mathematics. The point is this. In mathematics the comprehensions of terms coincide in reach with their denotations when the latter are available. For example, the comprehension of the concept «natural number» is «the natural numbers». But all the natural numbers are exactly what is actually *denoted* by the concept in question. In this respect mathematical concepts differ from such concepts as «fox» «man» etc. Of course, this does not abolish the distinction between comprehension and denotation even in mathematics. Firstly the concept of an odd-even number or of a round square has a comprehension but no denotation. Secondly, even when a denotation is available as, for example, in the case of the concept «the first four positive integers», the comprehension, though identical in *extent* with the denotation, is still distinct from it. The comprehension is simply the *notion* of the first four positive integers — a sort of mentally individuated envisagement of the given concept. Thus the distinctness of the comprehension from the denotation and, correspondingly, of the class from its members, is exactly reflected in the grammatical difference between talking of «the first four positive integers», on the one hand, and of the first four integers, on the other. And this is the intuitive rationale behind the set-theoretic distinction between '{1, 2, 3, 4}' and '1, 2, 3, 4.' However, on account of the fact that for most mathematical concepts comprehension coincides in reach with denotation, to talk of sets as the denotations of terms does not immediately sound as implausible in mathematics as elsewhere. The tendency for set theorists to think of classes or sets as the denotations of terms probably derives from this circumstance. Actually the situation in which the comprehension of a term coincides in extent with its denotation is not peculiar to mathematics. Whenever a term signifies things lying within an 'abstract' realm such a situation is apt to arise, whether in mathematics or outside it. For example, the comprehension of the term «finite verb» reaches no farther than its denotation. Again, wherever a class is of an explicitly 'closed' character such as is the case with unit classes and classes like the class of the first two presidents of independent Ghana, the same type of situation does tend to arise.

There are, then, at least two concepts of classes each with an associated concept of set. Both satisfy principles of extensionality *appropriate to them.* The denotational concept of classes satisfies the principle formulated as «Two classes are equal if and only if they have the same members» *where by a «member of a class», we mean an actual entity instantiating the appropriate class concept.* Thus a member of a set of books donated to a library is an actual book that can be picked up and read. This sense of «member» is probably the most natural meaning of the term. However, it is assuredly not the sense that is employed in logic and set theory and should not be understoodo to be ewhat is symbolised by the epsilon symbool. As we have seen, the denotational concept of classes to which this sense of membership is appropriate is not, and could not possibly be, the basis of classical set theory. Therefore, when formulating the principle of extensionality that is germane to the comprehensional view, we have a choice of either using the word «member» with appropriate qualification or adopting some other word altogether. The customary formulations — witness, for example, the one quoted from Quine earlier on — are in terms of membership, but owing to a confusion between the two concepts of classes which we have discriminated above, the need for a rider to the use of the word «member» in this context is not felt. Because of the deeply ingrained denotational complexion of the term «member», I prefer at this stage to restrict it to the denotational meaning. We may safely resort to Cantor's word «element» for comprehensional purposes. Corresponding to the interpretation of the principle of extensionality already given (see p. 172 above), I shouold give the following formulation: Two classes are identical if and only if they have the same elements.

Here, let it be repeated, by an element of a class we do not mean an actually existing entity falling under the relevant class concept. Thus Socrates is an element of the class of men but Socrates does not now exist. Moreover, even when actually existing entities fall under a given class concept, it is not the actual things themselves that are *elements* of the appropriate class; otherwise «closed» classes relating wholly to actually existing entities such as the class of the 1972 capital cities of Africa would be *constituted* by those actual cities, and the comprehensional concept of class would be lost. It is only the entities as *mentally individuated*, in Cantorite phraseology, the definite and separate objects of our thought, that are to be called *elements*. Element hood as a correlate of comprehension, then, rather than membership, is to be taken to be the idea expressed by the epsilon symbol. Naturally, no existing logical or set-theoretical symbol can be correctly considered as standing for (denotational) membership. In this matter we had better let sleeping dogs lie.

Finally it might be asked whether the comprehensional principle of extensionality is really *extensional* ? Is it not, in fact, intensional ? I do not think it is either urgent or very useful to try to affix onoe of these names rather than the other to the principle in question. What is important is the contention that to make sense of the principle that has been called the law of *extensionality* in the literature, the principle that is symbolically formulated as '$(S = T) \equiv (x)\,(x \in S \equiv x \in T)$', it must be interpreted comprehensionally. Whether it is really *extensional* depends on what one means by «extensional». It is by no means clear that one clear meaning is consistently attached to the word in the literature, as has already been remarked. On the other hand, to adopt the word «intensional» would seem to imply that the comprehensional concept of a class specially calls for some variety of non-classical logical foundations for set theory; which is not the case. It may well be that some form of the non-classical logics that are usually described as «intensional» is needed as a foundation for set theory, but the comprehensional interpretation of the concept of class does not *of itself* indicate such a need.