THE ITERATION OF DEONTIC MODALITIES

L. F. GOBLE

In contrast to investigations in alethic modal logic, in the construction of systems of deontic logic little attention has been paid to the iteration and reduction of the deontic modalities. We shall here present some new systems designed to avoid difficulties which beset most systems previously proposed. We then raise the question: how do these systems effect the iteration and reduction of modalities? And, since the deontic logic is to be a reconstruction of normative discourse, this question must be considered with one eye to the underlying problem: do we desire the logic to treat of iterations at all?

Iterated deontic modalities are questionable just because we do not know what to do with them. Confronted with a statement like '**OOPFOPP**p', we can hardly say it, much less provide it with an intelligible interpretation. Hence, I contend, such expressions are foreign to anything like our pre-formal intuitions (which must, after all, provide the starting point of reconstruction), and so they are suspect (1). Nevertheless, whether or not there actually are iterated modalities in our normative systems is an empirical question, properly left to substantive ethics; it is not a matter of logic. Consequently, we should not want our logic to commit us to their existence, as it should if the system contained such theorems as: **P**p $\rightarrow$ **PP**p, if something is permitted, then, necessarily, it is permitted to be permitted. On the other hand, neither should iterations be precluded by, for example, restrictions on the formation rules

(1) With some simple iterations it might be thought that plausible readings are available. For example, 'OOp' has been taken as 'there ought to be a law', and 'O(Op $\rightarrow$ p)' has been read as 'obligations ought to be fulfilled' or 'you ought to do your duty'. But such renderings depend, I suggest, for their plausibility on an equivocation on the different occurences of 'O'. When these expressions are read in such a way as to preclude ambiguity, we no longer know what to say.

of the language [2], or by the more subtle device of reducing all modalities to first degree [3]. Either course prejudices the logic on substantive issues.

I

The language of the systems to be considered below consists in: (i) propositional variables: p, q, r, ... etc.; (ii) logical constants: the familiar &, v, $\sim$, (, and), with $\prec$ or $\rightarrow$, representing the, intensional, relations of implication and entailment, as presented below; (iii) the alethic modes: N and M, interpreted as logical necessity and possibility, respectively; (iv) the deontic operators: **O**, **P**, and **F**, which are interpreted as 'it is obligatory that ...', 'it is permitted that ...', and 'it is forbidden that ...', respectively; and (v) the propositional constant: V, discussed below. (Some of these elements are eliminable by definition, as we shall see.) The usual formation rules are assumed, and we adopt Church's conventions for the elimination of parentheses.

In the manner of Parry [7] we introduce the following notions:

A *deontic modal function* (d.m.f.): propositional variables and V are d.m.f.'s; and if A and B are d.m.f.'s, so are $\sim$A, A & B, AvB, A $\prec$ B, A $\rightarrow$ B, NA, MA, **O**A, **P**A, and **F**A.

The *degree* of a deontic modal function: the degree of propositional variables and V is zero; if A is a d.m.f. of degree n, so are $\sim$A, NA and MA; if A and B are d.m.f.'s of degree m and n respectively, then the degree of A & B, AvB, A $\prec$ B, and A $\rightarrow$ B is max (m, n); and if A is a d.m.f. of degree n, the degree of **O**A, **P**A and **F**A is n + 1.

A *deontic modality*: propositional variables and V are deontic modalities; if A is a deontic modality, so are $\sim$A, **O**A, **P**A and **F**A.

A *proper deontic modality* is a deontic modality of degree greater than zero.

An *iterated deontic modal function* (*modality*) is a deontic modal function (modality) of degree greater than one.

(2) Cf. Prior, [9, p. 145].
(3) Cf. Anderson [1, pp. 72-74].

Two deontic modalities, A and B, are said to be *reducible to* each other if it is provable that each implies the other, otherwise they are *distinct*.

We shall, following Anderson, assume that there is an analytic connection between the statements: (i) it is obligatory that p, and (ii) if not-p, then V, where the constant 'V' is to represent some 'violation' or 'bad state of affairs' or 'wrong in the world'. We put no restrictions on the interpretation of 'V', but leave that to moral philosophy and the determination of a substantive ethical theory. Just what interpretation is taken is irrelevant to the logic, which must, again, remain neutral on substantive issues.

The connection between (i) and (ii) above is close enough that we shall, provisionally, adopt the following definitions of the deontic operators:

D.1 $\mathbf{O}p =_{df}$ if $\sim p$, then V

D.2 $\mathbf{F}p =_{df}$ $\mathbf{O}\sim p$

D.3 $\mathbf{P}p =_{df}$ $\sim \mathbf{F}p$

(in some appropriate sense of 'if ... then ...'). This procedure enables us to construct systems of deontic logic which not only stand parallel to the familiar modal logics, in the manner of Lemmon's D.1-D.5 [6], but are actually embedded in some alethic modal system (or some more sophisticated intensional logic) (4).

D.1-D.3 suit our intuitions, but for them to be applicable in formal systems, some determinate sense must be given to the context 'if ... then ...'. It is obvious that we should not take it as material 'implication', adding the definitions to the classical propositional calculus. For, in virtue of the familiar 'paradox': $A \supset . \sim A \supset B$, we should then have the ridiculous results: $p \supset \mathbf{O}p$ and $\sim p \supset \mathbf{F}p$ — whatever happens is obligatory and what ever does not happen is forbidden. A *prima facie* more plausible proposal would be to use strict 'implication', in the sense of the Lewis Modal Systems. This was the course originally taken by Anderson [1,2] with his systems OX, based on any standard alethic modal logic, X, such as M, S.4 or S.5, to which were added:

D.4 $\mathbf{O}p =_{df}$ $\sim p \prec V$

(and correspondingly for permission and prohibition) and either the

(4) See ANDERSON [1, 2] for the motivation and development of this approach.

axiom: $M \sim V$, or the definition: $V =_{df} B \,\&\, M \sim B$, for any arbitrary propositional constant 'B'. Obviously '$p \rightarrow\!\!\!\!\dashv\; \mathbf{O}p$' and '$\sim p \rightarrow\!\!\!\!\dashv\; \mathbf{F}p$' are not provable. However, in virtue of the 'paradox' of strict 'implication': $NA \rightarrow\!\!\!\!\dashv . \sim A \rightarrow\!\!\!\!\dashv B$, these systems do contain some similar counter-intuitive consequences:

(a) $Np \rightarrow\!\!\!\!\dashv \mathbf{O}p$

or

(a′) $N \sim p \rightarrow\!\!\!\!\dashv \mathbf{F}p$,

which are odd since we do not normally consider necessities and impossibilities in a normative context. Furthermore, in OS.4 and OS.5 (but not in OM) we have:

(b) $\mathbf{O}p \rightarrow\!\!\!\!\dashv N\mathbf{O}p$,

that obligations are necess ary, and hence:

(c) $\mathbf{O}p \rightarrow\!\!\!\!\dashv . q \rightarrow\!\!\!\!\dashv \mathbf{O}p$,

that when something is obligatory, then anything 'implies' that it is obligatory. In light of these results, OM, OS.4 and OS.5 cannot be considered satisfactory reconstructions of our normative discourse.

II

Noticing that the deontic systems mentioned above all fail just because their underlying logics do not formulate a reasonable relation of implication, since neither material nor strict 'implication' requires relevance between antecedent and consequent, we are led to consider a system free of this fault, namely OR, based on the system R of relevant implication. (5) R is defined by the following axiom schemata and rules:

A.1 $A \prec A$

A.2 $A \prec B \prec . B \prec C \prec . A \prec C$

A.3 $(A \prec . B \prec C) \prec . B \prec . A \prec C$

A.4 $(A \prec . A \prec B) \prec . A \prec B$

A.5 $\sim\sim A \prec A$

A.6 $A \prec \sim\sim A$

(5) This system, OR, was originally proposed by ANDERSON [3]. For a development of the implicative part of R (and also of E, considered below), see ANDERSON and BELNAP [4]; a more complete account will be available in their forthcoming book, *Entailment*.

A.7 $A \prec B \prec . \sim B \prec \sim A$
A.8 $A \prec \sim A \prec \sim A$
A.9 $A \,\&\, B \prec A$
A.10 $A \,\&\, B \prec B$
A.11 $(A \prec B) \,\&\, (A \prec C) \prec . A \prec (B \,\&\, C)$
A.12 $A \prec A\mathrm{v}B$
A.13 $B \prec A\mathrm{v}B$
A.14 $(A \prec B) \,\&\, (C \prec B) \prec . (A\mathrm{v}C) \prec B$
A.15 $A \,\&\, (B\mathrm{v}C) \prec (A \,\&\, B)\mathrm{v}C$

and

R.1 *Modus ponens* for '$\prec$': from A and $A \prec B$, to infer B
R.2 *Adjunction*: from A and B, to infer A & B.

'$A \prec B$' is read 'A is a sufficient *and relevant* condition for B' or 'A relevantly implies B'. The deontic system OR consists in R, so defined, with the definition:

D.5 $\mathbf{O}p =_{df} \sim p \prec V$

(and correspondingly for '**P**p' and '**F**p') and the axiom:

$$\sim p \prec V \prec . \sim(p \prec V)$$

(if $\sim p$ gets one in trouble, then p doesn't), which yields the thesis: $\mathbf{O}p \prec \mathbf{P}p$ (6).

Since R contains none of the 'paradoxes of implication', (a) and (c) above do not obtain in OR, and since it does not draw alethic modal distinctions, (b) becomes vacuous. However, all intuitively obvious deontic theses do hold in OR: in particular, it contains such theorems, discussed by von Wright [10], as:

$$\mathbf{P}p \mathrm{\ v\ } \mathbf{P}{\sim}p$$

and

$$\mathbf{P}(p\mathrm{v}q) \prec \mathbf{P}p \mathrm{\ v\ } \mathbf{P}q$$

(and conversely), along with the laws of 'commitment', such as:

$$\mathbf{O}p \,\&\, \mathbf{O}(p \prec q) \prec \mathbf{O}q.$$

As a result, OR presents itself as a, *prima facie*, plausible deontic system. We shall, however, consider some counter-intuitive, or at least non-intuitive, consequences of OR, having to do with iterated deontic modal functions.

OR commits us to the existence of iterated deontic modalities by

(6) This renders OR a *normal deontic logic*, in the sense of ANDERSON [1].

the following theorems. The Law of Contraction : (A —< . A —< B) —< . A —< B, (A.4 above), yields :

(1) **OOp** —< **Op**,

(substituting '∼V' for 'A' and 'p' for 'B' and contraposing ([7])), and dually :

(2) **P**p —< **PP**p.

From the theorem in R, Restricted Assertion : A —< B —< . A —< B —< C —< C, we have the theorem in OR :

(3) **F**p —< **FFF**p.

(Since we have the, somewhat implausible, thesis '**F**V' (from A.1), we also have '**FFF**V', by (3)).

For the consideration of the reduction of modalities, we require a Replacement Theorem, which we have for R :

If ⊢ A —< B and ⊢ B —< A, then ⊢ (... A ...) if and only if ⊢ (... B ...)

which is proved in the usual way.

The converse of (3), '**FFF**p —< **F**p', is provable in OR. Hence, in virtue of the equivalence, by definition, of '**O**p' with '**F** ∼p' and '**P**p with '∼**F**p' and of '∼ ∼p' with 'p', by A.5 and A.6, any deontic modality is reducible to a deontic modality with no two negation signs in immediate succession and no more than two '**F**''s in immediate succession. However, there are no further reductions. The converses of (1) and (2) are not provable in OR ([8]); this result may

([7]) We omit proofs in these cases since they are all easily effected.

([8]) This may be verified by the following matrix-group, which satisfies the (implication-negation) axioms and rules of OR (starred values are designated) :

—<	1	2	3	4	5	6	7	8	∼	
1	8	8	8	8	8	8	8	8	8	
2	1	7	1	7	1	1	7	8	7	
3	1	1	6	6	1	6	1	8	6	V
4	1	1	1	5	1	1	1	8	5	—
*5	1	2	3	4	5	6	7	8	4	4
*6	1	1	3	3	1	6	1	8	3	*5
*7	1	2	1	2	1	1	7	8	2	
*8	1	1	1	1	1	1	1	8	1	

When v(p) = 4, and v(V) = 5, '∼p —< V —< . ∼(∼p —< V) —< V' takes the undesignated value 1.

be extended to deontic modalities of arbitrary length by the theorem of R, concerning a generalized Contraction:

$$\vdash (A_m \prec \ldots \prec . A_1 \prec B) \prec . A_n \prec \ldots \prec . A_1 \prec B, \text{ if and only if, } m \geq n\ (^9)$$

where 'A' and 'B' are two distinct propositional variables, and the subscripts indicate the number of 'A''s to the left of 'B'. For OR this yields the result:

$$\vdash \mathbf{O}_m \ldots \mathbf{O}_1 p \prec \mathbf{O}_n \ldots \mathbf{O}_1 p, \text{ if and only if, } m \geq n$$

(and dually for '**P**'). Consequently, there can be no reductions of sequences of '**O**''s and '**P**''s; hence, OR distinguishes infinitely many deontic modalities.

One of the theorems in R: $A \prec . A \prec B \prec B$, the Law of Assertion, is of interest as it yields two sets of theorems in OR containing iterated deontic modal functions. First we have:

(4) $\mathbf{O}(\mathbf{O}p \prec p)$

and

(5) $\mathbf{O}(p \prec \mathbf{P}p)$,

which were discussed by Prior [8, pp. 225-6]. Secondly, and of more interest, we have what I shall call Fallacies of Deontic Assertion (named for their origin, and for their asserted connection between truth and obligation, between 'is' and 'ought'). From '$p \prec . p \prec V \prec V$' we infer '$p \prec . \sim \mathbf{P}p \prec V$', or:

(6) $p \prec \mathbf{OP}p$.

This may, at first sight, seem plausible: if p happens, then if it's not permitted there's trouble, or: if p happens, then it ought to be permitted. But (6) is equivalent to:

(7) $p \prec \mathbf{FF}p$

and hence to:

(8) $\mathbf{PF}p \prec \sim p$.

That is to say, if something, e.g. murder, is permitted to be forbidden, then it does not happen. That is, of course, nonsense.

Substituting '$\sim p$' for 'p' is (8) yields:

(9) $\mathbf{PO}p \prec p$,

from which, with '$\mathbf{OO}p \prec \mathbf{PO}p$', we infer by transitivity:

(9) A proof of this is available by means of a Gentzen formulation of R; see ANDERSON and BELNAP's book, *Entailment*, forthcoming.

(10) $\mathbf{OO}p \prec p$
and dually:
(11) $p \prec \mathbf{PP}p$.
We can here see that it is indeed fortunate that the converse of (1) was not provable, lest it with (10) yield the result: $\mathbf{O}p \prec p$, that 'ought' implies 'is'. Perhaps though (10) and (11) are bad enough.

Another proposition worth considering (it is not an iteration, but is still interesting) follows from the Law of Permutation (A.3 above). Substituting 'p' for 'A', '$\sim$q' for 'B' and 'V' for 'C', A.3 yields:
(12) $p \prec \mathbf{O}q \prec . \sim q \prec \mathbf{F}p$.
This, too, may seem plausible: e.g. if making a promise implies that we ought to keep it, then if it's not to be kept, we should not have made it. But (12) is equivalent to:
(13) $p \prec \mathbf{O}q \prec . \mathbf{P}p \prec q$,
by contraposition and definition. Reading '$p \prec \mathbf{O}q$' as 'p commits one to q', then (13) appears as: if p commits one to q, then if p is permitted, q happens; if making a promise, commits one to keep it, then if it is permitted to make the promise, then that promise must, necessarily, be kept. But that, too, is nonsense.

III

These really bad consequences of OR, (6) — (13), all follow from the two theses in R, Assertion and Permutation. But both of these are rejected in the system E of entailment developed by Anderson and Belnap [4, 5]. Hence, it will be fruitful to consider briefly the system OE, which is formed by the addition of:
D.6 $\mathbf{O}p =_{df} \sim p \rightarrow V$
(with '$\mathbf{P}p$' and '$\mathbf{F}p$' defined as before) and the deontic axiom:
$\sim p \rightarrow V \rightarrow . \sim(p \rightarrow V)$,
to E, as defined by the axiom schemata:
EA.1 $A \rightarrow A$
EA.2 $A \rightarrow B \rightarrow . B \rightarrow C \rightarrow . A \rightarrow C$
EA.3 $(A \rightarrow . A \rightarrow B) \rightarrow . A \rightarrow B$
EA.4 $A \rightarrow A \rightarrow B \rightarrow B$
EA.5 — EA.15 As for R (with the '$\prec$' transformed to the '$\rightarrow$')

EA.16 $NA \& NB \to N(A \& B)$ $[NA =_{df} A \to A \to A]$
and the rules:

ER.1 *Modus ponens* for the '$\to$': from A and $A \to B$, to infer B

ER.2 *Adjunction*: from A and B, to infer A & B.

'$A \to B$' is read 'A entails B' or 'A is a logically sufficient and relevant condition for B'.

Since Assertion and Permutation are not provable in E, we reject in OE the analogues of (4) — (13), and that is all to the good. However, since the system does contain Contraction (EA.3) and Restricted Assertion, OE does not avoid commitment to iterated deontic modalities; it contains the theorems:

(14) $\mathbf{OO}p \to \mathbf{O}p$

(15) $\mathbf{P}p \to \mathbf{PP}p$

(16) $\mathbf{F}p \to \mathbf{FF}\mathbf{F}p$.

Since nothing is provable in E which was not provable in R (under the obvious transformation), OE too distinguishes infinitely many deontic modalities.

Although OE resolves most of the worst results of OR, while preserving all proper deontic laws, it presents us with a new difficulty. E contains a theory of alethic modality, by the definition:

D.7 $NA =_{df} A \to A \to A$,

having the structure of S.4. One of its theorems is: $A \to B \to N(A \to B)$, from which:

(17) $\mathbf{O}p \to N\mathbf{O}p$

follows in OE. This is unfortunate, since we do not like to think our obligations are *logically* necessary.

IV

This result leads us to propose a new system, heretofore unconsidered, upon which to base our deontic logic. We call it E.M since it stands to the system M (of Feys and von Wright) as Anderson and Belnap's E stands to S.4 [10]. E.M is defined by adding to the axioms and rules of R:

[10] We submit that E. M is of interest not only as a basis for deontic logic, but also as a formulation of a theory of entailment.

A.16 $NA \mathbin{\&} NB \prec N(A \mathbin{\&} B)$ $[NA =_{df} A \to A \to A]$

A.17 $A \to B \prec . A \prec B$

A.18 $(A \prec B \to . C \prec D) \prec . A \to B \prec . C \to D$

and the rule:

R.3 *Necessitation*: if $\vdash A \prec B$, then $\vdash A \to B$.

The '$\to$' is taken as a primitive sign, in addition to the '$\prec$', and is used to represent entailment. (It is provable that '$A \to B$' is equivalent to '$N(A \prec B)$', taking D.7 over as the definition of necessity.) The pure entailment part of E.M differs from that of E by rejecting strict transitivity: $A \to B \to . B \to C \to . A \to C$, although it does contain the weaker thesis: $A \to B \to . B \to C \prec . A \to C$. All other axioms (and rules) of E are contained in E.M.

We construct the deontic system OE.M in the same manner as before, adding to E.M the definition D.6 (and the deontic axiom: $\sim p \to V \prec . \sim(p \to V)$). In this system the following desired results obtain: (A) All proper deontic laws are provable; (B) the theses (4) — (13) are rejected, since E.M like E rejects Assertion and Permutation for entailment. (C) Since E.M. is like M in rejecting the thesis: $A \to B \to N(A \to B)$, OE.M rejects (17), which led us to doubt OE. (D) In addition, OE.M rejects (16) above, since Restricted Assertion for entailment is not provable in the base system; thus we escape commitment to some iterations.

But lest it be thought that iterated deontic modalities are avoided altogether, we point out that since E.M. contains Contraction for entailment (EA.3), OE.M contains:

(14) $\mathbf{OOp} \to \mathbf{Op}$

and

(15) $\mathbf{Pp} \to \mathbf{PPp}$

as before. Again there are no reductions of these iterations, since no proposition constructed out of entailment and negation alone is provable in E.M. which was not provable in E.

These theorems, (14) and (15), will be with us so long as we work with definitions of the deontic operators such as given above and so long as the underlying systems are as strong as they have been. Two alternatives present themselves for the avoidance of iterations. We could reject the classical negation rules to disallow the elimination of double negations following contraposition; but to exchange the iteration of modalities for the irreducibility of negation seems a bad trade indeed. Alternatively, we could work with a system weakened in its implicative part such that it did not contain Contraction — such as E.M. without A.4. Such a system seems, in many respects, well worth exploring, but this is obviously not the place for it. We shall consider below another approach by which iterations can be avoided, an approach which is, moreover, more intuitively satisfying.

V

Since OE.M rejects all the really undesirable theses of OR and OE, while preserving all plausible consequences and properties of those systems, it seems the best suited system to serve as a reconstruction of our normative discourse. Nevertheless, we must now consider one line of attack which might be raised against OE.M (and OE as well) (11).

Suppose we define '**O**p' as in D.1 — if not-p, then V — and take 'if ... then ...' to be entailment in the sense of E.M (or E). We are then committed to the view that if p is obligatory, then V follows as a *logical consequence* of the failure of p. Suppose, for example, it is obligatory in chess not to open with P-KR6; then we should say that to open with this move logically entails V (a violation). But, it is argued, that is false; the move P-KR6 is consistent with the non-occurence of V, *for the rules of chess could have been otherwise*, such that P-KR6 was a permissible opener. Yet it is notorious that entailments, logical consequences, do not depend on the vagaries of authors of rule books, or even of moral codes. To put this another way, we should like to say that there is a contingent, even stipulative, element to our obligations, but we cannot stipulate entailments (as in the definitions).

However, we may accept the above points without discounting OE.M (or OE); what is needed is a modification of D.1. What seems to be contingent or stipulative in our obligations, is just that the rules are what they are. When we say it is forbidden in chess to open P-KR6, we mean that: *given this set of rules* (constituting chess), if you open P-KR6, then you've broken the rules. What is required

(11) This objection is due to ANDERSON [3].

in our formulation of obligation is just that the rules be specified. (Notice how in the above example we had to state, 'if it is forbidden *in chess* ...'; if the rules were otherwise, we should have a different game, in which case the set of obligations, permissions, and prohibitions would have to be re-evaluated.)

In light of these remarks, I now propose a new definition of the operator '**O**' (for which the other formulations might well be considered ellipses):

$$\text{D.8} \qquad \mathbf{O}p =_{df} \ R \rightarrow . \sim p \rightarrow V_R$$

(with '**P**p' and '**F**p' defined as before, in terms of '**O**p'). 'R' here is a new propositional constant to be interpreted as referring to a set of rules or principles. 'V_R' is the same as 'V' before, but the subscript points out its dependence on R. Again we leave the details of interpretation to substantive ethics. D.8 suits our intuitions well, and in that definition it makes perfectly good sense to interpret the '$\rightarrow$' as entailment, in the sense of E.M (or E). Thus, the above objection is obviated.

If we now consider the systems OR, OE and OE.M to be constructed by the addition to R, E and E.M respectively, of D.8 (under the obvious transformation for OR) rather than D.5 or D.6, we find that all desirable deontic theorems, such as von Wright's Laws, are still provable. '**F**V' is no longer a theorem of any system, which is an advance. In OR, however, the theorems (1) — (13) are preserved under the new definition; hence, we must reject it as a suitable deontic system. It is notable, however, that in OE and OE.M not only do (4) — (13) fail as before, but under D.8 the theses (14) — (16) are rejected as well. Consequently, in these systems there are no theorems committing us to the existence of iterated modalities. Nonetheless, OE preserves the theorem (17): **O**p $\rightarrow$ N**O**p, which is undesirable. This is, as pointed out, not a thesis of OE.M.

OE.M (modified by D.8) thus appears to be the only system free of the counter-intuitive results which infected the others, from (a) — (c) to (4) — (17), while preserving all plausible consequences. But the most outstanding feature of OE.M is that with regard to the iteration of deontic modalities, it leaves us free to accept or reject them as we like; it leaves the question of their existence to empirical investigation.

University of Pittsburgh L. F. GOBLE

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