A CLASSIFICATION OF CATEGORICAL PROPOSITIONS

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Syllogistical theory, which originally was a fundamental part of formal logic (1) is considered by modern logicians as a very special type in the theory of propositional functions, as exemplified by Russell and Whitehead's great work (2). In the 19th century, it was independently proposed by Bentham (3) and Hamilton (4) to extend the classical categorical propositions by quantification of the predicate. Venn (5) mentioned, among other possibilities, the 15 existential propositions on two properties, corresponding to 15 of the 16 of this paper. Analogous to Wittgenstein's method (6) of constructing all possible «molecular propositions», it will be shown here that the validity of syllogistic forms can be found by enumeration of the possible categorical propositions on the distribution of three properties in the universe of discourse. It is thus possible to avoid all the quite arbitrary «syllogistical rules» of classical theory, which amounted to little more than an explicit list of valid forms, if the premisses are treated extensionally (i.e. as asserting or denying the co-existence of properties in different classes, named «areas» in the universe of discourse). The present classification allows a translation of any other categorical proposition, e.g. a classical one, into a disjunction of one or more elementary propositions, which can be treated by a general method.

The principle of Wittgenstein can be stated:

If each of n elements is in one and only one of two states, there are 2^n different combinations possible (In polyvalent logics, the number 2 is substituted by 3, 4, etc).

The two states can, for instance, be «having a given property» or «not having a given property». Thus, n properties divide the universe of discourse into 2^n elementary classes. E.g. the two properties a and b ($n=2$) give the $2^2=4$ possible combinations ab, $a\bar{b}$, $\bar{a}b$, and $\overline{ab}$, where the lines denote negation. An elementary class can also be in two states, «empty» or «non-empty» in the universe of discourse. We will define an *elementary proposition on n properties* as one, where each of the 2^n elementary classes is asserted to be non-empty or empty. We will denote an elementary proposition of n properties by

$$\omega \; a_1 a_2 \ldots a_n$$

where ω is a characteristic *sign of quantity* and $\alpha_1, \alpha_2, \ldots \alpha_n$ are the considered properties. It is seen from Wittgenstein's principle that $2^{(2^n)}$ such elementary propositions are possible. In polyvalent logics, where $a^{(b^n)}$ elementary propositions are possible, a and b can independently assume different positive, integral values.

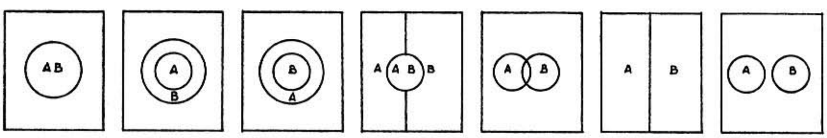
We will define a *composite proposition on n properties*

$$\omega_1 + \omega_2 + \ldots + \omega_m \alpha_1\alpha_2 \ldots \alpha_n$$

as an exclusive disjunction (either «$\omega_1 a_1 a_2 \ldots a_n$» or «$\omega_2 a_1 a_2 \ldots a_n$» or ... or «$\omega_m a_1 a_2 \ldots a_n$» and only one of these propositions). If $n=2$, there are $2^{(2^2)} = 16$ elementary propositions ωab possible. We will use the following signs of quantity, when the stated elementary classes are the only non-empty ones in the universe of discourse:

δ : $ab, \bar{a}\bar{b}$	λ : ab
ε : $ab, \bar{a}b, \bar{a}\bar{b}$	μ : $a\bar{b}$
ζ : $ab, a\bar{b}, \bar{a}\bar{b}$	ν : $\bar{a}b$
η : $ab, a\bar{b}, \bar{a}b$	ξ : $\bar{a}\bar{b}$
ϑ : $ab, a\bar{b}, \bar{a}b, \bar{a}\bar{b}$	o : $ab, a\bar{b}$
ι : $a\bar{b}, \bar{a}b$	π : $ab, \bar{a}b$
$\varkappa$: $a\bar{b}, \bar{a}b, \bar{a}\bar{b}$	ϱ : $a\bar{b}, \bar{a}\bar{b}$
	σ : $\bar{a}b, \bar{a}\bar{b}$
	υ : Every elementary class is empty.

The seven first in the left-hand row are denoted *heterogeneous* and the nine last in the right-hand row *homogeneous* propositions. Only in the first type, all four cases a, $\bar{a}$, b, and $\bar{b}$ are represented in the non-empty elementary classes. The Euler diagrams of the heterogeneous propositions were indicated by Keynes (7):



δ ε ζ η θ ι κ

These 16 elementary propositions on two properties present the remarkable feature that they can be converted biuniquely, $\equiv$ denoting equivalence:

$$\begin{array}{llll}
\delta & ab \equiv \delta \; ba & \qquad \lambda & ab \equiv \lambda \; ba \\
\varepsilon & ab \equiv \zeta \; ba & \qquad \mu & ab \equiv \nu \; ba \\
\zeta & ab \equiv \varepsilon \; ba & \qquad \nu & ab \equiv \mu \; ba \\
\eta & ab \equiv \eta \; ba & \qquad \xi & ab \equiv \xi \; ba \\
\vartheta & ab \equiv \vartheta \; ba & \qquad o & ab \equiv \pi \; ba \\
\iota & ab \equiv \iota \; ba & \qquad \pi & ab \equiv o \; ba \\
\varkappa & ab \equiv \varkappa \; ba & \qquad \varrho & ab \equiv \sigma \; ba \\
 & & \qquad \sigma & ab \equiv \varrho \; ba \\
 & & \qquad \upsilon & ab \equiv \upsilon \; ba
\end{array}$$

and all the other «immediate inferences», e.g. of the form $\omega_1 ab \equiv \omega_2 a\, b$, or $\omega_1 ab \equiv \omega_3 ab$, are also biunique. These operations transform the signs of quantity within six closed groups:

$$(\delta\iota)\ (\varepsilon\zeta\eta\pi)\ (\vartheta)\ (\lambda\mu\nu\xi)\ (o\varkappa\varrho\sigma)\ (\upsilon).$$

Four of the composite propositions on two properties are especially interesting and will be denoted by special signs of quantity:

$$\begin{array}{lll}
\alpha & ab \equiv & \delta + \varepsilon + \lambda + \pi \quad ab \\
\beta & ab \equiv & \zeta + \eta + \vartheta + o \quad ab \\
\gamma & ab \equiv & \iota + \varkappa + \mu + \varrho \quad ab \\
\tau & ab \equiv & \nu + \xi + \sigma + \upsilon \quad ab
\end{array}$$

In ordinary language, these composite propositions are:

$\alpha\ ab \equiv$ all A are B, and A exist.
$\beta\ ab \equiv$ some, but not all, A are B.
$\gamma\ ab \equiv$ no A are B, and A exist.
$\tau\ ab \equiv$ A do not exist.

It is seen that the classical a-proposition with existential implication is «$\alpha\, ab$», while the modern is «$\alpha + \tau\, ab$». The e-proposition is «$\gamma + \tau\, ab$», the i-proposition «$\alpha + \beta\, ab$», and the o-proposition «$\beta + \gamma\, ab$». All possible categorical propositions, e.g. with quantified predicates, can be expressed as elementary or composite propositions with the aid of this classification.

Three properties divide the universe of discourse into $2^3 = 8$ elementary classes in a divalent logic. These correspond to $2^8 = 256$ elementary propositions on 3 properties. Now, it is possible to interpret a proposition on the distribution of a, $\bar{a}$, m, $\bar{m}$, b, and $\bar{b}$ in the universe of discourse as a conjunction of three elementary propositions on two properties, $\omega_1 am$, $\omega_2 mb$, and $\omega_3 ab$. Table 1 shows this interpretation of the elementary propositions on three properties, numbered from 0 to 255, in terms of three signs of quantity from the δ, ε, ..., σ, υ group, arranged in the order $\omega_1\, \omega_2\, \omega_3$. This interpretation is not biunique, since a given set $\omega_1\, \omega_2\, \omega_3$ may be represented more than once in Table 1, e.g. $\vartheta\ \vartheta\ \vartheta$ is represented 35 times among the 256 propositions.

In discussion of the syllogistic forms with elementary propositions on two properties,

$$\begin{array}{c} \omega_1\, am \\ \omega_2\, mb \\ \hline \omega_3\, ab\,, \end{array}$$

it is not necessary to consider the four traditional figures of syllogisms independently, since the biunique conversion can translate them all to, e.g., the first figure studied here. For finding the valid forms, it is not necessary to arrange a deductive theory, but only to consider the possible propositions on three properties. We define «$\omega_3\, ab$» as a *possible conclusion* of «$\omega_1\, am$» and «$\omega_2\, mb$», if $\omega_1\, \omega_2\, \omega_3$ occurs at least once in the interpretation of the 256 propositions in Table 1, and we define the *total conclusion* as the composite proposition $\omega_3 + \omega'_3 + \omega''_3 + \ldots\, ab$ of all possible conclusions of $\omega_1\, am$ and $\omega_2\, mb$.

In this classification, two premises $\omega_1 am$ and $\omega_2 mb$ have no conclusion $\omega_3 ab$ only when they disagree with respect to the existence of m and $\bar{m}$. Thus, two heterogeneous premises have at least one possible conclusion. They can have more possible conclusions, e.g., the three cases ($\vartheta\, am$, $\vartheta\, mb$), ($\pi\, am$, $o\, mb$), and ($\varrho\, am$, $\sigma\, mb$) have all seven heterogeneous propositions as possible conclusions. But there never occurs a total conclusion, composite of all 16 elementary propositions, corresponding to the usual case of classical theory where «no conclusion is possible», i.e. all possibilities are compatible with the premises. The total conclusion of the heterogeneous premise-pairs are given in Table 2.

It is seen by comparison with Table 1 that the 49 total conclusions from Table 2 comprise 103 possible conclusions, corresponding to

193 of the 256 interpreted elementary propositions on three properties.

If syllogisms with composite premises are considered, e.g. with the classical categorical proposition, the total conclusions of the separate cases with elementary premises are united. For instance, the aai-syllogism in the fourth figure, «Bramantip»,

$$\begin{matrix} \alpha\, ma \\ \alpha\, bm \end{matrix} \equiv \begin{matrix} \delta + \varepsilon + \lambda + \pi\, ma \\ \delta + \varepsilon + \lambda + \pi\, bm \end{matrix}$$

can be biuniquely converted to the first figure:

$$\equiv \begin{matrix} \delta + \zeta + \lambda + o\, am \\ \delta + \zeta + \lambda + o\, mb \end{matrix}$$

The conclusions of the 16 premise-combinations are (where «-» denotes no conclusion possible):

δδδ	ζδζ	λδ-	οδο
δζζ	ζζζ	λζ-	οζο
δλ-	ζλ-	λλλ	ολ-
δσ-	ζο-	λοο	οο-

giving the total conclusion $\delta + \zeta + \lambda + o\, ab$. Since $\delta + \lambda$ are α-elements and $\zeta + o$ are β-elements, classical logic weakened the conclusion to $\alpha + \beta\, ab$, which might have contained 8 elementary propositions.

If the premises in the aai-syllogism are written $\alpha + \tau\, ma$ and $\alpha + \tau\, bm$, the total conclusion will be $\delta + \zeta + \lambda + \mu + \xi + o + \varrho + \upsilon ab$, which in the classical logic would be weakened to the tautology $\alpha + \beta + \gamma + \tau ab$.

Table 1 shows several features of binomial coefficient-structure. Thus, in the interpretations a given sign of quantity occurs in one of the three columns a certain number of times, as tabulated in Table 3.

Elementary propositions on more than 3 properties, or elementary propositions in polyvalent logics, show also a symmetrical distribution; but the mechanical difficulties increase immensely if the numerous propositions are treated individually. They might be an interesting subject for electronic computors in the future.

It is convenient to denote the elementary propositions on one property $\alpha\, a$, $\beta\, a$, $\gamma\, a$, and $\tau\, a$, respectively, when (a), $(a$ and $\bar{a})$, $(\bar{a})$, or no class is non-empty. It is worth remarking that the 16 propositions $\delta\, ab$, ... can be written unambigously as conjunctions (αab) $(\alpha \bar{a} b)$ (or (αab) $(\alpha \bar{a}\bar{b})$) etc.

Analogous to the possibility of translating different propositions into the scheme of the present classification, it is also possible to discuss the corresponding definitions of implication, which are closely connected with class-inclusion. The existential implication corresponds to $\alpha\, ab$, and the modern implication [2] to $\alpha + \tau\, ab$. (This is equivalent to defining the null-class as being included in every class). An intensional class-inclusion of classical origin maintains that, e.g., «All red-haired mermaids are red-haired» is in some formal way a tautological class-inclusion; while in our classification it has the form σab, when reality is treated as the universe of discourse. The corresponding «entailed implication» has been investigated by Lewis [8] and Moore [9]. This possibility can be treated in the present classification by defining an «entailed class-inclusion» of a in b, if all the defined properties of b also occur in the definition of a. From an extensional viewpoint, individuals are special one-member classes which do or do not have each given property; while the other classes have only some of all the possible properties and include those individuals which have the defining properties. Therefore, the general categorical propositions are symmetric with respect to subject and predicate, asserting only co-existence of properties. The singular propositions with characteristic individual-class relations do not occur in the classification. There is no inherent difference even between positive and negative properties for use in syllogisms, as demonstrated by the biunique translation $\omega_1 ab \equiv \omega_2 a\bar{b}$. These negative properties make the «entailed class-inclusion» quite confusing. Among other connections with propositional theory, we may in conclusion mention the seven possible relations between two propositions [10] which correspond in form to the heterogeneous propositions in two properties.

Appendix I. Epistemological Considerations.

It is often felt that the language necessary in the development of modern science is very far removed from that of formal logics, and that old speculations in the latter field now are quite obsolete. The present author believes that the formal logics provide us with a language we can put into quite different applications, which could not be foreseen a priori.

If we define things as special one-member classes, which definitely do have or do not have each given property, while the other classes lack definition on some properties (this does not, of course, prevent them from sometimes having infinitely many definite properties), it is a quite pertinent question whether we ever need to talk about things, or that we might restrict us to talk about (existing or non-existing) classes. It is clear that a thing according to the definition given above change completely to another thing (or class), if it changes at all. It is problematic, whether a thing thus defined would not be confined to a simple point of time (since its properties with regard to relation with other, changing things would change itself) and the present author proposes to call such things «Heraclitus things».

In practice, we need much more to use more or less extended classes for our propositions, and we may entirely forget the particular character of singular propositions, relating individuals to classes. If we assume that subject-predicate relations are completely convertible, as we say above for classes, it becomes a perhaps not meaningful, but at least very tempting question whether it is not a mistake to believe that things are «carriers» of properties, rather than properties hanging together without «internal» things.

The clue to the problem, what is the difference between things and classes, might be found in the following line of thought: our extensional description given in the first part applies to classes, all considering the same number of properties. But let us try to compare classes *abc*... wi'h a smaller number of definite properties than another type of classes *abc*...*AB*....

It is clearly seen that one of the second type of classes may belong to one of the former class, while the contrary would not be true. This is not quite the same as our sign of quantity ε (*abc*...*AB*...) (*abc*...), since we did not introduce the state existence E or non-existence $\overline{\text{E}}$ of the classes in our present example. In other words, *abc*...*AB*... $\overline{\text{E}}$ does not produce E in the class *abc*..., but *abc*...*AB*... is still formally (intensionally) included in *abc*... The characteristic asymmetry of singular propositions is based only on the presence of a larger number of definite properties *abc*...*AB*... in the subject «thing» than of definite properties *abc*...in the predicate «class», with all *abc*... being included in the larger number of properties *abc*...*AB*... . The comparison between classes having a different number of definite properties is a fairly complicated task; for instance, two classes may not show an inclusion, but a only partly overlap, such as $ab\bar{c}\bar{d}e$ and $ab\bar{c}fg$. Only the common parts $ab\bar{c}$ can be directly compared, and are identical in the two cases, as it would be in the case of the inclusion of $ab\bar{c}\bar{d}e$ in $ab\bar{c}$ or of $ab\bar{c}fg$ in $ab\bar{c}$. We do hardly have words in common language for denoting this type of complications. We may further on have completely irrelevant classes such as $\bar{a}b$ and $c\bar{d}$.

The foregoing analysis seems to show that the usual distinction between «things» and «classes» may reside only in a distinction between classes with numerous and with fewer definite properties. Hence, the entirely definite «Heraclitus things» become superfluous. There is one difference from the modern point of view, however: individuals are considered by modern authors ([2, 10]) as distinct from classes by the fact the state of existence E or non-existence $\bar{E}$ applies only to classes and not to individuals. This distinction would vanish, if our description of individuals is accepted.

There is a very sound background in much nominalistic thought that individuals are a somewhat more fundamental concept than the classes. However, the present author believes to have shown that an extreme point of view of nominalism, leading to definition of Heraclitus things, is about as difficult as the opposite extreme of realism, concentrating all the attention on classes as existing independently of their members (though these, in the author's point of view, are most conveniently described as classes with many more definite properties).

The present author does not suggest that one of the two points of view necessarily is the correct one, though the does not feel very convinced about the utility of introducing «modern» individuals, having the singular proposition type of relation to the single-member classes, the Heraclitus things, in the cases when the latter have the existence E. A quite interesting question is, whether the classes with a larger or smaller number of definite properties do not correspond effectively to deviations from divalent logic. It is often felt that properties may become meaningless, that a number is not not-red in the same sense as a green flask. The usual remedy is to define the class of red things as involving an observable colour; all things without an observable colour and those things having an observable colour, but not red, are both include in the class of not-red things. The discussion above might suggest that green bottles are $a\bar{b}$, red bottles ab, and numbers $\bar{a}$ simply (rather than $\bar{a}\bar{b}$), a being «having an observable colour», b being «red».

The critical philosophy has fought with great success against the idea that some propositions at the same time might be synthetic and necessarily valid. On the other hand, the adherents of synthetic a priori propositions have always found one of their last bastions in the syllogistic theory. We have probably all had, at some time of our occupation with formal logics, the feeling that the structure of valid conclusions in some way indicated a plan, according to which all possible forms of reality must obey. Though the present author to some extent make natural history of categorical propositions by inspection of Table 1 in the first part of this paper, he does no longer insist that he might discover necessary truths on the reality in this way. This is, however, much less due to a certain sceptical dogmatism, that such a necessary truth cannot be found, than because he is agnostic in a rather indifferent way with regard to such questions, as how the reality could be, if it was different. In our way of thinking, the reality could be more or less intelligible, and if it is intelligble to a certain extent, it seems as if it could not avoid to be described in accordance with formal logics. But the question whether we would possibly think in another way, if the reality was different, looks to the present author as void of interest as the sceptical epistemological ideas: he does not believe that a logical proof can be constructed against the philosopher who maintains that the world was created five minutes ago, with all the memories and souvenirs, or that it does not exist outside our mind, or that this philosopher himself does not exist. Our only reasonable response must be: allright, and then ? We can hardly make any difference between our attitude to reality, whether we are convinced (without logical proof) that there exist external things, or whether we doubt that this is the case. Some scepticists sometimes put barriers in the last moment, maintaining that a very few fundamental ideas, they will not doubt. The present author maintains that such limitations are only instructive on the psychology of the sceptical philosophers and not on the reality elsewhere. Returning to the question of formal logics supplying an useful language, there is no doubt that our theory of classes, either taken extensionally with a constant number of definite properties and inquiring into E or $\bar{E}$ of each class, or taken intensionally, with a varying number of definite properties and not always discussing existence, represents only the simpler and less sophisticated parts of the description possible. We have not at all discussed the properties of classes, as distinct from the properties of the members (e.g. the way in which the human beings are numerous, or the number of great planets in the Solar system nine) and we believe that Russell and Whitehead's theory of types is very adequate for this type of problems. However, the intensional inclusion presents rather intricate problems of properties of classes. While most class-inclusions are of the form all *A* are *B*, or no *A* are *B*, the proposition «Some, but not all, goblins are female» must be interpreted «the class of goblins is included in the class of beings, which (the class) has the property that either do both females and non-females exist, or neither of them exist». The present author believes that a finite catalogue is not possible to construct of all these examples, but that an infinite regression of complications can be found.

The characteristics of the natural sciences, from a logical point of view, is to find by observations that some properties always seem to occur together, even though no analytical relation subsists between these properties. Some parts of formal logics have been incorporated so thoroughly in the mind of the scientists that they often believe not to need further repetition of this trivial learning. However, sometimes, quite valuable suggestions and elucidations may come from the logicians. For instance, Jevons pointed out that in addition to the usual definitions, it is possible to make definitions from the common properties of a finite number of examples, e.g. that metals have the properties common to sodium, gold, iron and uranium. Very often, scientists have used this type of definition without making a clear reference to the theory of intensional classes, which elsewise is valuable in this case.

The idea of the «universe of discourse» as the field for a given set of propositions is a most agreable one to modern scientists, who do not have as clear-cut and short-sighted opinion of what are exactly the things, and who may concentrate on macroscopical or microscopical, continuous or quantized, descriptions of the matter. For them, the suggestion of individuals being classes with a larger number of definite properties than the usual classes formed from these individuals may look much more evident than for logicians a century ago.

Appendix II. Some remarks about elementary propositions on four and more properties in divalent logic.

Without doing the detailed interpretation of all the $2^{16} = 65536$ elementary propositions on four properties along the lines laid out in Part I, there can easily be recognized a close analogy to the symmetrical properties of the 256 elementary propositions on three properties.

Each elementary proposition on four properties can be interpreted as four different propositions on three properties or as six different propositions on two properties. In the former case, the four signs of quantity will be one of the numbers O to 255 from Table 1, whereas in the latter case, the six signs of quantity will be one of the series δ, ε, ..., σ, υ.

It seems that each of the four rows of sign of quantity of the first type is distributed in the following way:

Number of classes quoted:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Number of propositions of four properties:	1	16	120	560	1820	4368	8808	11440	12870	11440	8808	4368	1820	560	120	16	1
no. 0	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
no. 1-8 each	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—
no. 9-36 each	—	—	4	4	1	—	—	—	—	—	—	—	—	—	—	—	—
no. 37-92 each	—	—	—	8	12	6	1	—	—	—	—	—	—	—	—	—	—
no. 93-162 each	—	—	—	—	16	32	24	8	1	—	—	—	—	—	—	—	—
no. 163-218 each	—	—	—	—	—	32	80	80	40	10	1	—	—	—	—	—	—
no. 219-246 each	—	—	—	—	—	—	64	192	240	160	60	12	1	—	—	—	—
no. 247-254 each	—	—	—	—	—	—	—	128	448	672	560	280	84	14	1	—	—
no. 255	—	—	—	—	—	—	—	—	256	1024	1792	1792	1120	448	112	16	1

Thus, the total number of interpretations in each horizontal row is 3^n, where n is the number of non-empty classes in the propositions in the left-hand column.

In the way, each of the six rows of signs of quantity in propositions on two properties should be distributed:

$m = 0$ υ 1 time.

1 $\lambda + \mu + \nu + \xi$ each $9+2.3 = 15$ times.

2 $\delta + \iota + o + \pi + \varrho + \sigma$ each $81 + 27.4 + 9.4 =$ 225 times.

3 $\varepsilon + \zeta + \eta + \varkappa$ each $729+243.6+81.12+27.8 =$ 3375 times.

4 ϑ $6561+2187.8+729.24+243.32+81.16 = 50625$ times,

in other words 15^m times.

In analogy, one would expect that the 2^{32} elementary propositions on five properties would show the distribution according to propositions on four properties:

$$3^{16} + 16.3^{15} + 120.3^{14} + 560.3^{13} + 1820.3^{12} + \ldots = 4^{16} = 2^{32},$$

according to propositions on three properties:

$$15^8 + 8.15^7 + 28.15^6 + 56.15^5 + 70.15^4 + 56.15^3 + 28.15^2 + 7.15^2 + 1 = 16^8 = 2^{32}$$

and according to propositions on two properties:

$$255^4 + 4.255^3 + 6.255^2 + 4.255 + 1 = 256^4 = 2^{32}.$$

Continuing this binomial expression for the 2^{64} elementary propositions on six properties, it is expected that the distribution according to propositions on five properties would be:

$$3^{32} + 32.3^{31} + \frac{32.31}{2}\, 3^{30} + \frac{32.31.30}{2.3}.\ 3^{29} + \ldots = 4^{32} = 2^{64}$$

according to four properties

$$15^{16} + 16.15^{15} + 120.15^{14} + 560.15^{13} + 1820.15^{12} + \ldots = 16^{16} = 2^{64}$$

according to three properties

$$255^8 + 8.255^7 + 28.255^6 + 56.255^5 + 70.255^4 + 56.255^3 + 28.255^2 + 7.255 + 1 = 256^8 = 2^{64}$$

and finally, according to two properties

$$65535^4 + 4.65535^3 + 6.65535^2 + 4.65535 + 1 = 65536^4 = 2^{64}.$$

SUMMARY

The categorical propositions are classified from an existential-extensional point of view, and the validity of syllogistic forms shown by enumeration of the possible cases of co-existence of three properties, analogous to the construction of Wittgenstein's tables of the possible compound propositions.

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o	υ υ υ			
1	+ + +	λ λ λ		
2	+ + −	λ μ μ		
3	+ − +	μ ν λ		
4	+ − −	μ ξ μ		
5	− + +	ν λ ν		
6	− + −	ν μ ξ		
7	− − +	ξ ν ν		
8	− − −	ξ ξ ξ		
9	+ + +	+ + −	λ o o	
10	+ + +	+ − +	o π λ	
11	+ + +	+ − −	o δ o	
12	+ + +	− + +	π λ π	
13	+ + +	− + −	π o δ	
14	+ + +	− − +	δ π π	
15	+ + +	− − −	δ δ δ	
16	+ + −	+ − +	o ι o	
17	+ + −	+ − −	o ρ μ	
18	+ + −	− + +	π o ι	
19	+ + −	− + −	π μ ρ	
20	+ + −	− − +	δ ι ι	
21	+ + −	− − −	δ ρ ρ	
22	+ − +	+ − −	μ σ o	
23	+ − +	− + +	ι π π	
24	+ − +	− + −	ι ι δ	
25	+ − +	− − +	ρ ν π	
26	+ − +	− − −	ρ σ δ	
27	+ − −	− + +	ι δ ι	
28	+ − −	− + −	ι ρ ρ	
29	+ − −	− − +	ρ σ ι	
30	+ − −	− − −	ρ ξ ρ	
31	− + +	− + −	ν o σ	
32	− + +	− − +	σ π ν	
33	− + +	− − −	σ δ σ	
34	− + −	− − +	σ ι σ	
35	− + −	− − −	σ ρ ξ	
36	− − +	− − −	ξ σ σ	
37	+ + +	+ + −	+ − +	o η o
38	+ + +	+ + −	+ − −	o ζ o
39	+ + +	+ + −	− + +	π o η
40	+ + +	+ + −	− + −	π o ζ

Table 1.

Elementary propositions of 3 properties interpreted as conjunction of three propositions of 2 properties in the order

$\omega_1 am$, $\omega_2 mb$, and $\omega_3 ab$.

The stated elementary classes are in each case the only non-empty in the universe of discourse. For typographical reasons, they are denoted:

amb	+ + +
$am\bar{b}$	+ + −
$a\overline{m}b$	+ − +
$a\overline{mb}$	+ − −
$\bar{a}mb$	− + +
$\bar{a}m\bar{b}$	− + −
$\overline{am}b$	− − +
$\overline{amb}$	− − −

41	+ + +	+ + —	— — +	δ	η	η
42	+ + +	+ + —	— — —	δ	ζ	ζ
43	+ + +	+ — +	+ — —	ο	ε	ο
44	+ + +	+ — +	— + +	η	π	π
45	+ + +	+ — +	— + —	η	η	δ
46	+ + +	+ — +	— — +	ζ	π	π
47	+ + +	+ — +	— — —	ζ	ε	δ
48	+ + +	+ — —	— + +	η	δ	η
49	+ + +	+ — —	— + —	η	ζ	ζ
50	+ + +	+ — —	— — +	ζ	ε	η
51	+ + +	+ — —	— — —	ζ	δ	ζ
52	+ + +	— + +	— + —	π	ο	ε
53	+ + +	— + +	— — +	ε	π	π
54	+ + +	— + +	— — —	ε	δ	ε
55	+ + +	— + —	— — +	ε	η	ε
56	+ + +	— + —	— — —	ε	ζ	δ
57	+ + +	— — +	— — —	δ	ε	ε
58	+ + —	+ — +	+ — —	ο	κ	ο
59	+ + —	+ — +	— + +	η	η	η
60	+ + —	+ — +	— + —	η	ι	ζ
61	+ + —	+ — +	— — +	ζ	ι	η
62	+ + —	+ — +	— — —	ζ	κ	ζ
63	+ + —	+ — —	— + +	η	ζ	ι
64	+ + —	+ — —	— + —	η	ρ	ρ
65	+ + —	+ — —	— — +	ζ	κ	ι
66	+ + —	+ — —	— — —	ζ	ρ	ρ
67	+ + —	— + +	— + —	π	ο	κ
68	+ + —	— + +	— — +	ε	η	ι
69	+ + —	— + +	— — —	ε	ζ	κ
70	+ + —	— + —	— — +	ε	ι	κ
71	+ + —	— + —	— — —	ε	ρ	ρ
72	+ + —	— — +	— — —	δ	κ	κ
73	+ — +	+ — —	— + +	ι	ε	η
74	+ — +	+ — —	— + —	ι	κ	ζ
75	+ — +	+ — —	— — +	ρ	σ	η
76	+ — +	+ — —	— — —	ρ	σ	ζ
77	+ — +	— + +	— + —	ι	η	ε
78	+ — +	— + +	— — +	κ	π	π
79	+ — +	— + +	— — —	κ	ε	ε
80	+ — +	— + —	— — +	κ	ι	ε
81	+ — +	— + —	— — —	κ	κ	δ

82	+ − +	− − +	− − −		ρ σ ε
83	+ − −	− + +	− + −		ι ζ κ
84	+ − −	− + +	− − +		κ ε ι
85	+ − −	− + +	− − −		κ δ κ
86	+ − −	− + −	− − +		κ κ κ
87	+ − −	− + −	− − −		κ ρ ρ
88	+ − −	− − +	− − −		ρ σ κ
89	− + +	− + −	− − +		σ η σ
90	− + +	− + −	− − −		σ ζ σ
91	− + +	− − +	− − −		σ ε σ
92	− + −	− − +	− − −		σ κ σ
93	+ + +	+ + −	+ − +	+ − −	ο θ ο
94	+ + +	+ + −	+ − +	− + +	η η η
95	+ + +	+ + −	+ − +	− + −	η η ζ
96	+ + +	+ + −	+ − +	− − +	ζ η η
97	+ + +	+ + −	+ − +	− − −	ζ θ ζ
98	+ + +	+ + −	+ − −	− + +	η ζ η
99	+ + +	+ + −	+ − −	− + −	η ζ ζ
100	+ + +	+ + −	+ − −	− − +	ζ θ η
101	+ + +	+ + −	+ − −	− − −	ζ ζ ζ
102	+ + +	+ + −	− + +	− + −	π ο θ
103	+ + +	+ + −	− + +	− − +	ε η η
104	+ + +	+ + −	− + +	− − −	ε ζ θ
105	+ + +	+ + −	− + −	− − +	ε η θ
106	+ + +	+ + −	− + −	− − −	ε ζ ζ
107	+ + +	+ + −	− − +	− − −	δ θ θ
108	+ + +	+ − +	+ − −	− + +	η ε η
109	+ + +	+ − +	+ − −	− + −	η θ ζ
110	+ + +	+ − +	+ − −	− − +	ζ ε η
111	+ + +	+ − +	+ − −	− − −	ζ ε ζ
112	+ + +	+ − +	− + +	− + −	η η ε
113	+ + +	+ − +	− + +	− − +	θ π π
114	+ + +	+ − +	− + +	− − −	θ ε ε
115	+ + +	+ − +	− + −	− − +	θ η ε
116	+ + +	+ − +	− + −	− − −	θ θ δ
117	+ + +	+ − +	− − +	− − −	ζ ε ε
118	+ + +	+ − −	− + +	− + −	η ζ θ
119	+ + +	+ − −	− + +	− − +	θ ε η
120	+ + +	+ − −	− + +	− − −	θ δ θ
121	+ + +	+ − −	− + −	− − +	θ θ θ
122	+ + +	+ − −	− + −	− − −	θ ζ ζ

123	+ + +	+ — —	— — +	— — —	ζ ε θ
124	+ + +	— + +	— + —	— — +	ε η ε
125	+ + +	— + +	— + —	— — —	ε ζ ε
126	+ + +	— + —	— — +	— — —	ε ε ε
127	+ + +	— + —	— — +	— — —	ε θ ε
128	+ + —	+ — +	+ — —	— + +	η θ η
129	+ + —	+ — +	+ — —	— + —	η κ ζ
130	+ + —	+ — +	+ — —	— — +	ζ κ η
131	+ + —	+ — +	+ — —	— — —	ζ κ ζ
132	+ + —	+ — +	— + +	— + —	η η θ
133	+ + —	+ — +	— + +	— — +	θ η η
134	+ + —	+ — +	— + +	— — —	θ θ θ
135	+ + —	+ — +	— + —	— — +	θ ι θ
136	+ + —	+ — +	— + —	— — —	θ κ ζ
137	+ + —	+ — +	— — +	— — —	ζ κ θ
138	+ + —	+ — —	— + +	— + —	η ζ κ
139	+ + —	+ — —	— + +	— — +	θ θ ι
140	+ + —	+ — —	— + +	— — —	θ ζ κ
141	+ + —	+ — —	— + —	— — +	θ κ κ
142	+ + —	+ — —	— + —	— — —	θ ρ ρ
143	+ + —	+ — —	— — +	— — —	ζ κ κ
144	+ + —	— + +	— + —	— — +	ε η κ
145	+ + —	— + +	— + —	— — —	ε ζ κ
146	+ + —	— + +	— — +	— — —	ε θ κ
147	+ + —	— + —	— — +	— — —	ε κ κ
148	+ — +	+ — —	— + +	— + —	ι θ θ
149	+ — +	+ — —	— + +	— — +	κ ε η
150	+ — +	+ — —	— + +	— — —	κ ε θ
151	+ — +	+ — —	— + —	— — +	κ κ θ
152	+ — +	+ — —	— + —	— — —	κ κ ζ
153	+ — +	+ — —	— — +	— — —	ρ σ θ
154	+ — +	— + +	— + —	— — +	κ η ε
155	+ — +	— + +	— + —	— — —	κ θ ε
156	+ — +	— + +	— — +	— — —	κ ε ε
157	+ — +	— + —	— — +	— — —	κ κ ε
158	+ — —	— + +	— + —	— — +	κ θ κ
159	+ — —	— + +	— + —	— — —	κ ζ κ
160	+ — —	— + +	— — +	— — —	κ ε κ
161	+ — —	— + —	— — +	— — —	κ κ κ
162	— + +	— + —	— — +	— — —	σ θ σ

163	+ + +	+ + −	+ − +	+ − −	− + +	η	θ	η
164	+ + +	+ + −	+ − +	+ − −	− + −	η	θ	ζ
165	+ + +	+ + −	+ − +	+ − −	− − +	ζ	θ	η
166	+ + +	+ + −	+ − +	+ − −	− − −	ζ	θ	ζ
167	+ + +	+ + −	+ − +	− + +	− + −	η	η	θ
168	+ + +	+ + −	+ − +	− + +	− − +	θ	η	η
169	+ + +	+ + −	+ − +	− + +	− − −	θ	θ	θ
170	+ + +	+ + −	+ − +	− + −	− − +	θ	η	θ
171	+ + +	+ + −	+ − +	− + −	− − −	θ	θ	ζ
172	+ + +	+ + −	+ − +	− − +	− − −	ζ	θ	θ
173	+ + +	+ + −	+ − −	− + +	− + −	η	ζ	θ
174	+ + +	+ + −	+ − −	− + +	− − +	θ	θ	η
175	+ + +	+ + −	+ − −	− + +	− − −	θ	ζ	θ
176	+ + +	+ + −	+ − −	− + −	− − +	θ	θ	θ
177	+ + +	+ + −	+ − −	− + −	− − −	θ	ζ	ζ
178	+ + +	+ + −	+ − −	− − +	− − −	ζ	θ	θ
179	+ + +	+ + −	− + +	− + −	− − +	ε	η	θ
180	+ + +	+ + −	− + +	− + −	− − −	ε	ζ	θ
181	+ + +	+ + −	− + +	− − +	− − −	ε	θ	θ
182	+ + +	+ + −	− + −	− − +	− − −	ε	θ	θ
183	+ + +	+ − +	+ − −	− + +	− + −	η	θ	θ
184	+ + +	+ − +	+ − −	− + +	− − +	θ	ε	η
185	+ + +	+ − +	+ − −	− + +	− − −	θ	ε	θ
186	+ + +	+ − +	+ − −	− + −	− − +	θ	θ	θ
187	+ + +	+ − +	+ − −	− + −	− − −	θ	θ	ζ
188	+ + +	+ − +	+ − −	− − +	− − −	ζ	ε	θ
189	+ + +	+ − +	− + +	− + −	− − +	θ	η	ε
190	+ + +	+ − +	− + +	− + −	− − −	θ	θ	ε
191	+ + +	+ − +	− + +	− − +	− − −	θ	ε	ε
192	+ + +	+ − +	− + −	− − +	− − −	θ	θ	ε
193	+ + +	+ − −	− + +	− + −	− − +	θ	θ	θ
194	+ + +	+ − −	− + +	− + −	− − −	θ	ζ	θ
195	+ + +	+ − −	− + +	− − +	− − −	θ	ε	θ
196	+ + +	+ − −	− + −	− − +	− − −	θ	θ	θ
197	+ + +	− + +	− + −	− − +	− − −	ε	θ	ε
198	+ + −	+ − +	+ − −	− + +	− + −	η	θ	θ
199	+ + −	+ − +	+ − −	− + +	− − +	θ	θ	η
200	+ + −	+ − +	+ − −	− + +	− − −	θ	θ	θ
201	+ + −	+ − +	+ − −	− + −	− − +	θ	κ	θ
202	+ + −	+ − +	+ − −	− + −	− − −	θ	κ	ζ
203	+ + −	+ − +	+ − −	− − +	− − −	ζ	κ	θ

204	++−	+−+	−++	−+−	−−+		θ	η	θ
205	++−	+−+	−++	−+−	−−−		θ	θ	θ
206	++−	+−+	−++	−−+	−−−		θ	θ	θ
207	++−	+−+	−+−	−−+	−−−		θ	κ	θ
208	++−	+−−	−++	−+−	−−+		θ	θ	κ
209	++−	+−−	−++	−+−	−−−		θ	ζ	κ
210	++−	+−−	−++	−−+	−−−		θ	θ	κ
211	++−	+−−	−+−	−−+	−−−		θ	κ	κ
212	++−	−++	−+−	−−+	−−−		ε	θ	κ
213	+−+	+−−	−++	−+−	−−+		κ	θ	θ
214	+−+	+−−	−++	−+−	−−−		κ	θ	θ
215	+−+	+−−	−++	−−+	−−−		κ	ε	θ
216	+−+	+−−	−+−	−−+	−−−		κ	κ	θ
217	+−+	−++	−+−	−−+	−−−		κ	θ	ε
218	+−−	−++	−+−	−−+	−−−		κ	θ	κ
219	+++	++−	+−+	+−−	−++	−+−	η	θ	θ
220	+++	++−	+−+	+−−	−++	−−+	θ	θ	η
221	+++	++−	+−+	+−−	−++	−−−	θ	θ	θ
222	+++	++−	+−+	+−−	−+−	−−+	θ	θ	θ
223	+++	++−	+−+	+−−	−+−	−−−	θ	θ	ζ
224	+++	++−	+−+	+−−	−−+	−−−	ζ	θ	θ
225	+++	++−	+−+	−++	−+−	−−+	θ	η	θ
226	+++	++−	+−+	−++	−+−	−−−	θ	θ	θ
227	+++	++−	+−+	−++	−−+	−−−	θ	θ	θ
228	+++	++−	+−+	−+−	−−+	−−−	θ	θ	θ
229	+++	++−	+−−	−++	−+−	−−+	θ	θ	θ
230	+++	++−	+−−	−++	−+−	−−−	θ	ζ	θ
231	+++	++−	+−−	−++	−−+	−−−	θ	θ	θ
232	+++	++−	+−−	−+−	−−+	−−−	θ	θ	θ
233	+++	++−	−++	−+−	−−+	−−−	ε	θ	θ
234	+++	+−+	+−−	−++	−+−	−−+	θ	θ	θ
235	+++	+−+	+−−	−++	−+−	−−−	θ	θ	θ
236	+++	+−+	+−−	−++	−−+	−−−	θ	ε	θ
237	+++	+−+	+−−	−+−	−−+	−−−	θ	θ	θ
238	+++	+−+	−++	−+−	−−+	−−−	θ	θ	ε
239	+++	+−−	−++	−+−	−−+	−−−	θ	θ	θ
240	++−	+−+	+−−	−++	−+−	−−+	θ	θ	θ
241	++−	+−+	+−−	−++	−+−	−−−	θ	θ	θ
242	++−	+−+	+−−	−++	−−+	−−−	θ	θ	θ
243	++−	+−+	+−−	−+−	−−+	−−−	θ	κ	θ
244	++−	+−+	−++	−+−	−−+	−−−	θ	θ	θ

245	+ + −	+ − −	− + +	− + −	− − +	− − −	θ θ κ		
246	+ − +	+ − −	− + +	− + −	− − +	− − −	κ θ θ		
247	+ + +	+ + −	+ − +	+ − −	− + +	− + −	− − +	θ θ θ	
248	+ + +	+ + −	+ − +	+ − −	− + +	− + −	− − −	θ θ θ	
249	+ + +	+ + −	+ − +	+ − −	− + +	− − +	− − −	θ θ θ	
250	+ + +	+ + −	+ − +	+ − −	− + −	− − +	− − −	θ θ θ	
251	+ + +	+ + −	+ − +	− + +	− + −	− − +	− − −	θ θ θ	
252	+ + +	+ + −	+ − −	− + +	− + −	− − +	− − −	θ θ θ	
253	+ + +	+ − +	+ − −	− + +	− + −	− − +	− − −	θ θ θ	
254	+ + −	+ − +	+ − −	− + +	− + −	− − +	− − −	θ θ θ	
255	+ + +	+ + −	+ − +	+ − −	− + +	− + −	− − +	− − −	θ θ θ

Table 2

$\omega_1 \backslash \omega_2$	δ	ε	ζ	η	ϑ	ι	$\varkappa$
δ	δ	ε	ζ	η	ϑ	ι	$\varkappa$
ε	ε	ε	$\delta+\varepsilon+\zeta+\vartheta+\varkappa$	$\varepsilon+\eta+\vartheta+\iota+\varkappa$	$\varepsilon+\vartheta+\varkappa$	$\varkappa$	$\varkappa$
ζ	ζ	$\delta+\varepsilon+\zeta+\eta+\vartheta$	ζ	η	$\zeta+\eta+\vartheta$	η	$\zeta+\eta+\vartheta+\iota+\varkappa$
η	η	η	$\zeta+\eta+\vartheta+\iota+\varkappa$	$\delta+\varepsilon+\zeta+\eta+\vartheta$	$\zeta+\eta+\vartheta$	ζ	ζ
ϑ	ϑ	$\varepsilon+\eta+\vartheta$	$\zeta+\vartheta+\varkappa$	$\varepsilon+\eta+\vartheta$	$\delta+\varepsilon+\zeta+\eta+\vartheta+\iota+\varkappa$	ϑ	$\zeta+\vartheta+\varkappa$
ι	ι	η	$\varkappa$	ε	ϑ	δ	ζ
$\varkappa$	$\varkappa$	$\varepsilon+\eta+\vartheta+\iota+\varkappa$	$\varkappa$	ε	$\varepsilon+\vartheta+\varkappa$	ε	$\delta+\varepsilon+\zeta+\vartheta+\varkappa$

Table 3.

Proposition No	υ	$\lambda+\mu+\nu+\xi$ each	$\delta+\iota+o+\pi+\rho+\sigma$ each	$\varepsilon+\zeta+\eta+\kappa$ each	θ
0	1	—	—	—	—
1 — 8	—	2	—	—	—
9 — 36	—	1	4	—	—
37 — 92	—	—	4	8	—
93 — 162	—	—	1	12	16
163 — 218	—	—	—	6	32
219 — 246	—	—	—	1	24
247 — 254	—	—	—	—	8
255	—	—	—	—	1
Total Sum	1	3	9	27	81

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